The IUM report to the Simons foundation, 2013

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1 Introduction: list of awardees

The Simons foundation supported two programs launched by the IUM:
Simons stipends for students and graduate students;
Simons IUM fellowships.

11 applications were received for the Simons stipends contest. The selection committee consisting of *Yu.Ilyashenko (Chair), G.Dobrushina, G.Kabatyanski, S.Lando, I.Paramonova (Academic Secretary), A.Sossinsky, M.Tsfasman* awarded Simons stipends for 2013 year to the following students and graduate students:

1. Bufetov, Alexei Igorevich
2. Goncharuk, Nataliya Borisovna
3. Karpukhin, Mikhail Alexandrovich
4. Kubrak, Dmitry Vadimovich
5. Kurnosov, Nikon Mikhailovich
6. Naumov, Alexei Alexandrovich
7. Perepechko, Alexander Yurevich
8. Kharitonov, Mikhail Irogevich.

14 applications were received for the Simons IUM fellowships contest for the first half year of 2013 and 19 applications were received for the second half year. The selection committee consisting of *Yu.Ilyashenko (Chair), G.Dobrushina, B.Feigin, I.Paramonova (Academic Secretary), A.Sossinsky, M.Tsfasman, V.Vassiliev* awarded

Simons IUM-fellowships for the first half year of 2013 to the following researches:

1. Arzhantsev, Ivan Vladimirovich
2. Kuznetsov, Alexander Gennadevich
3. Kuyumzhiyan, Karine Georgievna
4. Olshanski, Grigori Iosifovich
5. Penskoi, Alexei Victorovich
6. Prokhorov, Yuri Gennadevich

7. Pushkar, Petr Evgenevich
8. Rybakov, Sergei Yurevich
9. Skopenkov, Arkady Borisovich
10. Skopenkov, Mikhail Borisovich
11. Vyugin, Ilya Vladimirovich
12. Zykin, Alexei Ivanovich

Simons IUM-fellowships for the second half year of 2013 to the following researches:

1. Bogachev Vladimir Igorevich
2. Burman Yurii Mikhailovich
3. Elagin Alexei Victorovich
4. Gorodentsev Alexei Lvovich
5. Kazarian Maxim Eduardovich
6. Kuznetsov, Alexander Gennadevich
7. Olshanski, Grigori Iosifovich
8. Penskoi, Alexei Victorovich
9. Pushkar, Petr Evgenevich
10. Skopenkov, Mikhail Borisovich
11. Smirnov Evgeny Yurevich
12. Shaposhnikov Stanislav Valerevich
13. Verbitsky Mikhail Sergeevich

The report below is split in two sections corresponding to the two programs above. The first subsection in each section is a report on the research activities. It consists of the titles of the papers published or submitted in the year of 2013, together with the corresponding abstracts. The second subsection of each section is devoted to conference and some most important seminar talks. The last subsection of the second section is devoted to the syllabi of the courses given by the winners of the Simons IUM fellowships. Most of these courses are innovative, as required by the rules of the contest for the Simons IUM fellowships.

The support of the Simons foundation have drastically improved the financial situation at the IUM, and the whole atmosphere as well. On behalf of the IUM, I send my deep gratitude and the best New year wishes to Jim Simons, David Eisenbud, with whom the program was started, Yuri Tschinkel, with whom the program is run, and the whole team of the Simons foundation.

Yulij Ilyashenko

President of the Independent University of Moscow

2 Program: Simons stipends for students and graduate students

2.1 Research

2.1.1 Alexei Bufetov

[1] A. Bufetov, *Kerov's interlacing sequences and random matrices*, Journal of Mathematical Physics, 54 113302 (2013) ; http://dx.doi.org/10.1063/1.4830024. `arXiv:1211.1507`

To a $N \times N$ real symmetric matrix Kerov assigns a piecewise linear function whose local minima are the eigenvalues of this matrix and whose local maxima are the eigenvalues of its $(N-1) \times (N-1)$ submatrix. We study the scaling limit of Kerov's piecewise linear functions for Wigner and Wishart matrices. For Wigner matrices the scaling limit is given by the Verhik-Kerov-Logan-Shepp curve which is known from asymptotic representation theory. For Wishart matrices the scaling limit is also explicitly found, and we explain its relation to the Marchenko-Pastur limit spectral law.

[2] With A. Borodin
Plancherel representations of $U(\infty)$ and correlated Gaussian Free Fields
`arXiv:1301.0511`, *submitted*

We study asymptotics of traces of (noncommutative) monomials formed by images of certain elements of the universal enveloping algebra of the infinite-dimensional unitary group in its Plancherel representations. We prove that they converge to (commutative) moments of a Gaussian process that can be viewed as a collection of simply yet nontrivially correlated two-dimensional Gaussian Free Fields. The limiting process has previously arisen via the global scaling limit of spectra for submatrices of Wigner Hermitian random matrices.

[3] With A. Borodin and G. Olshanski

Limit shapes for growing extreme characters of $U(\infty)$
`arXiv:1311.5697`, *submitted*

We prove the existence of a limit shape and give its explicit description for certain probability distribution on signatures (or highest weights for unitary groups). The distributions have representation theoretic origin — they encode decomposition on irreducible characters of the restrictions of certain extreme characters of the infinite-dimensional unitary group $U(\infty)$ to growing finite-dimensional unitary subgroups $U(N)$. The characters of $U(\infty)$ are allowed to depend on N. In a special case, this describes the hydrodynamic behavior for a family of random growth models in (2+1)-dimensions with varied initial conditions.

[4] With V. Gorin
Representations of classical Lie groups and quantized free convolution
`arXiv:1311.5780`, *submitted*

We study the decompositions into irreducible components of tensor products and restrictions of irreducible representations of classical Lie groups as the rank of the group goes to infinity. We prove the Law of Large Numbers for the random counting measures describing the decomposition. This leads to two operations on measures which are deformations of the notions of the free convolution and the free projection. We further prove that if one replaces counting measures with others coming from the work of Perelomov and Popov on the higher order Casimir operators for classical groups, then the operations on the measures turn into the free convolution and projection themselves. We also explain the relation between our results and limit shape theorems for uniformly random lozenge tilings with and without axial symmetry.

2.1.2 Natanliya Goncharuk

[1] With X.Buff. Complex rotation numbers
arXiv::1308.3510 *submitted to Journal of Modern Dynamics*

We investigate the notion of complex rotation number which was introduced by V.I.Arnold in 1978. Let $f\colon \mathbb{R}/\mathbb{Z} \to \mathbb{R}/\mathbb{Z}$ be an orientation preserving circle diffeomorphism and let $\omega \in \mathbb{C}/\mathbb{Z}$ be a parameter with positive imaginary part. Construct a complex torus by glueing the two boundary components of the annulus $\{z \in \mathbb{C}/\mathbb{Z} \mid 0 < Im(z) < Im(\omega)\}$ via the map $f+\omega$. This complex torus is isomorphic to $\mathbb{C}/(\mathbb{Z}+\tau Z)$ for some appropriate $\tau \in \mathbb{C}/\mathbb{Z}$.

According to Moldavskis (2001), if the ordinary rotation number $rot(f+\omega_0)$ is Diophantine and if ω tends to ω_0 non tangentially to the real axis, then τ tends to $rot(f+\omega_0)$. We show that the Diophantine and non tangential assumptions are unnecessary: if $rot(f+\omega_0)$ is irrational then τ tends to $rot(f+\omega_0)$ as ω tends to ω_0.

This, together with results of N.Goncharuk (2012), motivates us to introduce a new fractal set, given by the limit values of τ as ω tends to the real axis. For the rational values of $rot(f + \omega_0)$, these limits do not necessarily coincide with $rot(f + \omega_0)$ and form a countable number of analytic loops in the upper half-plane.

2.1.3 Mikhail Karpukhin

[1] With G. Kokarev and I. Polterovich
Multiplicity bounds for Steklov eigenvalues on Riemannian surfaces
arXiv:1209.4869, *to appear in Annales de l'Institute Fourier.*

We prove two explicit bounds for the multiplicities of Steklov eigenvalues σ_k on compact surfaces with boundary. One of the bounds depends only on the genus of a surface and the index k of an eigenvalue, while the other depends as well on the number of boundary components. We also show that on any given smooth Riemannian surface with boundary, the multiplicities of Steklov eigenvalues σ_k are uniformly bounded in k.

[2] Non-maximality of extremal metrics on torus and Klein bottle
arXiv:1210.8122, *to appear in Sbornik Math.*

El Soufi-Ilias' theorem establishes a connection between minimal submanifolds of spheres and extremal metrics for eigenvalues of the Laplace-Beltrami operator. Recently, this connection was used to provide several explicit examples of extremal metrics. We investigate the maximality of these metrics and prove that all of them are not maximal.

[3] Spectral properties of a family of minimal tori of revolution in five-dimensional sphere
arXiv:1301.2483, *to appear in Canadian Mathematical Bulletin.*

The normalized eigenvalues $\Lambda_i(M, g)$ of the Laplace-Beltrami operator can be considered as functionals on the space of all Riemannian metrics g on a fixed surface M. In recent papers several explicit examples of extremal metrics were provided. These metrics are induced by minimal immersions of surfaces in $\mathbb{S}^3$ or $\mathbb{S}^4$. In the present paper a family of extremal metrics induced by minimal immersions in $\mathbb{S}^5$ is investigated.

[4] Maximization of the first nontrivial eigenvalue on the surface of genus 2
arXiv:1309.5057.

The first nontrivial eigenvalue of the Laplacian can be considered as a functional on the space of all Riemannian metrics of unit volume on a fixed surface. In this paper we prove that for the surface of genus 2 the supremum of this functional is equal to 16π. This provides a positive answer to the conjecture by Jakobson, Levitin, Nadirashvili, Nigam and Polterovich.

2.1.4 Dmitry Kubrak

[1] With M.Finkelberg
Vanishing cycles on Poisson varieties.
arXiv:1212.3051, *submitted to Functional Analysis and Its Applications.*

We extend slightly the results of Evens-Mirkovic, and compute the character- istic cycles of Intersection Cohomology sheaves on the transversal slices in the double affine Grassmannian. We propose a conjecture relating the hyperbolic stalks and the microlocalization at a torus-fixed point in a Poisson variety.

2.1.5 Nikon Kurnosov

[1] The second Betti number of hyperkähler manifolds (joint work with M. Verbitsky)
arXiv:13*** (*to be submitted*)

Let M be a compact irreducible hyperkähler manifold, from Bogomolov inequality we obtain forbidden values of the second Betti number b_2 in arbitrary dimension.

2.1.6 Alexei Naumov

[1] With F. Goetze and A. Tikhomirov
Limit theorems for two classes of random matrices with dependent entries
arXiv: 1211.0389 *submitted to Probability theory and its applications*

In this paper we study ensembles of random symmetric matrices $\mathbf{X_n} = \{\mathbf{X_{ij}}\}_{\mathbf{i,j=1}}^{\mathbf{n}}$ with a random field type dependence, such that $\mathbb{E}X_{ij} = 0$, $\mathbb{E}X_{ij}^2 = \sigma_{ij}^2$, where σ_{ij} can be different numbers. Assuming that the average of the normalized sums of variances in each row converges to one and Lindeberg condition holds true we prove that the empirical spectral distribution of eigenvalues converges to Wigner's semicircle law.

[2] With F. Goetze and A. Tikhomirov
On the minimal singular value of large random matrices with correlated entries
arXiv: 1309.5711

Let $\mathbf{X}$ be a random matrix whose pairs of entries X_{jk} and X_{kj} are correlated and vectors (X_{jk}, X_{kj}), for $1 \le j < k \le n$, are mutually independent. Assume that the diagonal entries are independent from off-diagonal entries as well. We assume that $\mathbb{E}X_{jk} = 0$, $\mathbb{E}X_{jk}^2 = 1$, for any $j, k = 1, \ldots, n$ and $\mathbb{E}X_{jk}X_{kj} = \rho$ for $1 \le j < k \le n$. Let $\mathbf{M}_n$ be a non-random $n \times n$ matrix with $\|\mathbf{M}_n\| \le Kn^Q$, for some positive constants $K > 0$ and $Q \ge 0$. Let $s_n(\mathbf{X}+\mathbf{M}_n)$

denote the least singular value of the matrix $\mathbf{X} + \mathbf{M}_n$. It is shown that there exist positive constants A and B depending on K, Q, ρ only such that

$$\mathbb{P}(s_n(\mathbf{X} + \mathbf{M}_n) \leq n^{-A}) \leq n^{-B}.$$

As an application of this result we prove the elliptic law for this class of matrices with non identically distributed correlated entries.

2.1.7 Alexander Perepechko

[1] Flexibility of affine cones over del Pezzo surfaces of degree 4 and 5
Functional Analysis and Applications, 2013, Vol. 47, No. 4, pp. 45–52.

In this paper we prove that the action of the special automorphism group on affine cones over del Pezzo surfaces of degree 4 and 5 is infinitely transitive.

[2] With Ivan Arzhantsev and Hendrik Süß
Infinite transitivity on universal torsors
arXiv:1302.2309, *to appear in Journal of the London Mathematical Society.*

Let X be an algebraic variety covered by open charts isomorphic to the affine space and $q : X' \to X$ be the universal torsor over X. We prove that the automorphism group of the quasiaffine variety X' acts on X' infinitely transitively. Also we find wide classes of varieties X admitting such a covering.

2.1.8 Mikhail Kharitonov

[1] Estimates of a structure of piece-wise periodicity in Shirshov's height theorem
Vestnik Moskovskogo Universiteta, Seriya 1, Matematika.Mekhanika., 2013, No. 1, pp. 10 – 16.

The Gelfand-Kirillov dimension of l-generated general matrixes is $(l-1)n^2+1$. The minimal degree of the identity of this algebra is $2n$ as a corollary of Amitzur-Levitsky theorem. That is why the essential height of A being an l-generated PI-algebra of degree n over every set of words can be greater than $(l-1)n^2/4+1$. We prove that if A has a finite GK-dimension, then the number of lexicographically comparable subwords with the period $(n-1)$ in each monoid of A is not greater than $(l-2)(n-1)$. The case of the subwords with the period 2 is generalized to the proof of Shirshov's Height theorem.

[2] Estimates on the number of partially ordered sets
Vestnik Moskovskogo Universiteta, Seriya 1, Matematika.Mekhanika., In print.

We say, that $\epsilon_k(n)$ is the number of partially ordered sets such that
a) cardinality of each set sets is n,
b) dimension of each set is two,
c) length of the maximal antichain in each set is k.
We prove that $\epsilon_k(n) \leq \min\{\frac{k^{2n}}{(k!)^2}, \frac{(n-k+1)^{2n}}{((n-k)!)^2}\}$.
We denote by $\xi_k(n)$ the number of permutations from S_n such that the maximal decreasing chain of such permutation has length k. We prove that $\xi_k(n) \leq \frac{k^{2n}}{((k-1)!)^2}$.
We survey connections among the pairs of linear orders, the pairs Young diagrams, two-dimensional arrays of positive integers and matrices of nonnegative integers. This survey is based on papers [1], [2]. We show the generating function of $\xi_k(n)$. It was obtained in [3].

2.2 Scientific conferences and seminar talks

2.2.1 Alexei Bufetov

[1] Cornell Summer Probability School, Cornell University; June, 2013.

[2] Advanced Workshop and Conference on Random Matrices and Growth Models, ICTP, Trieste, September, 2013.

[3] Conference "Lomonosov"-2013, Moscow State University, Moscow; April 2013.

[4] Visit to MIT, Boston; March 2013.

[5] Several seminar talks at Higher School of Economics, Independent University of Moscow, Institute for Information Transmission Problems, Moscow State University.

2.2.2 Natanliya Goncharuk

[1] Summer School "Modern Mathematics" (Dubna, Russia), July 2013
Mini-course "Arnold's cat" (together with Yu. Kudryashov)

[2] Talk "Non-genericity of Liouville rotations" at Seminar "Dynamical Systems" (MSU, Moscow), October 2013

Talk "Complex and Diophantine rotation numbers" at Seminar "Dynamical Systems" (MSU, Moscow), December 2013.

[3] I plan a 5-month visit to Mexico, from January till June 2014, to Instituto de Matemáticas de la Universidad Nacional Autónoma de México (UNAM), namely to Ciudad Universitaria in Mexico City and Unidad de Cuernavaca. The main aim of this visit is to collaborate with Laura Ortiz Bobadilla and Alberto Verjovsky Sola in complex foliations.

2.2.3 Mikhail Karpukhin

[1] Conference ”Lomonosov”, Moscow, April 8 – 12.
Talk ”Multiplicity bounds for Steklov eigenvalues on Riemannian surfaces”

[2] Conference “Worshop on spectral theory and geometry”, Neuchâtel, June 4 – 8.
Talk ”Extremal metrics on torus and Klein bottle”

[3] Seminar ”Geometry, topology and mathematical physics” at Moscow State University
Talk ”Geometrical optimization of eigenvalues of the Laplacian on surfaces”

[4] Seminar ”Geometry of manifolds” at Higher School pf Economics
Talk ”Extremal properties of Laplacian spectrum”

[5] School ”Geomerical methods in mathematical physics”, Moscow, June 25 – 28.

[6] School ”School on geometry and physics”, Bialowieza, July 8 – 13.

2.2.4 Dmitry Kubrak

[1] Conference “Arithmetic, Geometry, Cryptography and Coding Theory”, Luminy, June, 3 – 7
Talk “Brauer-Siegel type theorems for reductive groups”

[2] Conference ” Arithmetic days”, St. Petersburg, May, 20 – 23
Talk ”Brauer-Siegel theorem for algebraic tori over function fields”

[3] Visit to Boston, 27 August – 1 November
Talk “Vanishing Cycles on Poisson Varieties” at “Representation Theory and Related Topics Seminar” (Northeastern University)

[4] Talk ”Brauer-Siegel type theorems for algebraic tori and reductive groups” at ”Seminar of Laboratory of Algebraic Geometry” (Higher School of Economics)

2.2.5 Nikon Kurnosov

[1] Conference “Differential geometry and applications”, Brno, August, 19 – August, 23
Talk “Connections on nilmanifolds”

[2] International Conference ”Geometry and Analysis on Metric Structures”, Novosibirsk, December, 4 – December, 7
Talk “The second Betti number of hyperkähler manifolds”

[3] Conference “Summer school-conference on problems of algebraic geometry and complex analysis”, Yaroslavl', May, 20 – May, 25
Talk “Connections on nilmanifolds”

[4] Master class: Around Torrelli's theorem for K3 manifolds, Strasbourg, October, 25 – November, 1

[5] Seminar "Geometric structures on manifolds", Department of mathematics, Moscow, April, 18
Talk "Holomorphic dynamics on K3 surfaces 3"

[6] Seminar "Geometric structures on manifolds", Department of mathematics, Moscow, June, 27
Talk "Futaki invariant"

[7] Seminar "Geometric structures on manifolds", Department of mathematics, Moscow, October, 10
Talk "Cohomological properties of kähler manifolds"

[8] Seminar "Geometric structures on manifolds", Department of mathematics, Moscow, November, 21
Talk "Hodge ring of kähler manifolds"

[9] Summer school "Contemporary mathematics", Dubna, July, 20 – July, 31
Lectures "Group theory and its applications to physics and chemistry"

2.2.6 Alexei Naumov

[1] Conference "Randomness in Physics and Mathematics. From Quantum Chaos to Free Probability", Bielefeld, August, 5 – August, 17
Talk "On the smallest singular value of large random matrices with dependent entries"
[2] Conference "Stochastic and Real World Models", Bielefeld, July, 15 – July, 19
Talk "Limit theorems for two classes of random matrices with dependent entries"
[3] Conference "International conference Lomonosov-2013", Moscow, April, 15 – April, 19
Talk "Semicircle law for a class of random matrices with dependent entries"
[4] Visit to Hangzhou, China, November
Talk "Limit theorems for random matrices" (Hangzhou University)
[5] Visit to Hefei, China, November
Talk "Limit theorems for random matrices" (Hefei University of Science and Technology)

2.2.7 Alexander Perepechko

[1] “2nd Swiss-French workshop on algebraic geometry”, Enney, Switzerland, February 18–22

[2] Autumn School “Power sum decompositions and apolarity, a geometric approach”, Lukecin, Poland, September 1–7

[3] Visit to Basel, Switzerland, April 5th

Talk “Flexibility of universal torsors” at the Seminar of Algebra and Geometry (Mathematisches Institut, Universitt Basel)

[4] Visit to Grenoble, France, December.

Writing two joint articles with Kevin Langlois and Mikhail Zaidenberg respectively.

PhD Thesis Defense ”Automorphismes des variétés affines” (Institut Fourier)

2.2.8 Mikhail Kharitonov

[1] Conference “Classical Aspects of Ring Theory and Module Theory”, Poland, Bedlewo, July, 14 – 20

Joint Talk with A. Belov “Subexponential estimates in the height theorem and estimates on numbers of periodic parts of small periods”

[2] Conference “Lomonosov-2013”, Moscow, April, 8 – 13

Talk “Estimates on the number of partially ordered sets”

[3] Visit to Israel, Ramat Gan

Talk “Subexponential estimates in the height theorem and estimates on numbers of periodic parts of small periods” at “Bar-Ilan Algebra Seminar” (Bar-Ilan University)

[4] Visit to Israel, Haifa

Talk “Subexponential estimates in the height theorem and estimates on numbers of periodic parts of small periods” at “Algebra Seminar” (Technion (Israel Institute of Technology))

3 Program: Simons IUM fellowships

3.1 Research

3.1.1 Ivan Arzhantsev

[1] With H. Flenner, S. Kaliman, F. Kutzschebauch, and M. Zaidenberg. Flexible varieties and automorphism groups. Duke Mathematical Journal, 2013, Vol. 162, No. 4, pp. 767–823.

Given an affine algebraic variety X of dimension at least 2, we let $\mathrm{SAut}(X)$ denote the special automorphism group of X i.e., the subgroup of the full automorphism group $\mathrm{Aut}(X)$ generated by all one-parameter unipotent subgroups. We show that if $\mathrm{SAut}(X)$ is transitive on the smooth locus of X then it is infinitely transitive on this locus. In turn, the transitivity is equivalent to the flexibility of X. The latter means that for every smooth point x of X the tangent space at x is spanned by the velocity vectors of one-parameter unipotent subgroups of $\mathrm{Aut}(X)$. We provide also different variations and applications.

[2] With D. Celik and J. Hausen. Factorial algebraic group actions and categorical quotients. Journal of Algebra, 2013, Vol. 387, pp. 87–98.

Given an action of an affine algebraic group with only trivial characters on a factorial variety, we ask for categorical quotients. We characterize existence in the category of algebraic varieties. Moreover, allowing constructible sets as quotients, we obtain a more general existence result which, for example, settles the case of a finitely generated algebra of invariants. As an application, we provide a combinatorial GIT-type construction of categorial quotients for actions on, e.g. complete varieties with finitely generated Cox ring via lifting to the characteristic space.

[3] With I.A. Bazhov. On orbits of the automorphism group on an affine toric variety. Central European Journal of Mathematics, 2013, Vol. 11, No. 10, pp. 1713–1724.

Let X be an affine toric variety. The total coordinates on X provide a canonical presentation of X as a quotient of a vector space by a linear action of a quasitorus. We prove that the orbits of the connected component of the automorphism group $\mathrm{Aut}(X)$ on X coincide with the Luna strata defined by the canonical quotient presentation.

[4] With H. Flenner, S. Kaliman, F. Kutzschebauch, and M. Zaidenberg. Infinite transitivity on affine varieties. In: Birational geometry, rational curves, and arithmetic - Simons symposium 2012, F. Bogomolov, B. Hassett, and Yu. Tschinkel (Editors), Springer Science-Business Media New York, 2013, pp. 1–13.

We survey recent results on automorphisms of affine algebraic varieties, infinitely transitive group actions and flexibility. We present related constructions and examples, and discuss geometric applications and open problems.

[5] With M.Zaidenberg. Acyclic curves and group actions on affine toric surfaces. In: Affine Algebraic Geometry, Proceedings of the Conference, Osaka, 3-6 March 2011, K. Masuda, H. Kojima, T. Hishimoto (Editors), World Scientific Publishing Co. Pte. Ltd. 2013, pp. 1–41.

We show that every irreducible, simply connected curve on a toric affine surface X over the field of complex numbers is an orbit closure of a multiplicative group action on X. It follows that up to the action of the automorphism group $\mathrm{Aut}(X)$ there are only finitely many non-equivalent embeddings of the affine line in X. A similar description is given for simply connected curves in the quotients of the affine plane by small finite linear groups. We provide also an analog of the Jung-van der Kulk theorem for affine toric surfaces, and apply this to study actions of algebraic groups on such surfaces.

[6] Torsors over Luna strata. In: Torsors, etale homotopy and applications to rational points. Proceedings of the ICMS workshop in Edinburgh, 10-14 January 2011, London Mathematical Society Lecture Note Series 405, A. Skorobogatov (Editor), Cambridge University Press, 2013, pp. 123–137.

Let G be a reductive group and X be a Luna stratum on the quotient space $V//G$ of a rational G-module V. We consider torsors over X with both non-commutative and commutative structure groups. It allows us to compute the divisor class group and the Cox ring of a Luna stratum under mild assumptions. This techniques gives a simple cause why many Luna strata are singular along their boundary.

[7] With J. Hausen, E. Herppich, and A. Liendo. The automorphism group of a variety with torus action of complexity one. arXiv:1202.4568, 34 pages, *to appear in Moscow Mathematical Journal.*

We consider a normal complete rational variety with a torus action of complexity one. In the main results, we determine the roots of the automorphism group and give an explicit description of the root system of its semisimple part. The results are applied to the study of almost homogeneous varieties. For example, we describe all almost homogeneous (possibly singular) del Pezzo k^*-surfaces of Picard number one and all almost homogeneous (possibly singular) Fano threefolds of Picard number one having a reductive automorphism group with two-dimensional maximal torus.

[8] With A.Yu. Perepechko and H. Suess. Infinite transitivity on universal torsors. arXiv:1302.2309, 17 pages, *to appear in the Journal of the London Mathematical Society.*

Let X be an algebraic variety covered by open charts isomorphic to the affine space and $q : X' \to X$ be the universal torsor over X. We prove that the automorphism group of the quasiaffine variety X' acts on X' infinitely transitively. Also we find wide classes of varieties X admitting such a covering.

[9] With A.B.Popovskiy. Additive actions on projective hypersurfaces. arXiv:1307.7341, 13 pages, *to appear in GABAG-2012, Springer Verlag, the Proceedings of the Conference Groups of Automorphisms in Birational and Affine Geometry, Levico Terme (Trento), 2012.*

By an additive action on a hypersurface H in the projective space P^{n+1} we mean an effective action of a commutative unipotent group on P^{n+1} which leaves H invariant and acts on H with an open orbit. Brendan Hassett and Yuri Tschinkel have shown that actions of commutative unipotent groups on projective spaces can be described in terms of local algebras with some additional data. We prove that additive actions on projective hypersurfaces correspond to invariant multilinear symmetric forms on local algebras. It allows us to obtain explicit classification results for non-degenerate quadrics and quadrics of corank one.

[10] With U. Derenthal, J. Hausen, and A. Laface. Cox rings. arXiv:1003.4229 *to appear in Cambridge Studies in Advanced Mathematics, Cambridge University Press, 2014.*

The aim of this monograph is to provide an introduction to Cox rings and their applications in algebraic and arithmetic geometry.

3.1.2 Vladimir Bogachev

[1] Bogachev V.I., Röckner M., Shaposhnikov S.V. On uniqueness of solutions to the Cauchy problem for degenerate Fokker–Planck–Kolmogorov equations. J. Evol. Equat. 2013. V. 13, N 3, P. 577–593.

Broad sufficient conditions for the uniqueness of solutions to the Cauchy problem for degenerate Fokker–Planck–Kolmogorov equations are presented.

[2] Bogachev V.I., Röckner M., Shaposhnikov S.V. On parabolic inequalities for generators of diffusions with jumps. Probability Theory and Related Fields. 2013; DOI 10.1007/s00440-013-0485-0

Broad sufficient conditions for the existence of densities of solutions for parabolic inequalities for integro-differential operators are found.

[3] Bogachev V.I., Kolesnikov A.V. Sobolev regularity for the Monge–Ampère equation in the Wiener space. Kyoto J. Math. 2013. V. 53, N 4, P. 713–738.

New Sobolev regularity results for the Monge–Ampère equation in the Wiener space are obtained.

[4] Bogachev V.I., Shaposhnikov A.V. On extensions of Sobolev functions on the Wiener space. Dokl. Akad. Nauk. 2013. V. 448, N 4, 379–383 (in Russian); English transl.: Dokl. Math. 2013. V. 87, N 1. P. 58–61.

An example is constructed of a Sobolev function on a convex infinite-dimensional domain in the Wiener space without Sobolev extensions to the whole space.

[5] Bogachev V.I., Rebrova E.A. Functions of bounded variation on infinite-dimensional spaces with measures. Dokl. Akad. Nauk. 2013. V. 449, N 2. P. 131–135 (in Russian); English transl.: Dokl. Math. 2013. V. 87, N 2. P. 1–4.

Two different classes of functions of bounded variation on infinite-dimensional spaces with differentiable measures are introduced and studied.

[6] Bogachev V.I., Pilipenko, A.Yu., Rebrova E.A. Classes of functions of bounded variation on infinite-dimensional domains. Dokl. Math. 2013. V. 450, N 6.

Two different classes of functions of bounded variation on domains in infinite-dimensional spaces with differentiable measures are introduced and studied.

[7] Bogachev V.I., Malofeev I.I. On the distributions of smooth functions on infinite-dimensional spaces with measures. Dokl. Math. 2014. V. 454. N 1.

A general sufficient condition for the existence of a distribution density of a smooth function on an infinite-dimensional space with a differentiable measure is obtained.

[8] Bogachev V.I., Kirillov A.I., Shaposhnikov S.V. The stationary Fokker–Planck–Kolmogorov equation with a potential. Dokl. Math. 2014. V. 454. N 2.

Existence and uniqueness problems are studied for the stationary Fokker–Planck–Kolmogorov equation with a potential.

[9] Bogachev V.I., Pilipenko, A.Yu., Shaposhnikov A.V. Sobolev functions on infinite-dimensional domains. MathArxiv: arXiv:1309.5475

Extensions of Sobolev and BV functions on infinite-dimensional domains with measures are studied.

BOOK:
Bogachev V.I., Krylov N.V., Röckner M., Shaposhnikov S.V. Fokker–Planck–Kolmogorov equations. Regular and Chaotic Dynamic: Moscow – Izhevsk, 2013; 592 pp.

The book gives a systematic exposition of the theory of Fokker–Planck–Kolmogorov equations.

3.1.3 Yurii Burman

[1] (with A.Ploskonosov and A.Trofimova) Matrix-tree theorems and discrete path integration, submitted to *Linear Algebra and Its Applications*.

In this paper we calculate characteristic polynomials of operators explicitly presented as polynomials of rank 1 operators. Corollaries of the main result include a generalization of the Forman's formula for the determinant of the graph Laplacian, the celebrated Matrix-tree theorem by G.Kirchhoff, and some its generalizations and analogs, both known (e.g. the Matrix-hypertree theorem by G.Masbaum and A.Vaintrob) and new.

[2] (with S.Lvovski) On projections of smooth and nodal plane curves, arXiv:1311.1904, submitted to *Journal of Algebraic Geometry*

Suppose that $C \subset \mathbb{P}^2$ is a general enough smooth plane curve of degree > 2 and that $\pi\colon C \to \mathbb{P}^1$ is a finite morphism simply ramified over the same set of points as a projection $\mathrm{pr}_p\colon C \to \mathbb{P}^1$, where $p \in \mathbb{P}^2 \backslash C$. We prove that the morphism π is equivalent to such a projection if and only if it extends to a finite morphism $X \to (\mathbb{P}^2)^*$ ramified over C^*, where X is a smooth surface. Actually we prove a similar result for nodal curves.

[3] Lie elements in the group algebra, arXiv:1309.4477.

Given a representation V of a group G, there are two natural ways of defining a representation of the group algebra $k[G]$ in the external power $V^{\wedge m}$. The set $\mathcal{L}(V)$ of elements of $k[G]$ for which these two ways give the same result is a Lie algebra and a representation of G. For the case when G is a symmetric group and $V = \mathbb{C}^n$ is a permutation representation, these spaces $\mathcal{L}(\mathbb{C}^n)$ are naturally embedded into one another. We describe $\mathcal{L}(\mathbb{C}^n)$ for small n and formulate questions and conjectures for future research.

3.1.4 Alexei Elagin

[1] "On equivariant triangulated categories", in preparation.

In this paper we present a construction of G-equivariant category for an action of a finite group G on a triangulated category, supposing that the action is induced by an action on a DG-enhancement.

3.1.5 Alexei Gorodentsev

[1] Algebra. Textbook for students specializing in math. Part I. Moscow MCCME (2013) 485 pp.

This is an intensive course of algebra for the 1st year students of mathematical faculties. It is based on my courses given at the Independent University of Moscow and at the Faculty of Mathematics of HSE. The book contains a lot of exercises, some of them are equipped with hints and answers, some others are for independent solution. Table of contents:

§1 Sets and maps.

§2 Examples of abelian groups, commutative fields and rings. Complex numbers. Divisibility in $\mathbb{Z}$.

§3 Rings and fields of residues. Chinese remainder theorem. Characteristic of a field.

§4 Polynomials: divisibility, roots, rings and fields $\Bbbk$. Fields of fractions, rational functions.

§5 Formal power series: differential calculus, exponential and logarithmic maps, binomial, expansion of elementary functions.

§6 Factor rings and ideals. Noetherian rings, Hilbert's theorem about a basis of an ideal. Principal ideal domains. Factorial rings, polynomial ring over a factorial ring is factorial too.

§7 Vector spaces, bases, dimension. Linear maps, kernel, image. Subspaces, sums and intersections.

§8 Dual spaces, isomorphism $V^{**} \simeq V$. Duality $U \leftrightarrow \operatorname{Ann} U$. Dual operators. A position of a subspace w.r.t. a basis. Gauss method, basis of a linear span and of a factor space.

§9 Algebra of matrices. Non commutative rings and algebras.

§10 Determinants and grassmannian polynomials. Laplace relations. Adjugate matrix. Cramer's rule.

§11 Modules. Generators and relations, torsion, decomposability. Homomorphisms, the Cayley-Hamilton identity. Factor modules. Rank of a free module.

§12 Classification of finitely generated modules over a principal ideals domain. Elementary divisors. Finitely generated abelian groups.

§13 Classification of vector spaces equipped with a linear endomorphism. Jordan's normal form and Jordan's decomposition. Commuting operators. Analytic functions of operators.

§14 Euclidean spaces. Gram matrices and orthogonalization. Orthogonal operators. Metrics and norms on a real vector space.

§15 Groups. Homomorphisms. Transformation groups and group actions. Length of an orbit.

§16 Cosets, Lagrange's theorem. Normal subgroups and factor groups. Simple groups. p-groups and Silov's theorems.

§17 Bilinear forms. Non-degenerated skew-symmetric forms, symplectic group. Symmetric and quadratic forms, isometries, reflections. Anisotropic and hyperbolic forms, Witt's lemma.

§18 Projective spaces. Projective geometry vs linear algebra. Linear projective group, cross-ratio.

§19 Projective quadrics: tangent space, singular points, polar mapping, dual quadric. Examples: Veronese, Segre, Plücker. Affine quadrics.

§20 Hermitean spaces and hermitean geometry. Normal operators, (anti) self-dual operators, unitary group. Polar decomposition.

§21 Real and complex structures. Complexification. Hermitian continuation of euclidean or symplectic structure, Kähler triples (I, g, ω). Siegel's upper half-space and Riemann's conditions.

§22 Quaternions. The universal covering $\mathrm{SU}_2 \twoheadrightarrow \mathrm{SO}_3$. Spinors and two families of complex structures on $\mathbb{H}$.

§23 Tensor products of modules, vector spaces and abelian groups. Universal properties, generators and relations. Standard isomorphisms.

§24 Tensor algebra of a vector space. Duality and contractions. Linear support of a tensor. Symmetric and exterior algebras.

§25 Symmetric and skew symmetric tensors. Polarization of commutative and grassmannian polynomials. Polar mappings and partial derivatives.

§26 Algebra of symmetric functions. Monomial, elementary, complete, Newton's, and Schuhr's bases and their interaction. Some determinantal identities.

§27 Calculus of massives, Young diagrams, and Young tableaux. Combinatorial Schuhr's polynomials. The Littlewood–Richardson rule.

3.1.6 Maxim Kazarian

[1] with P. Dunin-Barkowski, N. Orantin, S. Shadrin, L. Spitz

Polynomiality of Hurwitz numbers, Bouchard-Mariño conjecture, and a new proof of the ELSV formula

arXiv:1307.4729 *submitted to Advances in Mathematics*

In this paper we give a new proof of the ELSV formula. First, we refine an argument of Okounkov and Pandharipande in order to prove (quasi-)polynomiality of Hurwitz numbers without using the ELSV formula (the only way to do that before used the ELSV formula). Then, using this polynomiality we give a new prove of the Bouchard-Mariño conjecture. After that, using the correspondence between the Givental group action and the topological recursion coming from matrix models, we prove the equivalence of the Bouchard-Mariño conjecture and the ELSV formula (it is a refinement of an argument by Eynard).

3.1.7 Alexander Kuznetsov

[1] A. Kuznetsov, A. Polishchuk, Exceptional collections on isotropic Grassmannians, accepted in JEMS.

We introduce a new construction of exceptional objects in the derived category of coherent sheaves on a compact homogeneous space of a semisimple algebraic group and show that it produces exceptional collections of the length equal to the rank of the Grothendieck group on homogeneous spaces of all classical groups.

[2] Height of exceptional collections and Hochschild cohomology of quasiphantom categories, accepted in Crelle.

We define the normal Hochschild cohomology of an admissible subcategory of the derived category of coherent sheaves on a smooth projective variety X — a graded vector space which controls the restriction morphism from the Hochschild cohomology of X to the Hochschild cohomology of the orthogonal complement of this admissible subcategory. When the subcategory is generated by an exceptional collection, we define its new invariant (the height) and show that the orthogonal to an exceptional collection of height h in the derived category of a smooth projective variety X has the same Hochschild cohomology as X in degrees up to $h - 2$. We use this to describe the second Hochschild cohomology of quasiphantom categories in the derived categories of some surfaces of general type. We also give necessary and sufficient conditions of fullness of an exceptional collection in terms of its height and of its normal Hochschild cohomology.

[3] Scheme of lines on a family of 2-dimensional quadrics: geometry and derived category, accepted in Math. Zeitschrift.

Given a generic family Q of 2-dimensional quadrics over a smooth 3-dimensional base Y we consider the relative Fano scheme M of lines of it. The scheme M has a structure of a generically conic bundle $M \to X$ over a double covering $X \to Y$ ramified in the degeneration locus of $Q \to Y$. The double covering $X \to Y$ is singular in a finite number of points (corresponding to the points $y \in Y$ such that the quadric Q_y degenerates to a union of two planes), the fibers of M over such points are unions of two planes intersecting in a point. The main result of the paper is a construction of a semiorthogonal decomposition for the derived category of coherent sheaves on M. This decomposition has three components, the first is the derived category of a small resolution X^+ of singularities of the double covering $X \to Y$, the second is a twisted resolution of singularities of X (given by the sheaf of even parts of Clifford algebras on Y), and the third is generated by a completely orthogonal exceptional collection.

[4] A simple counterexample to the Jordan–Hölder property for derived categories, preprint math.AG/1304.0903.

A counterexample to the Jordan–Hölder property for semiorthogonal decompositions of derived categories of smooth projective varieties was constructed by Böhning, Graf von Bothmer and Sosna. In this short note we present a simpler example by realizing Bondals quiver in the derived category of a blowup of $\mathbb{P}^3$.

3.1.8 Karine Kuyumzhiyan

[1] with Fedor Bogomolov and Ilya Karzhemanov
Unirationality and existence of infinitely transitive models
Birational geometry, rational curves, and arithmetic, 2013, Ch. 4., p. 77–92. Boston: Birkhauser.

We study unirational algebraic varieties and the fields of rational functions on them. We show that after adding a finite number of variables some of these fields admit a so-called infinitely transitive model. The latter is an algebraic variety with the given field of rational functions and an infinitely transitive regular action of a group of algebraic automorphisms generated by unipotent algebraic subgroups. We expect that this property holds for all unirational varieties and in fact is a peculiar one for this class of algebraic varieties among those varieties which are rationally connected.

3.1.9 Grigori Olshanski

[1] Projections of orbital measures, Gelfand–Tsetlin polytopes, and splines.
Journal of Lie Theory, 2013, Vol. 23, No 4, pp. 1011-1022.

The unitary group $U(N)$ acts by conjugations on the space $H(N)$ of $N \times N$ Hermitian matrices, and every orbit of this action carries a unique invariant probability measure called an orbital measure. Consider the projection of the space $H(N)$ onto the real line assigning to an Hermitian matrix its $(1,1)$-entry. Under this projection, the density of the pushforward of a generic orbital measure is a spline function with N knots. This fact was pointed out by Andrei Okounkov in 1996, and the goal of the paper is to propose a multidimensional generalization. Namely, it turns out that if instead of the $(1,1)$-entry we cut out the upper left matrix corner of arbitrary size $K \times K$, where $K = 2, ..., N-1$, then the pushforward of a generic orbital measure is still computable: its density is given by a $K \times K$ determinant composed from one-dimensional splines. The result can also be reformulated in terms of projections of the Gelfand-Tsetlin polytopes.

[2] With A. Borodin

Markov dynamics on the Thoma cone: a model of time-dependent determinantal processes with infinitely many particles.

Electronic Journal of Probability, 2013, Vol. 18, No. 75, 1–43

The Thoma cone is an infinite-dimensional locally compact space, which is closely related to the space of extremal characters of the infinite symmetric group S_∞. In another context, the Thoma cone appears as the set of parameters for totally positive, upper triangular Toeplitz matrices of infinite size.

The purpose of the paper is to construct a family $\{X^{(z,z')}\}$ of continuous time Markov processes on the Thoma cone, depending on two continuous parameters z and z'. Our construction largely exploits specific properties of the Thoma cone related to its representation-theoretic origin, although we do not use representations directly. On the other hand, we were inspired by analogies with random matrix theory coming from models of Markov dynamics related to orthogonal polynomial ensembles.

We show that processes $X^{(z,z')}$ possess a number of nice properties, namely: (1) every $X^{(z,z')}$ is a Feller process; (2) the infinitesimal generator of $X^{(z,z')}$, its spectrum, and the eigenfunctions admit an explicit description; (3) in the equilibrium regime, the finite-dimensional distributions of $X^{(z,z')}$ can be interpreted as (the laws of) infinite-particle systems with determinantal correlations; (4) the corresponding time-dependent correlation kernel admits an explicit expression, and its structure is similar to that of time-dependent correlation kernels appearing in random matrix theory.

[3] Markov dynamics on the dual object to the infinite-dimensional unitary group.

arXiv:1310.6155, *to appear in Proceedings of the St. Petersburg Summer School "Probability and Statistical Physics".*

These are notes for a mini-course of 3 lectures given at the St. Petersburg School in Probability and Statistical Physics (June 2012). My aim was to explain, on the example of a particular model, how ideas from the representation theory of big groups can be applied in probabilistic problems. The material is based on the joint paper arXiv:1009.2029 by

Alexei Borodin and myself; a broader range of topics is surveyed in the lecture notes by Alexei Borodin and Vadim Gorin arXiv:1212.3351.

[4] With A. Borodin
An interacting particle process related to Young tableaux
arXiv:1303.2795, *to appear in Journal of Mathematical Sciences (New York)*

We discuss a stochastic particle system consisting of a two-dimensional array of particles living in one space dimension. The stochastic evolution bears a certain similarity to Hammersley's process, and the particle interaction is governed by combinatorics of the Young tableaux.

3.1.10 Alexei Penskoi

[1] Extremal spectral properties of Otsuki tori
Mathematische Nachrichten, 2013, Vol. 286, No. 4, pp. 379-391.

In this paper Otsuki tori are investigated. Otsuki tori form a countable family of immersed minimal two-dimensional tori in the unitary three-dimensional sphere. According to the El Soufi-Ilias theorem, the metrics on the Otsuki tori are extremal for some unknown eigenvalues of the Laplace-Beltrami operator. Despite the fact that the Otsuki tori are defined in quite an implicit way, we find explicitly the numbers of the corresponding extremal eigenvalues. In particular we provide an extremal metric for the third eigenvalue of the torus.

[2] Metrics extremal for eigenvalues of Laplace-Beltrami operator on surfaces.
Uspekhi Mat. Nauk, 2013, Vol 68, No. 6 (614), pp. 107-168.
English translation to appear in Russian Math. Surveys.

In this paper we give a short review of known results in the problem of geometric optimization of eigenvalues of Laplace-Beltrami operator and a detailed review of recent results in the theory of extremal metrics on surfaces.

[3] Generalized Lawson tori and Klein bottles
arXiv:1308.1628 *submitted to the Journal of Geometric Analysis*

Using Takahashi theorem we propose an approach to extend known families of minimal tori in spheres. As an example, the well-known two-parametric family of Lawson tau-surfaces including tori and Klein bottles is extended to a three-parametric family of tori and Klein bottles minimally immersed in spheres. Extremal spectral properties of the metrics on these surfaces are investigated. These metrics include i) both metrics extremal for the first non-trivial eigenvalue on the torus, i.e. the metric on the Clifford torus and the metric on the equilateral torus and ii) the metric maximal for the first non-trivial eigenvalue on the Klein bottle.

3.1.11 Yuri Prokhorov

[1] Fano threefolds of large Fano index and large degree.

Sb. Math. 2013, Vol. 204, No. 3, pp. 347–382.

We classify $\mathbb{Q}$-Fano threefolds of Fano index > 2 and large degree.

[2] On birational involutions of $\mathbb{P}^3$.

Izv. Math. 2013, Vol, 77, No. 3, pp. 627–648.

Let X be a rationally connected three-dimensional algebraic variety and let τ be an element of order two in the group of its birational selfmaps. Suppose that there exists a non-uniruled divisorial component of the τ-fixed point locus. Using the equivariant minimal model program we give a rough classification of such elements.

[3] G-Fano threefolds, I.

Adv. Geom. 2013, Vol. 13, No. 3, pp. 389–418.

We classify Fano threefolds with only terminal singularities whose canonical class is Cartier and divisible by 2, and satisfying an additional assumption that the G-invariant part of the Weil divisor class group is of rank 1 with respect to an action of some group G. In particular, we find a lot of examples of Fano 3-folds with "many" symmetries.

[4] G-Fano threefolds, II.

Adv. Geom. 2013, Vol. 13, No. 3. pp. 419–434.

We classify Fano threefolds with only Gorenstein terminal singularities and Picard number greater than 1 satisfying an additional assumption that the G-invariant part of the Weil divisor class group is of rank 1 with respect to an action of some group G.

[5] With F. Bogomolov.

On stable conjugacy of finite subgroups of the plane Cremona group, I.

Cent. European J. Math. 2013, Vol. 11, No. 12, pp. 2099–2105.

We discuss the problem of stable conjugacy of finite subgroups of Cremona groups. We show that the group $H^1(G, \mathrm{Pic}(X))$ is a stable birational invariant and compute this group in some cases.

[6] With T. Kishimoto and M. Zaidenberg.

$\mathbf{G}_a$-actions on affine cones

Transformation Groups. 2013, Vol. 18, No. 4, pp. 1137–1153.

We give a criterion of existence of a unipotent group action on the affine cone over a projective variety or, more generally, on the affine quasicone over a variety which is projective over another affine variety.

[7] With T. Kishimoto and M. Zaidenberg.

Unipotent group actions on del Pezzo cones

Algebraic Geometry, Vol. 2014, No. 1, pp. 46–56.

`http://www.algebraicgeometry.nl/2014-1/2014-1-003.pdf`

In a previous paper we established that for any del Pezzo surface Y of degree at least 4, the affine cone X over Y embedded via a pluri-anticanonical linear system admits an effective $\mathbf{G}_a$-action. In particular, the group $\mathrm{Aut}(X)$ is infinite dimensional. In contrast, we show in this note that for a del Pezzo surface Y of degree at most 2 the generalized cones X as above do not admit any non-trivial action of a unipotent algebraic group.

[8] With S. Mori.

Threefold extremal contractions of types (IC) and (IIB)

to appear in *Proc Edinburgh Math. Soc.,* 2014.

Let (X, C) be a germ of a threefold X with terminal singularities along an irreducible reduced complete curve C with a contraction $f : (X, C) \to (Z, o)$ such that $C = f^{-1}(o)_{\mathrm{red}}$ and $-K_X$ is ample. Assume that (X, C) contains a point of type (IC) or (IIB). We complete the classification of such germs in terms of a general member $H \in |-K_X|$ containing C.

[9] With T. Kishimoto and M. Zaidenberg.

Affine cones over Fano threefolds and additive group actions

ArXiv:1106.1312, to appear in *Osaka J. Math.*,

`http://www.math.sci.osaka-u.ac.jp/ojm/pdf/3380.pdf`

We address the following question: When an affine cone over a smooth Fano threefold admits an effective action of the additive group? In this paper we deal with Fano threefolds of index 1 and Picard number 1. Our approach is based on a geometric criterion from our previous paper, which relates the existence of an additive group action on the cone over a smooth projective variety X with the existence of an open polar cylinder in X. Non-trivial families of Fano threefolds carrying a cylinder were found in loc. cit. Here we provide new such examples.

[10] 2-elementary subgroups of the space Cremona group

ArXiv:1308.5628, to appear in *Groups of Automorphisms in Birational and Affine Geometry*, Springer Proceedings in Mathematics and Statistics, 2014.

We give a sharp bound for orders of elementary abelian 2-groups of birational automorphisms of rationally connected threefolds.

[11] On stable conjugacy of finite subgroups of the plane Cremona group, II

ArXiv:1308.5698.

We prove that, except for a few cases, stable linearizability of finite subgroups of the plane Cremona group implies linearizability.

[12] With C. Shramov.

Jordan property for groups of birational selfmaps

ArXiv:1307.1784.

Assuming Borisov-Alexeev-Borisov conjecture and Minimal Model Program, we prove that finite subgroups of the automorphism group of a finitely generated field over $\mathbb{Q}$ have bounded orders. Further, we investigate which algebraic varieties have groups of birational selfmaps satisfying the Jordan property.

3.1.12 Petr Pushkar'

I am preparing the following three papers:

[1] Counterexamples to lifting of Hamiltonian and contact isotopies

Preprint is available on my home page http://www.hse.ru/org/persons/23534083 http://www.hse.ru/data/2013/12/13/1339708735/Counter.pdf

In this paper I construct counterexamples to the lifting properties claimed by Eliashberg and Gromov.

[2] Chekanov-type theorem for spherized cotangent bundle

Preprint is available on my home page http://www.hse.ru/org/persons/23534083

That paper is devoted to the proof of Chekanov-type theorem for Legendrian manifolds in spherized cotangent bundles

[3] Morse theory for manifolds with boundaries, preprint

Preprint is available on my home page http://www.hse.ru/org/persons/23534083 http://www.hse.ru/data/2013/12/15/1338379877/morall6.pdf

Main subject of the paper is a study of (strong) Morse function on a compact manifold with boundary. We generalize classical Morse inequalities.

3.1.13 Sergei Rybakov

[1] DG-modules over de Rham DG-algebra. arxiv.org/abs/1311.7503

For a morphism of smooth schemes over a regular affine base we define functors of derived direct image and extraordinary inverse image on coderived categories of DG-modules over de Rham DG-algebras. Positselski proved that for a smooth algebraic variety X over a field k of characteristic zero the coderived category of DG-modules over $\Omega^\bullet_{X/k}$ is equivalent to the unbounded derived category of quasi-coherent right D_X-modules. We prove that our functors correspond to the functors of the same name for D_X-modules under Positselski equivalence.

3.1.14 Arkady Skopenkov

[1] D. Gonçalves and A. Skopenkov, A useful lemma on equivariant maps, submitted to Homology, Homotopy and Applications, part of http://arxiv.org/abs/1207.1326

The purpose of this note is to present a short proof of the following known result.

Lemma. *Suppose X, Y are finite connected CW-complexes with free involutions, $f : X \to Y$ is an equivariant map and l is a non-negative integer. If $f^* : H^i(Y) \to H^i(X)$ is an isomorphism for each $i > l$ and is onto for $i = l$, then*

(a_l) $f^\sharp : \pi^i_{eq}(Y) \to \pi^i_{eq}(X)$ is a 1-1 correspondence for $i > l$ and is onto for $i = l$.

Here $H^i(X)$ is (non-equivariant) cohomology with integral coefficients and $\pi^i_{eq}(X)$ is the set of equivariant homotopy classes of equivariant maps from X to S^i with the antipodal involution.

We could not find a proof in standard textbooks and references. So we feel obliged to present a short proof of such a basic and useful result.

[2] D. Crowley and A. Skopenkov, Classification of smooth embeddings of non-simply-connected 4-manifolds into R^7, preprint, 2013

Let N be a closed connected orientable 4-manifold with torsion free integral homology. The main result is *a complete readily calculable classification of embeddings* $N \to R^7$, in the smooth and in the piecewise-linear (PL) categories. Such a classification was earlier known only for simply-connected N, in the PL case by Boéchat-Haefilger-Hudson 1970, in the smooth case by the authors 2008. In particular, for $N = S^1 \times S^3$ we define geometrically a 1–1 correspondence between the set of PL isotopy classes of PL embeddings $S^1 \times S^3 \to R^7$ and the quotient set of $Z \oplus Z_6$ up to equivalence $(l, b) \sim (l, b')$ for $b \equiv b' \mod 2GCD(3, l)$. This particular case allows us to disprove the conjecture on the completeness of the Multiple Haefliger-Wu invariant, as well as the Melikhov informal conjecture on the existence of a geometrically defined group structure on the set of PL isotopy classes of PL embeddings in codimension 3. For $N = S^1 \times S^3$ and the smooth case we identify the isotopy classes of embeddings with an explicitly defined quotient of $Z_{12} \oplus Z \oplus Z$.

[3] A. Skopenkov, Classification of knotted tori

We describe the group of (smooth isotopy classes of smooth) embeddings $S^p \times S^q \to R^m$ for $p \le q$ and $m \ge 2p+q+3$. Earlier such a description was known only for $2m \ge 3p+3q+4$. We use (and reprove) a recent exact sequence of M. Skopenkov.

[4] A. Skopenkov, On the action of autodiffeomorphisms on embeddings

We work in the smooth category. For an n-manifold N denote by $E^m(N)$ the set of isotopy classes of embeddings $N \to R^m$. The following problem was suggested by E. Rees in 2002: describe the action of self-diffeomorphisms of $S^p \times S^{n-p}$ on $E^m(S^p \times S^{n-p})$.

Let $g : S^p \times S^{n-p} \to R^m$ be an embedding such that $g|_{a \times S^{n-p}} : a \times S^{n-p} \to R^m - g(b \times S^{n-p})$ is null-homotopic for some different points $a, b \in S^p$ and $m \ge n+2+\frac{1}{2}\max\{p, n-p\}$.

Theorem. *For a map $\varphi : S^p \to SO_{n-p}$ define an autodiffeomorphism φ' of $S^p \times D^{n-p}$ by $\overline{\varphi}(a, b) := (a, \varphi(a)b)$. Let φ'' be the S^{n-p-1}-symmetric extension of φ to an autodiffeomorphism of $S^p \times S^{n-p}$. Then for each map $\varphi : S^p \to SO_{n-p}$ embedding $g \circ \varphi''$ is isotopic to embedded connected sum $g\#u$ for some embedding $u : S^n \to S^m$.*

Let N be an oriented n-manifold and $f : N \to R^m$ an embedding. Denote by $E^m(N)/\#$ the quotient set of $E^m(N)$ by embedded connected sum with embeddings $S^n \to R^m$. As a corollary we obtain that under certain conditions *for orientation-preserving embeddings* $s : S^p \times D^{n-p} \to N$ *the class of* S^p*-parametric embedded connected sum* $f\#_s g$ *in* $E^m(N)/\#$ *depends only on* f, g *and the isotopy (the homotopy or the homology) class of* $s|_{S^p\times 0}$.

Expository publications for university students.

[5] A. Skopenkov, A two-page disproof of the Borsuk partition conjecture, Mat. Prosveschenie, 17 (2013), http://arxiv.org/abs/0712.4009, v2

It is presented the simplest known disproof of the Borsuk conjecture stating that if a bounded subset of n-dimensional Euclidean space contains more than n points, then the subset can be partitioned into $n + 1$ nonempty parts of smaller diameter. The argument is due to N. Alon and is a remarkable application of combinatorics and algebra to geometry. This note is purely expository and is accessible for students.

[6] M. Ilyinskiy, A. Kupavskiy, A. Raygorodskiy and A. Skopenkov, Discrete analysis for mathematician and programmers, Mat. Prosveschenie, 17 (2013).

We present sequences of problems on combinatorics and graph theory (including random graphs).

[7] A. Skopenkov, Algebraic topology from geometric viewpoint, http://arxiv.org/abs/0808.1395 v2, Moscow, MCCME, to appear

This book is purely expositional and is in Russian. It is shown how in the course of solution of interesting geometric problems (close to applications) naturally appear main notions of algebraic topology (homology groups, obstructions and invariants, characteristic classes). Thus main ideas of algebraic topology are presented with minimal technicalities. Familiarity of a reader with basic notions of topology (such as 2-dimensional manifolds and vector fields) is desirable, although definitions are given at the beginning. The book is accessible to undergraduates and could also be an interesting easy reading for mature mathematicians.

[8] A. Skopenkov, Some more proofs from the Book: solvability and insolvability of equations in radicals, in Russian, submitted, http://arxiv.org/abs/0804.4357 v5

This paper is purely expository and is in Russian. We present short elementary proofs of

• the Gauss Theorem on constructibility of regular polygons;
• the existence of a cubic equation unsolvable in real radicals;
• the existence of a quintic equation unsolvable in complex radicals (Galois Theorem).

We do not use the terms 'Galois group' or even 'group'. However, our presentation is a good way to learn (or recall) starting idea of the Galois theory. The paper is accessible for students familiar with elementary algebra (including complex numbers), and could be an interesting easy reading for mature mathematicians. The material is presented as a sequence of problems, which is peculiar not only to Zen monasteries but also to serious mathematical education; most problems are presented with hints or solutions.

[9] V.V. Prasolov, A.B. Skopenkov, Some reflections on why Lobachevsky geometry was recognized, http://arxiv.org/abs/1307.4902

Sometimes arguments that preceded recognition of non-Euclidean (Lobachevsky) geometry are represented in a simplified 'black and white' pattern: 'conservators made nonsense of genius'. Although there is something in this point of view, the real situation was more complicated, and up to some time there were decent grounds for not recognizing the importance of the new theory. We try to explain why non-Euclidean geometry was not recognized at once. We show how important for such recognition was discovery of applications of the new geometry. These reflections have practical importance for modern mathematics because they are related to the question: how a mathematician should choose directions for research?

[10] A. Rukhovich, A. Skopenkov, M. Skopenkov, A. Zimin, Realizability of hypergraphs, preprint. http://www.turgor.ru/lktg/2013/1/index.htm

The following problem is well known: can a given graph be embedded in the plane, i.e., can the graph be drawn on the plane so that its edges have no pairwise intersections and no self intersections except at end points? The present cycle of problems is about the embedding of the two-dimensional analogs of graphs (called hypergraphs) in three-dimensional space and even in four-dimensional space. The most important results are simple proofs of the non-realizability in four-dimensional space of the complete hypergraph on seven vertices and (solution of the Menger Problem) of the product $K_5 \times K_5$. The proofs are based on reduction to *Ramsey linking theory* resuls for graphs in three-dimensional space.

[11] A. Skopenkov, Basic Differential Geometry As a Sequence of Interesting Problems, in Russian, MCCME, Moscow, 2009. http://arxiv.org/abs/0801.1568 (new version completed in 2013)

This paper is purely expositional and is in Russian (sample English translation of two pages is given). It is shown how in the course of solution of interesting geometric problems (close to applications) naturally appear different notions of *curvature*, which distinguish given geometry from the 'ordinary' one. Direct elementary definitions of these notions are presented. The paper is accessible for students familiar with analysis of several variables, and could be an interesting easy reading for mature mathematicians. The material is presented as a sequence of problems, which is peculiar not only to Zen monasteries but also to serious mathematical education.

[12] A. Skopenkov, Algorithms for recognition of the realizability of hypergraphs, in Russian, www.mccme.ru/circles/oim/algor.pdf (new versions completed in 2013)

This is an exposition of recent result of J. Matoušek, M. Tancer and U. Wagner on hardness of embedding simplicial complexes in R^d (http://arxiv.org/abs/0807.0336), and of related topics.

[13] A. Daynyak, A. Glibichuk, M. Ilyinskiy, A. Kupavskiy, A. Raygorodskiy and A. Skopenkov, Elements of discrete mathematics as a sequence of problems, draft of a book.

We present sequences of problems on combinatorics and graph theory (including random graphs).

[14] A. Skopenkov and M. Skopenkov, Some short proofs of the unrealizability of hypergraphs, preprint.

We present short elementary proofs of van Kampen–Flores and Ummel's theorems on unrealizability of certain hypergraphs in four-dimensional space. The proofs are based on reduction to *Ramsey linking theory* resuls for graphs in three-dimensional space.

3.1.15 Mikhail Skopenkov

[1] M. Skopenkov, The boundary value problem for discrete analytic functions, Adv. Math. 240 (2013) 61-87. http://arxiv.org/abs/1110.6737

This paper is on further development of discrete complex analysis introduced by R. Isaacs, R. Duffin, and C. Mercat. We consider a graph lying in the complex plane and having quadrilateral faces. A function on the vertices is called discrete analytic, if for each face the difference quotients along the two diagonals are equal.

We prove that the Dirichlet boundary value problem for the real part of a discrete analytic function has a unique solution. In the case when each face has orthogonal diagonals we prove that this solution converges to a harmonic function in the scaling limit. This solves a problem of S. Smirnov from 2010. This was proved earlier by R. Courant–K. Friedrichs–H. Lewy for square lattices, by D. Chelkak–S. Smirnov and implicitly by P.G. Ciarlet–P.-A. Raviart for rhombic lattices.

In particular, our result implies uniform convergence of finite element method on Delauney triangulations. This solves a problem of A. Bobenko from 2011. The methodology is based on energy estimates inspired by alternating-current networks theory.

[2] F. Nilov, M. Skopenkov, A surface containing a line and a circle through each point is a quadric, Geom. Dedicata 163:1 (2013), 301-310; http://arxiv.org/abs/1110.2338

We prove that a surface in 3-dimensional Euclidean space containing a line and a circle through each point is a quadric. We also give some particular results on the classification of surfaces containing several circles through each point.

[3] A. Bobenko, M. Skopenkov, Discrete Riemann surfaces: linear discretization and its convergence, submitted to J. für die reine und angewandte Mathematik (2013). http://arxiv.org/abs/1210.0 (a new version submitted in 2013)

We develop linear discretization of complex analysis, originally introduced by R. Isaacs, J. Ferrand, R. Duffin, and C. Mercat. We prove convergence of discrete period matrices and discrete Abelian integrals to their continuous counterparts. We also prove a discrete counterpart of the Riemann–Roch theorem. The proofs use energy estimates inspired by electrical networks.

[4] M. Skopenkov, When the set of embeddings is finite?, submitted to Intern. J. Math (2013). http://arxiv.org/abs/1106.1878 (a new version submitted in 2013)

Given a manifold N and a number m, we study the following question: *is the set of isotopy classes of embeddings* $N \to S^m$ *finite*? In case when the manifold N is a sphere the answer was given by A. Haefliger in 1966. In case when the manifold N is a disjoint union of spheres the answer was given by D. Crowley, S. Ferry and the author in 2011.

We consider the next natural case when N is a product of two spheres. In the following theorem, $FCS(i,j) \subset \mathbb{Z}^2$ is a concrete set depending only on the parity of i and j which is defined in the paper.

Theorem. Assume that $m > 2p+q+2$ and $m < p+3q/2+2$. Then the set of isotopy classes of smooth embeddings $S^p \times S^q \to S^m$ is infinite if and only if either $q+1$ or $p+q+1$ is divisible by 4, or there exists a point (x,y) in the set $FCS(m-p-q, m-q)$ such that $(m-p-q-2)x + (m-q-2)y = m-3$.

Our approach is based on a group structure on the set of embeddings and a new exact sequence, which in some sense reduces the classification of embeddings $S^p \times S^q \to S^m$ to the classification of embeddings $S^{p+q} \sqcup S^q \to S^m$ and $D^p \times S^q \to S^m$. The latter classification problems are reduced to homotopy ones, which are solved rationally.

[5] A. Pakharev, M. Skopenkov, A. Ustinov, Through the net of resistors, submitted to Mat. Prosv. 3rd ser. (2013);

This is a popular science paper devoted to an elementary proof of the following beautiful folklore result:

Theorem. (a) A man which is randomly walking in a 2-dimensional square lattice will hit the right neighbor of the initial point before returning to the initial point with probability 1/2.

(b) The resistance between neighboring nodes of an infinite 2-dimensional square lattice of unit resistances equals 1/2.

Parts (a) and (b) turn out to be equivalent to each other. The approach to the proof is based on a physical interpretation.

[6] A. Skopenkov, M. Skopenkov, Some short proofs of the unrealizability of hypergraphs, unpublished preprint.

We present short elementary proofs of van Kampen–Flores and Ummel's theorems on unrealizability of certain hypergraphs in four-dimensional space. The proofs are based on reduction to *Ramsey linking theory* resuls for graphs in three-dimensional space.

In addition to [1]–[5], several talk abstracts have been published in 2013.

3.1.16 Evgeni Smirnov

[1] Young diagrams, plane partitions and alternating sign matrices
Book (64 pp., in Russian), MCCME publishing house, 2014 (to appear)

These are extended notes of a four-lecture minicourse given at the summer school "Contemporary mathematics" (Dubna, Russia) in July 2013. These notes contain an introduction to the theory of Young diagrams and plane partitions. We also discuss relation between plane partitions and alternating sign matrices; this relation was the key ingredient in D. Zeilberger's proof of the alternating sign matrix conjecture.

[2] With V. Kleptsyn
Plane curves and bialgebra of Lagrangian subspaces
In preparation

We study multicomponent plane curves with possible singularities of selftangency type. To each such curve we assign a so-called L-space, which is a Lagrangian subspace in an even-dimensional vector space with the standard symplectic form. This invariant generalizes the notion of the intersection matrix for the framed chord diagram of a one-component plane curve. Moreover, the actions of Morse perestroikas and Vassiliev moves are reinterpreted nicely the language of L-spaces, becoming changes of bases in this vector space. Finally, we define a bialgebra structure on the set of L-spaces.

[3] With G. Merzon
Determinantal identities for flagged Schur and Schubert polynomials
In preparation

We prove new determinantal identities for a family of flagged Schur polynomials. As a corollary of these formulas we obtain determinantal expressions of Schubert polynomials for certain vexillary permutations. Principal specializations and specializations at the identity of these formulas have interesting applications to combinatorics of plane partitions; in particular, they imply Crattenthaller and Fomin–Kirillov identities for the number of plane partitions inside a triangular prism.

3.1.17 Stanislav Shaposhnikov

1. Bogachev V.I., Röckner M., Shaposhnikov S.V. On uniqueness of solutions to the Cauchy problem for degenerate Fokker–Planck–Kolmogorov equations. J. Evol. Equat. 2013. V. 13, N 3, P. 577–593.

Broad sufficient conditions for the uniqueness of solutions to the Cauchy problem for degenerate Fokker–Planck–Kolmogorov equations are presented.

2. Bogachev V.I., Röckner M., Shaposhnikov S.V. On parabolic inequalities for generators of diffusions with jumps. Probability Theory and Related Fields. 2013; DOI 10.1007/s00440-013-0485-0

Broad sufficient conditions for the existence of densities of solutions for parabolic inequalities for integro-differential operators are found.

3. Manita O.A., Shaposhnikov S.V. Nonlinear parabolic equations for measures. St. Petersburg Math. J. 2014. V. 25 N 1. P. 43–62.

A new existence result is established for weak parabolic equations for probability measures. Sufficient conditions are given for the existence of local and global-in-time probability solutions of the Cauchy problem for such equations. Some conditions under which global-in-time solutions do not exist are indicated.

4. Shaposhnikov S.V. On estimates of solutions of Fokker–Planck–Kolmogorov equations with potential terms and non uniformly elliptic diffusion matrices. Transactions of the Moscow Mathematical Society. 2013. V. 74. N 1. P.1–18.

We consider Fokker–Planck–Kolmogorov equations with unbounded coefficients and obtain upper estimates of solutions. We also obtain new estimates involving Lyapunov functions.

5. Bogachev V.I., Kirillov A.I., Shaposhnikov S.V. The stationary Fokker–Planck–Kolmogorov equation with a potential. Dokl. Math. 2014. V. 454. N 2.

Existence and uniqueness problems are studied for the stationary Fokker–Planck–Kolmogorov equation with a potential.

6. Manita O.A., Shaposhnikov S.V. On the well-posedness of the Cauchy problem for Fokker–Planck–Kolmogorov equations with potential terms on arbitrary domains. Math-Arxiv: arXiv:1307.3662 [math.AP]

We study the Cauchy problem for Fokker–Planck–Kolmogorov equations with unbounded and degenerate coefficients. Sufficient conditions for the existence and uniqueness of solutions are indicated.

BOOK:

Bogachev V.I., Krylov N.V., Röckner M., Shaposhnikov S.V. Fokker–Planck–Kolmogorov equations. Regular and Chaotic Dynamic: Moscow – Izhevsk, 2013; 592 pp.

The book gives a systematic exposition of the theory of Fokker–Planck–Kolmogorov equations.

3.1.18 Mikhail Verbitsky

[1] With Ornea L., Vuletescu V.

Blow-ups of locally conformally Kahler manifolds // International Mathematics Research Notices. 2013. No. 12. P. 2809-2821.

A locally conformally Kahler (LCK) manifold is a manifold which is covered by a Kahler manifold, with the deck transform group acting by homotheties. We show that the blow-up of a compact LCK manifold along a complex submanifold admits an LCK structure if and only if this submanifold is globally conformally Kahler. We also prove that a twistor space (of a compact 4-manifold, a quaternion-Kahler manifold or a Riemannian m anifold) cannot admit an LCK metric, unless it is Kahler.

[2] With Grantcharov G.

Calibrations in hyper-Kähler geometry // Communications in Contemporary Mathematics. 2013. Vol. 15. No. 2. P. 1-27.

We describe a family of calibrations arising naturally on a hyperkähler manifold M. These calibrations calibrate the holomorphic Lagrangian, holomorphic isotropic and holomorphic coisotropic subvarieties. When M is an HKT (hyperkaehler with torsion) manifold with holonomy $SL(n, \mathbb{H})$, we construct another family of calibrations Φ_i, which calibrates holomorphic Lagrangian and holomorphic coisotropic subvarieties. The calibrations Φ_i are (generally speaking) not parallel with respect to any torsion-free connection on M.

[3] With Ornea L.

Locally conformally Kahler manifolds admitting a holomorphic conformal flow // Mathematische Zeitschrift. 2013. Vol. 273. No. 3-4. P. 605-611.

A manifold M is locally conformally Kahler if it admits a Kahler covering with monodromy acting by holomorphic homotheties. Let M be an LCK manifold admitting a holomorphic conformal flow of diffeomorphisms, lifted to a non-isometric homothetic flow on the covering. We show that M admits an automorphic potential, and the monodromy group of its conformal weight bundle is $\mathbb{Z}$.

[4] Mapping class group and a global Torelli theorem for hyperkahler manifolds // Duke Mathematical Journal. 2013. Vol. 162. No. 15 (2013). P. 2929-2986.

A mapping class group of an oriented manifold is a quotient of its diffeomorphism group by the isotopies. We compute a mapping class group of a hypekahler manifold M, showing that it is commensurable to an arithmetic subgroup in $SO(3, b_2 - 3)$. A Teichmuller space of M is a space of complex structures on M up to isotopies. We define a birational Teichmuller space by identifying certain points corresponding to bimeromorphically equivalent manifolds, and show that the period map gives an isomorphism of the birational Teichmuller space and the corresponding period space $SO(b_2 - 3, 3)/SO(2) \times SO(b_2 - 3, 1)$. We use this result to obtain a Torelli theorem identifying any connected component of birational moduli space with a quotient of a period space by an arithmetic subgroup. When M is a Hilbert scheme of n points on a K3 surface, with n-1 a prime power, our Torelli theorem implies the usual Hodge-theoretic birational Torelli theorem (for other examples of hyperkahler manifolds the Hodge-theoretic Torelli theorem is known to be false).

[5] Pseudoholomorphic curves on nearly Kahler manifolds // Communications in Mathematical Physics. 2013. Vol. 324. No. 1. P. 173-177.

Let M be an almost complex manifold equipped with a Hermitian form such that its de Rham differential has Hodge type (3,0)+(0,3), for example a nearly Kahler manifold. We prove that any connected component of the moduli space of pseudoholomorphic curves on M is compact. This can be used to study pseudoholomorphic curves on a 6-dimensional sphere with the standard (G_2-invariant) almost complex structure.

[6] Degenerate twistor spaces for hyperkahler manifolds// arXiv:1311.5073

Let M be a hyperkaehler manifold, and η a closed, positive (1,1)-form which is degenerate everywhere on M. We associate to η a family of complex structures on M, called a degenerate twistor family, and parametrized by a complex line. When η is a pullback of a Kaehler form under a Lagrangian fibration L, all the fibers of degenerate twistor family also admit a Lagrangian fibration, with the fibers isomorphic to that of L. Degenerate twistor families can be obtained by taking limits of twistor families, as one of the Kahler forms in the hyperkahler triple goes to η.

[7] With Ljudmila Kamenova, Steven Lu

Kobayashi pseudometric on hyperkahler manifolds// arXiv:1308.5667

The Kobayashi pseudometric on a complex manifold M is the maximal pseudometric such that any holomorphic map from the Poincare disk to M is distance-decreasing. Kobayashi has conjectured that this pseudometric vanishes on Calabi-Yau manifolds. Using ergodicity of complex structures, we prove this result for any hyperkaehler manifold if it admits a deformation with a Lagrangian fibration, and its Picard rank is not maximal. The SYZ conjecture claims that any parabolic nef line bundle on a deformation of a given hyperkaehler manifold is semi-ample. We prove that the Kobayashi pseudometric vanishes for all hyperkaehler manifolds satisfying the SYZ property. This proves the Kobayashi conjecture for K3 surfaces and their Hilbert schemes.

[8] With Taras Panov, Yuri Ustinovsky

Complex geometry of moment-angle manifolds// arXiv:1308.2818

Moment-angle manifolds provide a wide class of examples of non-Kaehler compact complex manifolds. A complex moment-angle manifold Z is constructed via certain combinatorial data, called a complete simplicial fan. In the case of rational fans, the manifold Z is the total space of a holomorphic bundle over a toric variety with fibres compact complex tori. In general, a complex moment-angle manifold Z is equipped with a canonical holomorphic foliation F and a C*-torus action transitive in the transverse direction. Examples of moment-angle manifolds include Hopf manifolds of Vaisman type, Calabi-Eckmann manifolds, and their deformations.

We construct transversely Kaehler metrics on moment-angle manifolds, under some restriction on the combinatorial data. We prove that all Kaehler submanifolds (or, more generally, Fujiki class C subvarieties) in such a moment-angle manifold lie in a compact complex torus contained in a fibre of the foliation F. For a generic moment-angle manifold Z in its combinatorial class, we prove that all subvarieties are moment-angle manifolds of smaller dimension. This implies, in particular, that the algebraic dimension of Z is zero.

[9] Ergodic complex structures on hyperkahler manifolds// arXiv:1306.1498

Let M be a compact complex manifold. The corresponding Teichmuller space $Teich$ is a space of all complex structures on M up to the action of the group of isotopies.

The group Γ of connected components of the diffeomorphism group (known as the mapping class group) acts on $Teich$ in a natural way. An ergodic complex structure is the one with a Γ-orbit dense in $Teich$. Let M be a complex torus of complex dimension ≥ 2 or a hyperkahler manifold with $b_2 > 3$. We prove that M is ergodic, unless M has maximal Picard rank (there is a countable number of such M). This is used to show that all hyperkahler manifolds are Kobayashi non-hyperbolic.

[10] With Frédéric Campana, Jean-Pierre Demailly

Compact Kähler 3-manifolds without non-trivial subvarieties// arXiv:1304.7891

We prove that any compact Kähler 3-dimensional manifold which has no non-trivial complex subvarieties is a torus. This is a very special case of a general conjecture on the structure of 'simple manifolds', central in the bimeromorphic classification of compact Kähler manifolds. The proof follows from the Brunella pseudo-effectivity theorem, combined with fundamental results of Siu and of the second author on the Lelong numbers of closed positive (1,1)-currents, and with a version of the hard Lefschetz theorem for pseudo-effective line bundles, due to Takegoshi and Demailly-Peternell-Schneider. In a similar vein, we show that a normal compact and Kähler 3-dimensional analytic space with terminal singularities and nef canonical bundle is a cyclic quotient of a simple non-projective torus if it carries no effective divisor. This is a crucial step to complete the bimeromorphic classification of compact Kähler 3-folds

[11] With Andrey Soldatenkov

Holomorphic Lagrangian fibrations on hypercomplex manifolds// arXiv:1301.0175

A hypercomplex manifold is a manifold equipped with a triple of complex structures satisfying the quaternionic relations. A holomorphic Lagrangian variety on a hypercomplex manifold with trivial canonical bundle is a holomorphic subvariety which is calibrated by a form associated with the holomorphic volume form; this notion is a generalization of the usual holomorphic Lagrangian subvarieties known in hyperkaehler geometry. An HKT (hyperkaehler with torsion) metric on a hypercomplex manifold is a metric determined by a local potential, in a similar way to the Kaehler metric. We prove that a base of a holomorphic Lagrangian fibration is always Kaehler, if its total space is HKT. This is used to construct new examples of hypercomplex manifolds which do not admit an HKT structure.

3.1.19 Ilya Vyugin

[1] (With R.R. Gontsov) Solvability of linear differential systems in the Liouvillian sense *arXiv:1312.2518* , 2013, (submited to journal).

The paper concerns the solvability by quadratures of linear differential systems, which is one of the questions of differential Galois theory. We consider systems with regular singular points as well as those with (non-resonant) irregular ones and propose some criteria of solvability for systems whose (formal) exponents are sufficiently small.

[2] On the linear independence of some system of functions
Proceedings of the young mathematician conference, 2013, p. 14-18 (an extended version will be submited to journal soon).

The question of linear independece of some system of functions has been studied. The system consists of products of the functions $x^{\alpha t}$ and powers of fundumental solutions $y_i^{\alpha_i}(x)$ of some linear differential equation. Such system is linearly independent if t is sufficiently large. The estimate of t has been obtained.

3.1.20 Alexei Zykin

[1] Asymptotic properties of zeta functions over finite fields
arXiv:1310.6107 *submitted to Israel Journal of Mathematics*

In this paper we study asymptotic properties of families of zeta and L-functions over finite fields. We do it in the context of three main problems: the basic inequality, the Brauer–Siegel type results and the results on distribution of zeroes. We generalize to this abstract setting the results of Tsfasman, Vlăduţ and Lachaud, who studied similar problems for curves and (in some cases) for varieties over finite fields. In the classical case of zeta functions of curves we extend a result of Ihara on the limit behaviour of the Euler–Kronecker constant. Our results also apply to L-functions of elliptic surfaces over finite fields, where we approach the Brauer–Siegel type conjectures recently made by Kunyavskii, Tsfasman and Hindry.

3.2 Scientific conferences and seminar talks

3.2.1 Ivan Arzhantsev

[1] Conference "Christmas meetings with Pierre Deligne", Moscow, January 8-11, 2013

Talk ”The automorphism group of a variety with torus action of complexity one”

[2] 2nd Swiss-French Workshop on Algebraic Geometry, Enney, Switzerland, February, 2013,

Lecture Course ”Homogeneous Spaces and Cox Rings” (5 lectures)

[3] Visit to the University of Bern, Switzerland, February 23-26, 2013,

Colloquium Talk ”Cox Rings, Universal Torsors, and Infinite Transitivity”

[4] Visit to the Eberhard Karls University, Tuebingen, Germany, February 2013

[5] Conference ”Combinatorial Algebraic Geometry”, Levico Terme, Italy, June 10-15, 2013

[6] Conference ”Algebraic Topology and Abelian Functions”, Moscow, June 18-22, 2013

Talk ”Infinite transitivity on universal torsors”

[7] 13th Summer School ”Modern Mathematics”, Dubna, Moscow, July 2013,

Lecture Course ”Automorphisms of Affine Spaces” (4 lectures)

[8] 8th Summer School Achievements and Applications of Contemporary Informatics, Mathematics and Physics (AACIMP-2013), Kiev, Ukraine, August 2013,

Lecture Course ”Linear Programming: Theory and Applications” (4 lectures)

[9] Visit to Institute Fourier, Grenoble, France, December 14-18, 2013,

Talk ”Infinite transitivity on universal torsors”

3.2.2 Vladimir Bogachev

[1] ERC School on Geometric Measure Theory and Real Analysis, Centro di Ricerca Matematica Ennio De Giorgi, Scuola Normale Superiore di Pisa, Italy, invited lecturer, series of lectures ”Sobolev classes over infinite-dimensional spaces with measures”, October, 2013,

[2] ERC Workshop on Geometric Measure Theory, Analysis in Metric Spaces and Real Analysis, Centro di Ricerca Matematica Ennio De Giorgi, Scuola Normale Superiore di Pisa, Italy, invited speaker, talk ”Extensions of Sobolev functions and BV functions in infinite dimensions”, October, 2013,

[3] The Meeting of the Mathematical Section of the Moscow Scientist's Club dedicated to Gelfand's centennial, invited speaker, talk "Gelfand's work in functional analysis", October, 2013,

[4] A Joint Meeting of the Sankt Petersburg Mathematical Society and the Mathematical Section of the Sankt Petersburg Scientist Club dedicated to the 125th birthday of Fichtenholz, invited speaker, talk "Mathematical research of Fichtenholz", November, 2013,

[5] Visit to the University of Bielefeld, Germany, International seminar "Infinite-dimensional stochastic analysis", talk "Nonlinear Fokker–Planck–Kolmogorov equations for measures", August, 2013.

3.2.3 Yurii Burman

[1] The 2013 Meeting of the Israel Mathematical Union, Haifa, June 16.

Talk "Higher matrix-tree theorems".

3.2.4 Alexei Elagin

[1] International Conference dedicated to the 90th anniversary of I.R.Shafarevich, Moscow, 2013, June 3-5.

[2] International Conference "Geometry of algebraic varieties" dedicated to the memory of V.A.Iskovskikh, Moscow, 2013, October 22-25.

[3] Christmas meetings with Pierre Deligne, Moscow, 2013, January 8-11.
Talk "Coherent sheaves on 1-dimensional stacks"

3.2.5 Alexei Gorodentsev

[1] XII International school in theorethical and mathematical physics, Sevastopol, Ukraine, May, 1 – 11 2013.
Site: `http://wwwth.itep.ru/mathphys/conf/kiev-2013/`.
Series of lectures "Massives and representations of classical groups".
Supervision of sections "Homological algebra" and "Riemann–Roch theorem"

[2] IV International conference "Geometry and Quantization GEOQUANT 2013", ESI, Vienna, Austria, August 19 – 30, 2013.
Site: `http://math.uni.lu/geoquant2013/`.
Talk "Mukai lattices: known structures and open questions", transparencies:
`http://gorod.bogomolov-lab.ru/ps/math/mukai_lattices_2013_08_27_Wien.pdf`

3.2.6 Maxim Kazarian

[1] School-Conference "Contemporary problems of mathematics", IMM, Ekaterinburg, February 25 – March 02

Mini-course "Symplectic geometry"

[2] Conference "Integrable Systems and Moduli Spaces", BIRS, Vancouver, Canada, August, 25 – August, 30

Talk "Symplectic Geometry of topological recursion"

[3] School-Conference "Analytic, probabilistic, and algebraic methods in mathematical physics", St.Petersburg, Repino, December 8 – December 13

Mini-course "Mathematical physics of Hurwitz numbers"

[4] Visit to Dijon University, France, March 05 – Marcch 30,

Talk "Topological recursion for the genus zero descendant Hurwitz potential"

[5] Talks at Moscow HSE seminars Talk "Topological recursion for the genus zero descendant Hurwitz potential" (Math department of HSE)

Talk "Symplectic Geometry of topological recursion" (Math department of HSE)

3.2.7 Alexander Kuznetsov

[1] Conference "Higher Dimensional Algebraic Geometry", Tokyo, January 7–11, 2013.

Talk "Categorical resolutions of singularities"

[2] Conference "Géométrie Algébrique et Géométrie Complexe", Luminy, January 28–February 1, 2013.

Minicourse "Derived categories of coherent sheaves in birational geometry"

[3] Conference "Birational Geometry and Geometric Invariant Theory", Vienna, May 21–24, 2013.

Talk "On a noncommutative blowup"

[4] Conference "Géométrie Algébrique en Liberté ", Stockholm, June 24–28, 2013.

Minicourse "Derived Categories of Coherent Sheaves"

[5] Conference "Geoquant 2013", Vienna, August 19–30, 2013.

Minicourse "Derived Categories of Coherent Sheaves and Moduli Spaces"

[6] Conference "Geometry of Algebraic Varieties", Moscow, October 22–25, 2013.

Talk "Higher blowups"

[7] Visit to SISSA, Trieste, August 2013.

3.2.8 Karine Kuyumzhiyan

[1] Combinatorial Algebraic Geometry, Levico Terme, Italy, June 9-15, 2013, "Normality of maximal torus orbits closures in simple modules of algebraic groups"

[2] Second International Conference “Mathematics in Armenia: Advances and Perspectives”, Tsaghkadzor, Armenia, August 25-31, 2013, ”Normality of maximal torus orbits closures in simple modules of algebraic groups”

[3] Christmas meetings with Pierre Deligne, Independent University of Moscow, January 8-11, 2013, “Unirationality and existence of infinitely transitive birational models”

3.2.9 Grigori Olshanski

[1] Visit to Paris, June 23-29, 2013

Invited talk “Boundaries of branching graphs” at the 25th International Conference on Formal Power Series and Algebraic Combinatorics (Université Paris 7).

[2] Conference “Israel Gelfand Centenary (1913-2009)”, Moscow, July, 20–25.

Talk “Representations and probability”

3.2.10 Alexei Penskoi

[1] Conference / Summer School ”XXXII Workshop on Geometric Methods in Physics / The School on Physics and Geometry”, Białowieża, Poland, June 30 – July 13, 2013

Lectures “Spectral Geometry and Mathematical Physics”

[2] Visit to Centre de Recherche Mathématiques, Montréal, Canada, February-March

Talk “Constructing explicitly parametrized minimal tori in spheres via Takahashi’s theorem” at “Analysis Seminar (McGill & Concordia)” (McGill University)

3.2.11 Yuri Prokhorov

[1] Workshop on Algebraic Geometry, 22-24 May, 2013, Imperial College, London

Talk: “On threefold extremal contractions”

[2] Symposium on Projective Algebraic Varieties and Moduli, February 18 - 21, 2013, Yeosu, Korea

Invited Talk: “Cylindricity of Fano varieties”

[3] Workshop on Algebra Combinatorics Dynamics and Applications, September 2-5, 2013, Belfast, UK

Talk:“Jordan property for groups of birational selfmaps”

[4] Birational geometry and Galois groups, June 10-14, 2013, University of Edinburgh, UK

Talk: "On stable conjugacy of finite subgroups of the plane Cremona group"

[5] Edge Days, June 7-9, 2013, University of Edinburgh, UK

Talk: "G-Fano threefolds and Cremona groups"

[6] Annual memorial conference dedicated to the memory of A. N. Tyurin, October 28, 2013, Steklov Institute, Moscow

Talk: "Cylindrical property of Fano varieties"

[7] Workshop "Quantum and motivic cohomology, Fano varieties and mirror symmetry", Euler Institute, St. Petersburg, 26-28 September, 2013

Talk: "Automorphisms of Fano varieties and Cremona groups"

[8] International conference dedicated to the 90th anniversary of I. R. Shafarevich, June 3-5, 2013, Steklov Institute, Moscow

Talk: "On stable equivalence of subgroups of Cremona groups"

[9] Visit to International Centre for Theoretical Physics, Trieste, Italy, August 2013

Talk: "Cremona Groups" at Mathematics Seminar, ICTP, 6 August, 2013

`http://cdsagenda5.ictp.trieste.it/full_display.php?ida=a13273`

[10] Visit to National Taiwan University, Taipei, April 27 – May 15, 2013

[11] Visit to RIMS, Kyoto, February 2013

Talk: "Boundedness results for birational automorphisms" at Algebraic Geometry Seminar, Kyoto University, February 15, 2013

`http://www.math.kyoto-u.ac.jp/~abetaku/agseminar.htm`

[12] IUM general seminar "Globus", April 4, 2013

Talk: "Groups of birational automorphisms, and minimal models"

`http://mccme.cde.ru/video/Globus-prohorov-part1-401-2013-4-4-15-43-17.1mbps.mp4`

[13] Meetings of the Moscow Mathematical Society, March 26, 2013

Talk: “Cremona Group and its subgroups”

http://www.mathnet.ru/php/seminars.phtml?option_lang=eng&presentid=6467

[14] Grour theory seminar, Moscow State University, April 5, 2013

Talk: “Cremona group”

http://halgebra.math.msu.su/groups//

[15] Seminar of the Department of Algebra and Number Theory and of the Department of Algebraic Geometry (Shafarevich Seminar), September 17, 2013

Talk: “On stable conjugacy of finite subgroups in the Cremona group”

http://www.mathnet.ru/php/seminars.phtml?option_lang=eng&presentid=7538

[16] Scientific session of the Steklov Mathematical Institute dedicated to the results of 2013 November 20, 2013

Talk: “G-Fano threefolds”

http://www.mathnet.ru/php/presentation.phtml?option_lang=eng&presentid=7836

3.2.12 Petr Pushkar’

[1] Talk “Morse complexes” on seminar on Singularity theory at Math deparment of HSE (spring 2013)

[2] Talk “On a dencity of Morse functions” on seminar on Morse theory and Smooth manifolds at Math deparment of HSE (october 2013)

3.2.13 Sergei Rybakov

[1] Conference “Algebraic Geometry and Coding Theory 14”, Marseille, June, 3 – 7, 2013 Talk “Groups of points on abelian varieties over finite fields”

[2] Conference “Algebraic Geometry and Coding Theory, India”, Mumbai, December, 2 – 6, 2013

[3] Conference “Global fields”, Moscow, September 2 - 6, 2013

[4] Conference “Diophantine Geometry”, Moscow, May 13 - 17, 2013

3.2.14 Arkady Skopenkov

[1] Conference of Moscow Institute of Physics and Technology, November, 2013
Talk “Classification of knotted tori”

[2] Postnikov Memorial Seminar, Moscow State University,
Talk “Classification of knotted tori”

[3] Topology Seminar, Faculty of Mathematics, Higher School of Economics,
Talk “Classification of knotted tori”

[4] Seminar ‘Lie algebras, Rimann surfaces and mathematical physics’, The Independent University of Moscow,
Talk “Classification of knotted tori”

3.2.15 Mikhail Skopenkov

[1] International conference “Discrete curvature”, Marseille, France, 18.11-22.11.
Talk “Discrete Riemann surfaces: convergence results”
[2] International conference ”Multidimensional continued fractions”, Graz, Austria, 22.06-26.06.
Talk “Tiling of a rectangle, alternating current, and continued fractions”.
[3] I.M. Gelfand Centennial Conference, Moscow, 22.07-25.07.
Talk “Discrete complex analysis: convergence results”
[4] Christmas mathematical meetings of the “Dynasty” Foundation, Moscow, 8.01-11.01.
Talk “Discrete analytic functions: convergence results”.
[5] Algebra and number theory, Saratov, 9-14.09.
Talk “Triangulations of surfaces by circular arcs”
[6] Visit to Technical University of Vienna, Austria, 2-5.07.
[7] Visit to Technical University of Linz, Austria, 4.07.
[8] Talks at several seminars in Moscow.

3.2.16 Evgeni Smirnov

[1] Maurice Auslander Distinguished Lectures and International Conference, Woods Hole, MA, USA, April 18–23, 2013
Talk “Schubert calculus and Gelfand–Zetlin polytopes
[2] International Conference on Algebraic Geometry and Coding Theory, Mumbai, India, December 2–6, 2013
Talk “Schubert decomposition for double Grassmannians”

[3] Visit to University of Edinburgh, Scotland, September 21–25, 2013
Mini-course “Grassmannians and flag varieties”
[4] Visit to St. Petersburg Department of Steklov Mathematical Institute
Talk “Plane curves and Lagrangian subspaces”, December 13, 2013

3.2.17 Stanislav Shaposhnikov

1)The Fourth International Conference dedicated to Corresponding Member of RAS, Member of European Academy of Sciences, Professor L.D.Kudryavtsev on the occasion of his 90th anniversary, talk “Nonlinear Fokker–Planck–Kolmogorov equations”, March, 2013.

2) Visit to the University of Bielefeld, Germany, International seminar “Infinite-dimensional stochastic analysis”, talk “Nonlinear Fokker–Planck–Kolmogorov equations for measures”, August, 2013.

3.2.18 Mikhail Verbitsky

[1] International Conference on Analytic and Algebraic Geometry related to bundles 18.03.2013 - 22.03.2013, India, Mumbai

Talk: “Instanton bundles on P3 and rational curves in twistor spaces of Nakajima quivers”

[2] International Conference “Moduli Spaces and their Invariants in Mathematical Physics”. Montreal, 3.06.2013 - 14.06.2013

Talk: “Holography principle and Moishezon twistor spaces”

[3] Workshop on Deformation and Moduli in Complex Geometry 25.03.2013 - 29.03.2013, Korea, Seoul,

Talk: “Ratner theorem and ergodic complex structures on hyperkaehler manifolds”

[4] Workshop ”Symplectic Algebraic Geometry” 30.09.2013 - 4.10.2013, Kyoto, Japan

Talk: “Ergodic complex structures and Kobayashi metric”

[5] The 6th meeting of ”The locally free geometry seminars” 8.03.2013 - 8.03.2013, Spain, Madrid

Talk: “Ergodic complex structure on hyperkahler manifolds”

[6] Special Lectures in Real and Complex Geometry 11.07.2013 - 15.07.2013, Tel-Aviv University, Israel

Two talks:

(a) “Global Torelli theorem for hyperkaehler manifolds

(b) “Ergodic complex structures”

3.2.19 Ilya Vyugin

[1] Workshop “Formal and Analytic Solutions of Differential, Difference and Discrete Equations”, Warsaw, 25.08.2013-31.08.2013,

Talk “On solvability of linear differential systems by quadratures” (joined with R.R. Gontsov).

[2] International conference dedicated to the centenary of Israel Gelfand, Moscow, 22.07.2013-15.07.2013,

Talk “Linear independence of functions and intersections of subsets of $\mathbb{Z}_p$”.

[3] Conference “Geometric Days in Novosibirsk-2013”, Novosibirsk, 28.08.1013-1.09.1013,

Talk “Local form of solutions of some Painlevé equations”.

[4] Seminar “Analitic theory of differential equations” (D.V Anosov, V.P. Lexin), MIAN RAS

Talk “Linear independence of functions and the orders of zeros”.

3.2.20 Alexei Zykin

[1] Conference “Arithmetic days 2013”, St. Petersburg, May, 20 – 23
Talk: “Asymptotic properties of global fields and varieties over them”

[2] Visit to Institut de Mathématiques de Toulouse, 06.2012
Talk: “On M-functions of modular forms”

3.3 Teaching

3.3.1 Ivan Arzhantsev

[1] Linear Algebra and Geometry. Lomonosov Moscow State University, I year students, February-June 2013, 4 hours per week.

Program.

1. Finite-dimensional vector spaces. Linear functions and dual spaces. Subspaces and flags.
2. Linear operators. Eigenvalues and eigenspaces. The Jordan normal form.
3. Bilinear symmetric and skew-symmetric forms. Euclidean and Hermit spaces.
4. Tensor algebra. Symmetric and skew symmetric tensors. Interior algebra.

[2] Algebra-3. Independent University of Moscow, II year students, September-December 2013, 4 hours per week.

Program.

1. Basic commutative algebra and algebraic geometry. Noetherian rings and modules. Hilbert's Nullstellensatz. Products, tensor products and fiber products. Dimension. Integral extensions and finite morphisms.
2. Groebner bases and computational algebraic geometry.
3. Lie algebras and their representations. Semisimple Lie algebras, their structures and complete reducibility of representations.

[3] Algebra. Lomonosov Moscow State University, II year students, September-December 2013, 6 hours per week.

Program.

1. Finite group theory. Solvable and simple groups. Sylow's Theorems.
2. Representations of finite groups. Complete reducibility. Classification of complex irreducible representations. Characters and their applications.
3. Rings, algebras and fields. Ideals. Field extensions and roots of polynomials. Finite fields. Division algebras and Frobenius Theorem.

3.3.2 Vladimir Bogachev

[1] Gaussian measures. Moscow State University, 2-5 year students and PhD students, September – December 2013, 2 hours per week.

Program

1. Gaussian random variables.
2. Gaussian measures on finite-dimensional spaces.
3. Gaussian measures on infinite-dimensional spaces and Gaussian processes.
4. The Wiener process and Wiener measure.
5. Measurable linear functionals.
6. The Cameron–Martin space.
7. The Cameron–Martin formula.
8. The Wiener stochastic integral.
9. The Tsirelson theorem.

[2] Transformations and convergence of measures, conditional measures, and Rohlin's theory. Independent University of Moscow, 2-5 year students and PhD students, September – December 2013, 2 hours per week.

Program

1. Introductory remarks on basic problems of measures theory.

2. Borel sets. Borel measures.

3. Souslin sets and their basic properties. Non-Borel Souslin sets. Measures on Souslin sets.

4. The image of a measure under a mapping.

5. The measurable choice theorem.

6. Weak convergence of measures. The Alexandrov and Prohorov theorems.

7. The Kantorovich metric.

8. Conditional expectations and conditional measures.

9. Lebesgue – Rohlin spaces and Rohlin measurable partitions.

[3] Sobolev classes over infinite-dimensional spaces with measures, ERC School on Geometric Measure Theory and Real Analysis, Centro di Ricerca Matematica Ennio De Giorgi, Scuola Normale Superiore di Pisa, Italy, 4 lectures, October, 2013,

Program

1. Weighted Sobolev classes on finite-dimensional spaces.

2. Differentiable measures on infinite-dimensional spaces.

3. Sobolev classes over differentiable measures on infinite-dimensional spaces.

4. Functions of bounded variation on on infinite-dimensional spaces with measures.

3.3.3 Yurii Burman

[1] Differential geometry, Independent University of Moscow, 2nd year students, September to December 2013, 2 hours/week lectures and 2 hours/week exercise sessions.

Program.

1. Smooth manifolds and smooth mappings.

   Manifolds, submanifolds, smooth mappings and implicit function theorem. Paracompactness and partition of unity. Corollaries (the diffeomorphism group acts transitively on k-tuples of points, tangent and cotangent bundles are isomorphic, etc.)

2. Tangent bundle.

   Hadamard lemma, equivalent definitions of the tangent functor (tangent bundle of a manifold and derivative of a smooth mapping). Correctness of the initial value problem for a differential equation. Commutator of vector fields; commuting fields have commuting flows. Orientability.

3. Vector bundles.

   Main operations over vector bundles (inverse image, direct sum, tensor product). Feldbau lemma.

4. Transversality.

   Thom's transversality theorem. Every manifold possesses a Morse function. Morse lemma.

5. Differential forms.

   De Rham complex, inverse image of a form, Poincaré lemma. Integral of a form, Stokes' theorem. Lie derivative and the $\iota d + d\iota = \mathcal{L}$ identity.

6. Frobenius' theorem.

[2] Basic notions of mathematics, seminar for 1st year students, January to May and September to December 2013, Higher School of Economics.

Program (list of topics of the talks).

1. Cubic equations.
2. Quadratic reciprocity law.
3. Principal section of a 4-cube.
4. Projective plane.
5. Hilbert's third problem.
6. p-adic numbers.
7. Euler's pentagonal theorem.
8. Tangent vector fields on a sphere.
9. Sharkovskiy's theorem.
10. Trees and parking functions.
11. Families of lines in a 3-space.
12. $\mathrm{SO}(3) = \mathbb{R}P^3$.
13. Electrical networks and the matrix-tree theorem.

3.3.4 Alexei Elagin

[1] Algebraic geometry - 1. Independent University of Moscow & Moscow Institute of Physics and Technology, III year students/master students, September-December 2013, 2 hours per week.

Program:

[1] Commutative rings, ideals, prime and maximal ideals. Quotient-rings. Radical of a ring. Intersection and sum of ideals. Null-radical. Integral domains.

[2] Spectrum of a ring, Zariski topology.

[3] Ring homomorphisms, contraction and extension of ideals, of prime ideals. The induced morphism on spectra.

[4] Affine algebraic varieties, subvarieties, regular mappings. Correspondence between algebraic and geometric points of view.

[5] Points of variety and maximal ideals, residue field. Nullstellensatz.

[6] Finite field extensions. Algebraic elements, their minimal polynomials.

[7] Finite fields.

[8] Modules. Submodules and quotient-modules. Direct sums. Kernels, theorems on homomorphism, Chinese remainder theorem.

[9] Tensor product of modules. Universal bilinear map.

[10] Exact sequences. Hom and tensor product as functors. Projective and injective modules. Exact functors.

[11] Quotient field of a ring. Rational functions, functions regular on a subvariety. Localization of rings. Ideals behavior under localization.

[12] Localization of modules. Localization as exact functor. Local properties of rings and modules.

[13] Finitely generated modules. Noetherian rings and modules. Hilbert's basis theorem.

[14] Integral and finite extensions of rings. Rings of algebraic integers. Finite coverings of varieties. Fibers of finite morphisms of spectra.

[15] Primary ideals and primary decomposition. Primary decomposition in 1-dimensional domains. Dedekind domains.

Topics of exercise sheets:

[1] Rings and ideals

[2] Spectrum of a ring

[3] Homomorphisms of rings

[4] Affine algebraic varieties

[5] Finite fields

[6] Field extensions

[7] Modules

[8] Exact sequences

[9] Localization

[10] Modules - 2

[11] Finite ring extensions

See *http://ium.mccme.ru/f13/elagin-f13.html* for the lecture drafts and exercise sheets.

3.3.5 Alexei Gorodentsev

[1] Some geometric faces of algebra and algebraic faces of geometry. São Carlos branch of the Universidade de São Paulo (USP, Brasil). III-VI year students, July 2013, 6 hours per week, totally 4 weeks.
Site: `http://gorod.bogomolov-lab.ru/ps/stud/geom_sao-carlos/list.html`.
Lecture notes:
`http://gorod.bogomolov-lab.ru/ps/stud/geom_sao-carlos/lecture_notes.pdf`

Program:

[1] Fields (especially finite ones). Vector spaces and bases: samples of usage. Could a field of 27 elements contain a subfield of 9 elements?

[2] Points and figures. Affine space. Polynomial functions. Algebraic varieties. Given $\dim V$, find $\dim S^n V$.

[3] Projective space. Cell decomposition and affine charts. Topological degression: $\mathbb{RP}_1 = S^1$, $\mathbb{CP}_1 = S^2$, $\mathbb{RP}_2 = D^2 \underset{S^1}{\#} \texttt{Möbius tape}$, $\pi_1(\mathrm{SO}_3 = \mathbb{RP}_3) = \mathbb{Z}/(2)$. Projective varieties.

[4] Space of hypersyrfaces. Basic example: $S^d\mathbb{P}_1 = \mathbb{P}_d$. Veronese curve. How to present Veronese varieties by quadratic equations

[5] Projections. Complementary subspaces and joins. Projecting Quadric, rational parametrization of conic. Projecting Veronese curve: rational curves, smooth cubic is not rational.

[6] Projective linear group, especially $\mathbb{P}\mathrm{GL}_2$. Coss-ratio, quadrangle, the epimorphism $S_4 \twoheadrightarrow S_3$, harmonic pairs of points. Homographies: games with lines, games with conics, Pascal's theorem, Ponselet's porism. Space of conics, pencils of conics.

[7] Projective quadrics: polarization, correlation, tangent space, smoothness, polar mapping. Linear subspaces on a smooth quadric. Space of quadrics. Segre's quadric in $\mathbb{P}_3$, Schläflische doppelsechs. How many lines do cross 4 given mutually skew lines in 3D-space?

[8] Orthogonal geometry of quadratic form over an arbitrary field. Orthogonal group is spanned by reflections. Orthogonal decomposition into hyperbolic and anisotropic parts.

[9] Tensor products and Segre varieties. Tensor algebra of a vector space: contractions, linear span of a tensor.

[10] Grassmannian algebra. Linear change of basis and Laplace relations. Grassmannian quadratic forms. Polarization and partial derivatives. Linear span of skew polynomial and Plücker relations.

[11] $\mathrm{Gr}(2,4)$ and lines in $\mathbb{P}_3$. Plücker quadric in $\mathbb{P}_5$. Planes and lines on the Plücker quadric. Schubert cells.

[12] Plücker–Segre–Veronese interaction and spinors for SO_4 and SO_6 .

[13] Spinors for SO_5 and the lagrangian grassmannian $\mathrm{LGr}(2,4)$

[14] Grassmannians in general: cell decomposition, affine charts, Plücker embedding, introduction to the Schubert calculus.

[15] Barycentric combinations, barycentric coordinates as local affine coordinates on projective space. Convexity. Interaction with topology

[16] Supporting hyperplanes and properties of affine-linear maps.

[17] Convex figures: faces and extreme points. Convex hulls vs intersections of half-spaces.

[18] Compact convex polytopes, Minkowski–Weyl theorem.

[19] Convex polyhedral cones, Farkas lemma and variations.

[20] Gale duality and variations.

[21] Geometric aspects of linear optimization.

[2] Algebra-1. Independent University of Moscow, I year students, September–December 2013, 4 hours per week.
Site: `http://gorod.bogomolov-lab.ru/ps/stud/algebra-1/1314/list.html`.
Lecture notes:
`http://gorod.bogomolov-lab.ru/ps/stud/algebra-1/1314/lec-total.pdf`

Program:

[1] Sets and maps. Binary relations. Multinomial coefficients, Young diagrams and other combinatorics.

[2] Definitions and simple properties of abelian groups, commutative rings, and fields and their homomorphisms. Direct products. Rings and fields $\mathbb{Z}/(m)$. Chinese remainder theorem, GCD and coprime elements in $\mathbb{Z}$. Simple subfield, characteristic, Frobenius mapping.

[3] Polynomials and formal power series. Chinese remainder theorem, GCD and coprime elements in $\Bbbk[x]$. Roots of polynomials. Rings and fields $\Bbbk[x]/(f)$. Complex numbers. Finite fields. Quadratic reciprocity.

[4] Rings and fields of fractions. Rational functions: partial fraction and power series expansions, linear recursions. Exponent, logarithm, binomial. The Todd series and the Bernoulli numbers. Laurent and Puiseux series, Gensel's lemma and Newton's method.

[5] Ideals and factor rings. Noetherian rings, Hilbert's theorem about a basis of an ideal. Euclidean rings and PIDs and. Factorial rings, $\Bbbk[x]$ is factorial. Factorization in $\mathbb{Z}[x]$.

[6] Vector spaces, bases, dimnsion. Linear maps: kernel, image and their dimensions. Subspaces: sums, intersections and their dimensions. Direct sums vs direct products. Affine spaces and their affine subspaces. Factor spaces and linear spans.

[7] Dual spaces, dual bases, $V^{**} \simeq V$. Duality between annihilators. Dual operators, rank of a matrix. Gauss method.

[8] Algebras over a field. Matrix algebra. Matrices of linear maps. Inverse matrices. Matrices over non-commutative rings.

[9] Volume and multilinear skew forms. Sign of a permutation. Determinant. Grassmannian polynomials and minors, the Laplace relations. Adjugate matrix. Cramer's rules.

[10] Modules over commutative rings. Generators and relations, torsion, decomposability. Homomorphisms, the Cayley-Hamilton identity. Factor modules. Rank of a free module. Finitely generated modules over PIDs. Elementary divisors. Finitely generated abelian groups.

[11] Classification of vector spaces equipped with a linear endomorphism. Jordan's normal form and Jordan's decomposition. Commuting operators. Analytic functions of operators..

[3] Geometric introduction to algebraic of geometry. Faculty of mathematics at HSE. II-V year students, September–December 2013, 4 hours per week.
Site: `http://gorod.bogomolov-lab.ru/ps/stud/giag_ru/list.html`.
Lecture notes: `http://gorod.bogomolov-lab.ru/ps/stud/giag_ru/giag.pdf`

Program:

[1] Projective spaces and projective varieties.

[2] Working example: Plücker–Segre–Veronese interaction.

[3] Working example: Grassmannians.

[4] Projective quadrics.

[5] Tensor guide: polarizations and contractions.

[6] Commutative algebra draught: integer extensions, Hilbert's theorems, transcendence degree.

[7] Affine algebraic varieties, affine algebraic – geometric dictionary. Zarisski topology.

[8] Rational functions. Dominant morphisms. Finite morphisms.

[9] Algebraic manifolds.

[10] Dimension.

[11] Working example: lines on surfaces.

[12] Abstract nonsense appendix.

[13] Vector bundles, Picard group. Linear systems.

[14] Coherent sheaves.

3.3.6 Maxim Kazarian

[1] Calculus. Independent University of Moscow, I year students, September-December 2013, 2 hours per week.

Program

Rational and real numbers.

Limit of a sequence, limit of a series. *Fundamental sequences, metric spaces.*

Topology of a real line. *Open and closed subsets, compact spaces, connectness.*

cardinality of a set. *Countable sets. Continuum.*

Continuous functions. *Inverse function, Maximum and minimum, intermediate value theorem, elementary functions.*

Power series. *Radius of convergency. Cauchy and d'Alembert formulas.*

Derivative. *Derivative of a product, quotient, inverse function, finite increment formula, Taylor expansion.*

[2] Differential Geometry, Higher School of Economics, III year students, September-December 2013, 4 hours per week.

Program.

Vector fields and differential forms. *Lie bracket, exterior differential and wedge product, distributions, Frobenius criterium of integrability.*

Plane and space curves. *Length, curvature, focal set of a plane curve, normal and geodesic curvature of a space curve on a surface.*

Surface geometry. *Riemann structure, IInd quadratic form, principal curvatures, Gaussian curvature.*

Gauss' Theorema Egregium. *Connection and curvature forms of a metric on a surface, Euclidean coordinates of a flat metric.*

Topological connection. *Fiber bundles, trivializations, parallel transport, curvatur as an infinitesimal holonomy.*

Covariant derivative. *Vector bundles, sections, connection matrix, structure Cartan equation, curvature tensor.*

Riemann manifolds. *Levi-Civita connection, Riemann tensor, geodesics.*

3.3.7 Alexander Kuznetsov

[1] "Enumerative geometry on the projective plane", summer school "Modern Mathematics", Dubna, July 2013, 5 lectures.

The problem of Apollonius on the number of circles that are tangent to three given circles dates back to third centuary BC. Now it is a school level problem and the maximal possible answer is wel known to be 8. Actually, from the algebraic point of view the number of solutions is always 8, the only thing that happens is that sometimes some solutions are complex (e.g. when one circle lies inside the other).

A natural algebro-geometric generalization of Appolonius' problem is the question about the number of degree 2 curves on a complex projective plane that are tangent to 5 given curves of degree 2 (the number of curves increased from 3 to 5 because generic curve of degree 2 depends on 5 parameters). This problem, however, turns out to be unexpectedly nontrivial. A naive computation based on Bezout Theorem gives 7776 as the answer, but this answer is wrong!

The reasons behind this phenomenon as well as means of dealing with them and getting the right answer will be discussed during the course. Among other things we will discuss the basic notions of algebraic geometry — projective space, projective variety, the Veronese variety, the blowup of a subvariety, cohomological intersection theory and others.

More detailed information is on the course web-page

`http://www.mccme.ru/dubna/2013/courses/kuznetsov.htm`

[2] "DG-categories", Science-educational center, Steklov Mathematical Institute, Fall 2013, 3 hours per week.

Program

- DG-algebras, DG-categories, and DG-functors.
- DG-modules.
- Trianguated structure on the homotopy category of DG-modules.
- Derived category of DG-modules. Homotopically projective and injective DG-modules.
- Perfect DG-modules, twisted complexes, tensor products.
- Derived tensor product and RHom.
- Drinfeld's quotient.
- Drinfeld's quotient. Enhancements of derived categories.
- H-injective and h-flat enhancements of the derived category of sheaves.
- Properness and smoothness of DG-categories. Hochschild homology and cohomology.
- Gluing of DG-categories. DG-categories of orbits.

More detailed information is on the course web-page

`http://www.mi.ras.ru/~akuznet/dgcat/index-dgcat-eng.htm`

3.3.8 Karine Kuyumzhiyan

[1] Lie Groups and Lie Algebras. Independent University of Moscow, II year students, February-May 2013, 3 hours per week, lectures and seminars.

Program

[1] Main definitions and examples: Lie group, Lie subgroup, Lie groups homomorphism, representations and actions of Lie groups.

[2] Orbits and stabilizers. Smooth structure on the set of cosets. Quotient groups.

[3] Left- and right-invariant tensors on a Lie group. Existence of the invariant volume form on a compact Lie group.

[4] Four ways to define a Lie algebra of a Lie group. Adjoint representation.

[5] Tangent homomorphism and tangent representation. Existence and uniqueness theorems for the homomorphisms of Lie groups.

[6] Exponential map. Description of all connected Lie groups with the given Lie algebra.

[7] Main classes of Lie groups and Lie algebras: solvable, nilpotent, simple, semisimple.

[8] Structure theory of semisimple Lie algebras: Cartan subalgebra, Cartan-Killing form, root system, Cartan matrix, Dynkin diagram.

[9] Classification of simple Lie algebras. Chevalley basis, Serre relations. Isomorphisms in small dimensions.

[10] Levi and Maltsev theorems.

[11] Introduction to the representation theory of Lie algebras: fundamental representations. Universal enveloping algebra. Representations of $\mathfrak{sl}(2)$. Highest weight representations. Weyl character formula.

[2] Probability Theory and Mathematical Statistics, Higher School of Economics, Management department, I year students, January-June 2013, 2 hours per week.

[3] Calculus III. Higher School of Economics & New Economic School, II year students, September-December 2013, 2 hours per week, seminars.

[4] Discrete Mathematics. Higher School of Economics, Philology department, I year students, September-December 2013, 4 hours per week, seminars.

3.3.9 Grigori Olshanski

[1] Representations and Probability. Independent University of Moscow, January–May 2013, 2 hours per week.

Program

Seven lectures at the research seminar on combinatorial, algebraic, and probabilistic aspects of representation theory.

[2] Representations and Probability. Independent University of Moscow, September-December 2013, 2 hours per week.

Program.

Asymptotic properties of the uniform distributions on the large symmetric groups. The space of virtual permutations. The Kingman graph and Gibbs measures on the path space. Kingman's theorem about the classification of partition structures. The Poisson-Dirichlet distributions. Introduction to harmonic analysis on the infinite symmetric group. Diffusion processes on the Kingman simplex.

[3] Analysis, Probability, Algebra. National Research University Higher School of Economics, September-December 2013, 2 hours per week.

Program

Projections of simplices and B-splines. The Weil algebra and its quotient skew field. De Finetti's theorem, Hausdorff moment problem, and Bernstein's polynomials. What is the invariant measure on the infinite-dimensional sphere? The Robinson-Schensted correspondence. The Plancherel measure on Young diagrams.

3.3.10 Alexei Penskoi

[1] Lectures and Exercise classes on Differential Geometry, Independent University of Moscow, 2nd year students, February-May 2013, 4 hours per week (lecture 2 hours + exersise class 2 hours).

Program.

[1] Curves and surfaces in the plane and the three-dimensional space. Curvature, torsion, Frenet frame. First and second fundamental forms. Principal curvatures, mean curvature and Gauß curvature. Mean curvature normal vector. Euler formula for the normal section curvature.

[2] Surfaces in n-dimensional space. First and second fundamental forms. Connections in the tangent and normals bundles on a surface. Second fundamental form and Weingarten operator. Gauß-Weingarten derivational equations. Gauß-Bonnet theorem for surfaces.

[3] Basic theory of Lie groups ans algebras.

[4] Vector bundles and gluing cocycles. Structure group. Euclidean and hermitian bundles. Natural operations with bundles. Orientable bundles.

[5] Connections in vector bundles. Connection local form, Christoffel symbols. Connections in euclidean and hermitian bundles. Connections compatible with metrics and their curvature.

[6] Riemannian manifolds. Curvature, torsion. Levi-Civita connection. Symmetries of curvature tensor. Ricci tensor. Scalar curvature.

[7] Riemannian manifolds II. Geodesics. Geodesic coordinates. Lagrangian approach to geodesics. Second variation.

[8] Submanifolds of Riemannian manifolds. First and second fundamental forms.

[9] Laplace-Beltrami operator and minimal submanifolds, Takahashi theorem.

[10] Characteristic classes. Chern-Weil construction of characteristic classes. Chern, Pontryagin and Euler classes and their properties.

[2] Lectures on Spectral Geometry, Independent University of Moscow, ≥3rd year students, September-December 2013, 2 hours per week.

Program.

[1] Laplace-Beltrami operator on Riemannian manifolds

[2] Eigenvalues of Laplace-Beltrami operator (Dirichlet problem, Neumann problem, problem on manifolds without boundary)

[3] Variational desription of eigenvalues, Rayleigh quotient

[4] Weyl function and its asymptotics

[5] Inequalities for eigenvalues, Dirichlet-Neumann bracketing

[6] Nodal domains, Courant nodal domain theorem

[7] Isoperimetric inequalities, symmetrization

[8] Cheeger isoperimetric constant, Cheeger inequality

[9] Geometric optimization of eigenvalues, extremal metrics

[10] Conformal volume, Yang-Yau inequality

[11] Extremal metrics and minimal submanifolds of spheres

[3] Partial Differential equations. Math in Moscow program of the Independent University of Moscow for undergraduate students from the U.S. and Canada, February-May 2013, 4 hours per week (lecture 2 hours + exersise class 2 hours).

[1] The string equation. Bounded and unbounded domains, Fourier approach and integral formulas.

[2] 1st order PDEs

[3] 2nd order PDE canonical form.

[4] Generalized functions (distributions). Convolution. Foundamental solutions.

[5] Fourier transform.

[6] Laplace operator, eigenvalues and eigenfunctions.

[7] Wave equation.

[8] Heat equation.

[9] Discontinuity of solutions and characteristics.

[4] Differential Geometry. Math in Moscow program of the Independent University of Moscow for undergraduate students from the U.S. and Canada, September-December 2013, 4 hours per week (lecture 2 hours + exersise class 2 hours).

[1] Plane and space curves. Curvature, torsion, Frenet frame.

[2] Surfaces in 3-space. Metrics and the second quadratic form. Curvature.

[3] Connections in tangent and normal bundles to a k-surfaces in $\mathbb{R}^n$.

[4] Parallel translations.

[5] Geodesics.

[6] Gauß and Codazzi formulas. “Theorema egregium” of Gauß.

[7] Gauß-Bonnet theorem.

[8] Extremal properties of geodesics. Minimal surfaces.

[9] Vector bundles, connection in vector bundles.

[10] Levi-Civita connection.

[11] Connection curvature. Riemann curvature tensor.

[5] Exercise classes for various courses at Moscow State University: Classical Differential Geometry, February-May 2013, 2 hours per week; Analytic Geometry, September-December 2013, 4 hours per week; Differential Geometry, September-December 2013, 2 hours per week.

[6] Exercise classes for various courses at National Research University “High School of Econmics”: Geometry-I, September-December 2013, 2 hours per week; Differential Geometry, September-December 2013, 2 hours per week.

3.3.11 Yuri Prokhorov

[1] Introduction to algebraic geometry, Moscow State University, 3 year students, September-December 2013, 2 hours per week.

Program: Varieties in mathematics. Maps between varieties. Sheaves. Sheaves on varieties. Exact sequences of sheaves. Affine algebraic varieties and prime ideals. Zariski topology. Irreducible components. Dimension. Local properties. Tangent space, smooth points, singular locus. Dimension. Affine schemes. Topology, structure sheaf, basic properties. Irreducible decomposition, dimension. Arbitrary schemes (definition). Products of schemes.

[2] Rational surfaces, Independent University of Moscow and Steklov Institute, 3 year students, September-December 2013, 2 hours per week.

Program: Rational surfaces. Ruled surfaces. Elementary transformations. Del Pezzo surfaces: existence of good curve, projective models, classification, existence of (-1)-curve, birational description. Deformations of curves. Bend and break. Cone theorem. Minimal models. Applications. G-surfaces. Equivariant MMP. Minimal models of G-surfaces. Criterion of birational triviality. Minimal conic bundles. Singularities of linear systems. Canonical thresholds. Noether-Fano inequality. Birational rigidity. Segre-Manin theorem. Sarkisov program. Cremona group.

[3] Rational and unirational varieties (teaching seminar, with D. Orlov and C. Shramov), Steklov Institute, 3-5 year students, September-December 2013, 2 hours per week. Program: Del Pezzo surfaces (in arbitrary characteristic). Rationality constructions of del Pezzo surfaces. Basic birational invariant. Unirationality of del Pezzo surfaces of degree ≥ 2. Estimates of the unirationality degree. Stable birational invariants. Artin-Mumford and Gross constructions. Intermediate Jacobian and Prym variety. Application to three-dimensional conic bundles. An example of non-rational variety with trivial intermediate Jacobian.

[4] Minicourse on G-varieties, National Taiwan University (Special Program in Algebraic Geometry and Representation Theory), May 1-8, 3 lectures, 6 hours.

Program: A G-variety is an algebraic variety equipped with an action of an algebraic group G. To study G-varieties it is very natural to use the equivariant version of MMP. Then the birational classification of rationally connected G-varieties is reduced to the birational classification of Fano-Mori fiber spaces. I give a complete modern proof of classification of two-dimensional Fano-Mori fiber spaces (Manin-Iskovskikh) and discuss known results in the three-dimensional case. Applications to rationality problems, classification of finite subgroups of Cremona groups and arithmetical questions are also discussed.

http://www.math.ntu.edu.tw/~ctsdev/download/activity/minicourse%20yuri.pdf

3.3.12 Petr Pushkar'

[1] Topology-1. Independent University of Moscow, 2 year students, February-May 2013, 4 hours per week.

Program

Metrical and topological spaces, continuous maps. Compactness Operations on topological spaces Homotopy, homotopical equivalence Cellular spaces Two-dimensional surfaces Fundamental group Covering spaces Higher homotopy groups

[2] Differential Geometry-2, High School of Economics, Dept of Math, 3-4 year students January-June 2013, 4 hours per week.

The following topics were considered

1. Integrability of distributions 2. Elements of Symplectic Jeometry 3. Elements of Contact geometry 4. Connection, curvature 5. Riemanian Geometry, geodesics, Morse theory

[3] Topology-2. Independent University of Moscow, 2 year students, September-December 2013, 4 hours per week.

Program

1. Sungular homology

Definition, properties, induced maps, long exact sequence of pairs and triples, coefficients

2. Cellular homologies

3. Calculation of homologies of some spaces.

4. Cohomologies, multiplication

5. Morse theory

6 Poincare duality, intersection theory

[4] Also I assist on courses: Dinamical systems, Topology for 2 year students and Singularity Theory for 3-4 course and magistrants, September-December 2013 6 hours per week totally

3.3.13 Sergei Rybakov

[1] Algebra. Independent University of Moscow, I year students, September-December 2013, 2 hours per week.

Program

[1] Groups. Lagrange's theorem. Isomorphism theorems.

[2] Rings, ideals, quotient rings. Isomorphism theorems. Chinese remainder theorem.

[3] Euclidean and Principal ideal domains. Unique factorization domains. Gauss Lemma.

[4] Modules over Euclidean rings. Classification of finitely generated abelian groups. Jordan normal form of a matrix.

[5] Extensions of fileds. Galois theorey. Main theorem. Cyclotomic and Kummer extensions.

[6] Noetherian rings. Hilbert's basis theorem.

[7] Dedekind domains. Ring of integers of a number field.

[8] Bonus lecture: projective limits, p-adic numbers, Witt vectors.

3.3.14 Arkady Skopenkov

[1] Combinatorial topology, III year students, September-December 2013, 4 hours per week. Moscow Institute of Physics and Technology

Program.

It is shown how in the course of solution of interesting geometric problems (close to applications) naturally appear main notions of algebraic topology (homology groups, obstructions and invariants, characteristic classes). Thus main ideas of algebraic topology are presented with minimal technicalities. Familiarity of a reader with basic notions of topology (such as 2-dimensional manifolds and vector fields) is desirable, although definitions are given at the beginning.

Detailed information in Russian: http://www.mccme.ru/circles/oim/home/combtop13.htm

[2] Differential Geometry (exercises), III year students, September-December 2013, 4 hours per week. Moscow Institute of Physics and Technology

Program.

It is shown how in the course of solution of interesting geometric problems (close to applications) naturally appear different notions of *curvature*, which distinguish given geometry from the 'ordinary' one. Direct elementary definitions of these notions are presented. The paper is accessible for students familiar with analysis of several variables, and could be an interesting easy reading for mature mathematicians. The material is presented as a sequence of problems, which is peculiar not only to Zen monasteries but also to serious mathematical education.

Detailed information in Russian: http://www.mccme.ru/circles/oim/home/difgeom13.htm

[3] Discrete analysis (exercises), II year students, September-December 2013, 2 hours per week. Moscow Institute of Physics and Technology

Program. We study certain topics in combinatorics and graph theory (including random graphs).

Detailed information in Russian: http://www.mccme.ru/circles/oim/home/discran1314.htm

[4] Homotopy and geometric topology of manifolds Independent University of Moscow, February-May 2013, 2 hours per week.

Program. We study the set of homotopy classes of continuous maps from one manifold to another. We exhibit applications to geometric topology.

Detailed information in Russian: http://www.mccme.ru/circles/oim/home/zadanienmu.htm

[5] Differential and geometric topology of manifolds Independent University of Moscow, September-December 2013, 2 hours per week.

Program. We study important methods of differential topology such as Pontryagin construction, homology, characteristic classes, vector bundles, surgery.

Detailed information in Russian: http://www.mccme.ru/circles/oim/home/nmuaut13sko.htm

[6] Faculty of Mathematics, Higher School of Economics, October-December 2013, 2 hours per week.

Program. It is shown how in the course of solution of interesting geometric problems (close to applications) naturally appear main notions of algebraic topology (homology groups, obstructions and invariants). Thus main ideas of algebraic topology are presented with minimal technicalities. In particular, we study (in low dimensions) embeddings, approximation by embeddings and thickenings.

Detailed information in Russian: http://www.mccme.ru/circles/oim/home/mfaut13sko.htm

[7] Thickenings of graphs and hypergraphs (minicourse), Summer School 'Modern Mathematics', July 2013, 8 hours.

Program. We study 2-dimensional thickenings of graphs and 3-dimensional thickenings of hypergraphs

Detailed information in Russian: http://www.mccme.ru/dubna/2013/courses/skopenkov.htm

3.3.15 Mikhail Skopenkov

[1] Complex analysis. Independent University of Moscow, II year students, February-May 2013, 4 hours per week.

Program (short version).

1. Survey of applications of complex analysis.
2. Simple properties of complex numbers and quaternions.
3. Analytic and holomorphic functions.
4. Integration of holomorphic functions.
5. Order of zeroes of holomorphic functions.
6. Conformal mappings.
7. Harmonic functions.
8.* Discrete harmonic functions.
9. Meromorphic functions.
10. Liuville' s theorem.
11.* Elliptic functions.

[2] Geometry. Independent University of Moscow, I year students, September- December 2013, 4 hours per week + distant excercise class http://dist-math.ru.

Program (short version).

1. Projective geometry.
2. Möbius geometry.
3. Spherical geometry.
4. Hyperbolic geometry.
5. Nine Caley-Klein planar geometries. Klein's concept of geometry.

[3] Distant courses for mathematical olympiads winners (http://math.olymp.mioo.ru). Since 2013 working without any financial support.

3.3.16 Evgeni Smirnov

[1] Symmetric Functions, Independent University of Moscow & Higher School of Economics, course for 2nd–4th year students, September–December 2013, 2 hours per week

Course outline:

1. Symmetric polynomials. Elementary and complete symmetric polynomials, duality. Skew-symmetric polynomials. Schur polynomials.

2. Pieri, Giambelli, Jacobi–Trudi formulas.

3. Combinatorial description of Schur polynomials. Standard and semistandard Young tableaux. Kostka numbers. Hook length formula.

4. Representations of symmetric groups (Okounkov–Vershik approach). Branching rule.

5. Characters of representations of symmetric groups. Murnaghan–Nakayama rule.

6. Frobenius characteristic map. Relation between representations of symmetric groups and Schur polynomials. Positivity of Littlewood–Richardson coefficients.

[2] Algebra, 2nd year, Higher School of Economics, 1st quarter (September–October 2013), 4 hours per week

1. Ring of polynomials. Gauss Lemma. Polynomials over a UFD form a UFD.

2. Symmetric polynomials. Fundamental theorem on symmetric polynomials.
3. Resultant and discriminant.
4. Groups. Direct and semidirect product. Group automorphisms.
5. Derived subgroup and its properties.
6. Group actions. Sylow theorems.
7. Simple groups. Jordan–Hölder theorem.

3.3.17 Stanislav Shaposhnikov

1) Elliptic second order partial differential equations. Independent University of Moscow, 3-5 year students and PhD students, September – December 2013, 2 hours per week.

Program

1. Classical maximum principle.
2. Bernstein estimates.
3. Hölder spaces.
4. Solvability in Hölder spaces.
5. Sobolev spaces. Weak solutions.
6. Maximum principle for weak solutions.
7. Mosers Harnack inequality.
8. Smoothness of weak solutions.
9. Alexandrov's maximum principle.
10. Boundedness of strong solutions.

3.3.18 Mikhail Verbitsky

[1] 2013 (Spring): Galois theory (a half-semester course; second year undergraduate). 16 lecture hours, 32 seminar hours. `http://bogomolov-lab.ru/KURSY/GALOIS-2013/` Higher School of Economics.

Program: First introduction to Galois theory, from algebraic numbers and algebraic closures to the fundamental theorem of Galois theory and Abel's theorem.

[2] 2013 (Spring): Differential Geometry, Math in Moscow, (24 lecture hours, 30 seminar hours) `http://bogomolov-lab.ru/KURSY/GEOM-2013/`

INTRODUCTION TO DIFFERENTIAL GEOMETRY AND GEOMETRIC ANALYSIS

Spring 2013, Math in Moscow and HSE.

Differential geometry is the study of smooth manifolds by means of vector bundles and the Lie group theory. I will give a gentle introduction to some of the most basic notions of differential geometry: vector bundles, tangent spaces, sheaves, connections and differential operators.

Approximate syllabus.

1. Smooth manifolds, partition of unit, Hausdorff dimension and Hausdorff measure. Whitney embedding theorems.

2. Sheaves, categories, limits, colimits, and germs of functions. Smooth manifolds as ringed spaces.

3. Derivations on the ring of smooth functions; vector fields as derivations. Vector bundles; equivalence of different definitions of vector bundles. Serre-Swan theorem.

4. Differential operators and their symbols. De Rham algebra and de Rham differential. Lie derivative and Cartan's formula.

5. Elliptic equations and their properties. Weak maximum principle. Harmonic functions, mean value property of harmonic functions.

6. Stokes' theorem, de Rham cohomology, applications to topology.

7. Definition of a connection. Construction of connections on vector bundles. Parallel transport along a connection.

8. Torsion and curvature of a connection. Existence and uniqueness of the Levi-Civita connection.

Prerequisites: some knowledge of linear algebra (tensor product, polylinear, symmetric, anti-symmetric forms, self-adjoint and anti-self adjoint operators) analysis (manifolds, coordinates, Taylor series, partition of unit) and topology (topological spaces, continuous maps, limits, compactness).

[3] 2013 (Summer): Symplectic capacity and pseudoholomorphic curves 6 lecture hours, Summer School of Laboratory of Algebraic Geometry (Yaroslavl) `http://bogomolov-lab.ru/SHKOLA2013/talks/verbit.html`

Program: I defined symplectic capacities according to Gromov and Ekeland-Hofer and proved that Gromov's capacity is finite. As an application I proved the symplectic camel theorem and closedness of the symplectomorphism group.

[4] 2013 (Fall): Differential geometry and vector bundles (HSE and IUM). 26 lecture hours, 32 seminar hours. `http://bogomolov-lab.ru/KURSY/BUNDLES-2013/`

Program: Basic results and concepts related to principal bundles and G-structures on manifolds: vector bundles and principa bundles, connections, curvature, torsion, Levi-Civita connection, sheaves, differential operators.

3.3.19 Ilya Vyugin

[1] Calculus (lectures and seminars). Independent University of Moscow, I year students, Feburary-May 2013, 2+2 hours per week.

Program

1. Functions of several variables.
2. Implicit function theorem and its corollaries. Morse lemma.
3. Jordan measure and Lebesgue measure.
4. Measurable functions.
5. Lebesgue integral.
6. Fubini's theorem and the theorem of Radon-Nikodym.
7. Space L_2.
8. Orthogonal system of functions. Fourier series.

[2] Analitic theory of differential equations (lectures) Special course in HSE and IUM.

Program

Linear differential equations

1. Linear differential equations: monodromy, singular points. Levelt decomposition.
2. Elements of global theory.
3. Hypergeometric equation and hypergeometric functions.

Painlevé property for first order nonlinear equations.

4. Fuchs conditions.
5. Riccati equation.
6. Equations of the genius zero are equivalent to Riccati equation.
7. Some of second order nonlinear differential equations. Painlevé equations.

Theory of the local normal forms.

8. Formal normal forms. Poincaré-Dulac theorem.
9. Poincaré and Siegel domains. Theorem of Poincaré.

[3] Calculus (seminars-exercises) Higher School of Economics, II year students, January-June 2013, 2 hours per week.

Program

1. Jordan measure and Lebesgue measure.

4. Measurable functions.
5. Lebesgue integral.
6. Absolutely continuous functions.
7. Space L_2.
8. Orthogonal system of functions. Fourier series.
9. Spatial integrals.

[4] PDE (seminars-exercises). Higher School of Economics, II year students, January-June 2013, 2 hours per week.

1. Canonical form of a linear second order PDE.
2. The wave equation. The Dalamber formula.
3. The heat equation.
4. The Poisson equation.
5. Fourier method.
6. The wave equation in the second and third dimensional spaces.
7. Harmonic functions.

3.3.20 Alexei Zykin

[1] Introduction to Algebraic Number Theory. Independent University of Moscow and Mathematical Department of the Higher School of Economics, 2–4 year students, September–May 2012–2013, 2 hours per week.

Program

[1] **Introduction.** Diophantine equations (Fermat's last theorem, congruent numbers and elliptic curves).

[2] **Galois theory and finite fields.** Basic facts from Galois theory. The structure of finite fields. Equations over finite fields. Quadratic reciprocity law.

[3] p**-adic numbers.** Congruences and p-adic numbers. Hensel's lemma. Ostrowski's theorem.

[4] **Quadratic forms.** Representation of numbers by quadratic forms over $\mathbb{Q}_p$ and $\mathbb{Q}$. Minkowski–Hasse theorem.

[5] **Algebraic number fields.** Prime ideal decomposition, ramification, discriminant, class number, units.

[6] **Elliptic curves.** Basic properties. Mordell-Weil theorem.

[7] **Zeta-functions.** Distribution of primes and Riemann zeta function. Dirichlet theorem on primes in arithmetic progressions. Functional equation for Dedekind zeta-function and residue formula.

[2] Modular Forms (together with O. Shvartsman). Mathematical Department of the Higher School of Economics, 2–6 year students, September–May 2012–2013, 2 hours per week.

Program

Algebra

[1] Lattices. Bases and sublattices. Invariant factors for pairs (lattice, sublattice). Linear transformations of lattices. Quadratic lattices. Dual lattice.

[2] Reduction theory for definite quadratic lattices in dimension 2. Reduction theory for indefinite lattices in dimension 2.

[3] General theory of Hecke algebras.

Number theory

[4] Algebraic numbers. Algebraic integers. Finite extensions of $\mathbb{Q}$. Lattice of algebraic integers. Discriminants and integral bases. Quadratic extensions of $\mathbb{Q}$.

[5] Factorization in rings of integers of algebraic number fields. Ideals. Ideal class groups of quadratic fields and quadratic lattices.

[6] Pell's equation and Dirichlet's unit theorem for real quadratic fields.

[7] Zeta-functions and the analytic class number formula.

Analysis

[8] Periodic functions. Elliptic functions. Lattices and elliptic curves.

[9] Modular functions and modular forms.

[10] Theta-functions and quadratic forms. Modular forms of half-integral weight.

Discrete transformation groups

[11] Transformation groups and quotient spaces.

[12] The modular group. Fuchsian groups.

[3] Introduction to Discrete Mathematics. Mathematical Department of the Higher School of Economics, 1 year students, January–June 2013, 2 hours per week.

Program

[1] Introduction.

[2] Generating functions.

[3] Balls and Boxes: the twelvefold way.

[4] Rational generating functions.

[5] Catalan numbers. Dick and Motzkin paths.

[6] Graphs. Adjacency matrix. Generating function for paths in graphs.

[7] Generating functions of several variables. Bernulli–Euler numbers.

[8] Inclusion–exclusion principle. Dirichlet generating functions.

[9] Enumeration of trees. Matrix tree theorem and Cayley formula.

[10] Graph invariants. Chromatic polynomial. Cycles, cocycles, cyclotomic number.

[11] Deletion–contraction recurrence. Tutte theorem.

[12] Graphs and permutations. Hurwitz numbers.

[13] Formal languages. Finite automata.

[14] Context-free grammars, regulars languages and automata with stacks.