The IUM report to the Simons foundation, 2014

Contents

1 Introduction: list of awardees

The Simons foundation supported two programs launched by the IUM:
Simons stipends for students and graduate students;
Simons IUM fellowships.

11 applications were received for the Simons stipends contest. The selection committee consisting of *Yu.Ilyashenko (Chair), G.Dobrushina, G.Kabatyanski, S.Lando, I.Paramonova (Academic Secretary), A.Sossinsky, M.Tsfasman* awarded Simons stipends for 2013 year to the following students and graduate students:

1. Bufetov, Alexei Igorevich
2. Bychkov, Boris Sergeevich
3. Voynov, Andrei Sergeevich
4. Karpukhin, Mikhail Alexandrovich
5. Oganesyan, Dmitry Alexeevich
6. Okunev, Alexei Vladimirovich
7. Rudenko, Daniil Glebovich
8. Shilin, Ivan Sergeevich.

14 applications were received for the Simons IUM fellowships contest for the first half year of 2014 and 17 applications were received for the second half year. The selection committee consisting of *Yu.Ilyashenko (Chair), G.Dobrushina, B.Feigin, I.Paramonova (Academic Secretary), A.Sossinsky, M.Tsfasman, V.Vassiliev* awarded

Simons IUM-fellowships for the first half year of 2014 to the following researches:

1. Gorodentsev, Alexei Lvovich
2. Domrin, Andrei Victorovich
3. Kazarian, Maxim Eduardovich
4. Khoroshkin, Anton Sergeevich
5. Olshanski, Grigori Iosifovich
6. Penskoi, Alexei Victorovich

7. Prokhorov, Yuri Gennadevich
8. Pushkar, Petr Evgenevich
9. Skopenkov, Arkady Borisovich
10. Skopenkov, Mikhail Borisovich
11. Smirnov, Evgeni Yurevich
12. Vyugin, Ilya Vladimirovich

Simons IUM-fellowships for the second half year of 2014 to the following researches:

1. Burman, Yurii Mikhailovich
2. Feigin, Evgeny Borisovich
3. Fonarev, Anton Vyacheslavovich
4. Gorodentsev, Alexei Lvovich
5. Elagin, Alexei Dmitrievich
6. Levin, Andrei Mikhailovich
7. Olshanski, Grigori Iosifovich
8. Penskoi, Alexei Victorovich
9. Pushkar, Petr Evgenevich
10. Rybakov, Sergey Yurevich
11. Savin, Anton Yurevich
12. Shaposhnikov, Stanislav Valerevich
13. Skopenkov, Mikhail Borisovich

The report below is split in two sections corresponding to the two programs above. The first subsection in each section is a report on the research activities. It consists of the titles of the papers published or submitted in the year of 2014, together with the corresponding abstracts. The second subsection of each section is devoted to conference and some most important seminar talks. The last subsection of the second section is devoted to the syllabi of the courses given by the winners of the Simons IUM fellowships. Most of these courses are innovative, as required by the rules of the contest for the Simons IUM fellowships.

The support of the Simons foundation have drastically improved the financial situation at the IUM, and the whole atmosphere as well. On behalf of the IUM, I send my deep gratitude and the best New year wishes to Jim Simons, Yuri Tschinkel, and the whole team of the Simons foundation.

Yulij Ilyashenko

President of the Independent University of Moscow

2 Program: Simons stipends for students and graduate students

2.1 Research

2.1.1 Alexei Bufetov

[1] With A. Borodin
Plancherel representations of $U(\infty)$ and correlated Gaussian Free Fields,
Duke Mathematical Journal, vol. 163, no. 11 (2014), 2109-2158; `arXiv:1301.0511`.

We study asymptotics of traces of (noncommutative) monomials formed by images of certain elements of the universal enveloping algebra of the infinite-dimensional unitary group in its Plancherel representations. We prove that they converge to (commutative) moments of a Gaussian process that can be viewed as a collection of simply yet nontrivially correlated two-dimensional Gaussian Free Fields. The limiting process has previously arisen via the global scaling limit of spectra for submatrices of Wigner Hermitian random matrices.

[2] With V. Gorin
Representations of classical Lie groups and quantized free convolution
to appear in *Geometric And Functional Analysis*, `arXiv:1311.5780`.

We study the decompositions into irreducible components of tensor products and restrictions of irreducible representations of classical Lie groups as the rank of the group goes to infinity. We prove the Law of Large Numbers for the random counting measures describing the decomposition. This leads to two operations on measures which are deformations of the notions of the free convolution and the free projection. We further prove that if one replaces counting measures with others coming from the work of Perelomov and Popov on the higher order Casimir operators for classical groups, then the operations on the measures turn into the free convolution and projection themselves. We also explain the relation between our results and limit shape theorems for uniformly random lozenge tilings with and without axial symmetry.

[3] With A. Borodin and G. Olshanski
Limit shapes for growing extreme characters of $U(\infty)$
to appear in *Annals of Applied Probability*, `arXiv:1311.5697`.

We prove the existence of a limit shape and give its explicit description for certain probability distribution on signatures (or highest weights for unitary groups). The distributions have representation theoretic origin — they encode decomposition on irreducible characters of the restrictions of certain extreme characters of the infinite-dimensional unitary group U(infinity) to growing finite-dimensional unitary subgroups $U(N)$. The characters of $U(\infty)$ are allowed to depend on N. In a special case, this describes the hydrodynamic behavior for a family of random growth models in (2+1)-dimensions with varied initial conditions.

[4] With L. Petrov
Law of Large Numbers for Infinite Random Matrices over a Finite Field
`arXiv:1402.1772`, submitted

Asymptotic representation theory of general linear groups $GL(n, q)$ over a finite field leads to studying probability measures ρ on the group U of all infinite uni-uppertriangular matrices over F_q, with the condition that ρ is invariant under conjugations by arbitrary infinite matrices. Such probability measures form an infinite-dimensional simplex, and the description of its extreme points (in other words, ergodic measures ρ) was conjectured by Kerov in connection with nonnegative specializations of Hall-Littlewood symmetric functions. Vershik and Kerov also conjectured the following Law of Large Numbers. Consider an n by n diagonal submatrix of the infinite random matrix drawn from an ergodic measure coming from the Kerov's conjectural classification. The sizes of Jordan blocks of the submatrix can be interpreted as a (random) partition of n, or, equivalently, as a (random) Young diagram $\lambda(n)$ with n boxes. Then, as n goes to infinity, the rows and columns of $\lambda(n)$ have almost sure limiting frequencies corresponding to parameters of this ergodic measure. Our main result is the proof of this Law of Large Numbers. We achieve it by analyzing a new randomized Robinson-Schensted-Knuth (RSK) insertion algorithm which samples random Young diagrams $\lambda(n)$ coming from ergodic measures. The probability weights of these Young diagrams are expressed in terms of Hall-Littlewood symmetric functions. Our insertion algorithm is a modified and extended version of a recent construction by Borodin and the second author (arXiv:1305.5501). On the other hand, our randomized RSK insertion generalizes a version of the RSK insertion introduced by Vershik and Kerov (1986) in connection with asymptotic representation theory of symmetric groups (which is governed by nonnegative specializations of Schur symmetric functions).

[5] With V. Gorin

Stochastic monotonicity in Young graph and Thoma theorem
`arXiv:1411.3307`, submitted.

We show that the order on probability measures, inherited from the dominance order on the Young diagrams, is preserved under natural maps reducing the number of boxes in a diagram by 1. As a corollary we give a new proof of the Thoma theorem on the structure of characters of the infinite symmetric group. We present several conjectures generalizing our result. One of them (if it is true) would imply the Kerov's conjecture on the classification of all homomorphisms from the algebra of symmetric functions into R which are non-negative on Hall–Littlewood polynomials.

2.1.2 Boris Bychkov

[1] On the decomposition of the cyclic permutation into the product of a given number of permutations (in russian)
submitted to Functional Analysis and Its Applications

The problem of the decompositions permutation into the product of permutations playing the main role in the studying of meromorphic functions or in other words ramified coverings of the two-dimensional sphere; it goes back to the papers of Hurwitz at the end of XIX century. In 2000 M. Bousquet-Mélou and J. Shaeffer got the elegant formula for the quantity of the decompositions the permutation into the product of a given number of permutations, corresponding to the genus 0 coverings. Generalizations of their formula for coverings of the two-dimensional sphere with higher genus are unknown. In this paper we present a new proof of the Bousquet-Mélou–Shaeffer formula in the case of decompositions of the cyclic permutation, which hopefully could works in the cases of positive genus.

2.1.3 Andrei Voynov

[1] With V.Protasov
On the noncontractive compact affine operator semigroups.
to appear in Mat.Sbornik.

We study compact multiplicative semigroups of affine operators acting on a finite-dimensional real space. The main result states that either such semigroup is contractive (i.e. contains an element of an arbitrary small operator norm) or all of its elements possess a common invariant affine subspace and the restriction of the semigroup to this subspace is contractive. The main tool used in the proof is the functional equations with a contraction of an argument. We cover the applications to the self-affine subdivisions of convex bodies, to the classification of the finite affine semigroups and give a new proof of the criteria of a nonnegative matrix semigroup to be primitive.

[2] With V.Protasov
Matrix semigroups with constant spectral radius
arXiv:1407.6568 *submitted to Advances in Mathematics.*

Multiplicative matrix semigroups with constant spectral radius (c.s.r.) are studied and applied to several problems of algebra, combinatorics, functional equations, and dynamical systems. We show that all such semigroups are characterised by means of irreducible ones. Each irreducible c.s.r. semigroup defines walks on Euclidean sphere, all its nonsingular elements are similar (in the same basis) to orthogonal. We classify all nonnegative c.s.r. semigroups and arbitrary low-dimensional semigroups. For higher dimensions, we describe five classes and leave an open problem on completeness of that list. The problem of algorithmic recognition of c.s.r. property is proved to be polynomially solvable for irreducible semigroups and undecidable for reducible ones.

2.1.4 Mikhail Karpukhin

[1] Spectral properties of bipolar surfaces to Otsuki tori.
Journal of Spectral Theory, 2014, Vol. 4, No. 1, pp. 87-111.

The i-th eigenvalue λ_i of the Laplace-Beltrami operator on a surface can be considered as a functional on the space of all Riemannian metrics of unit volume on this surface. Surprisingly only few examples of extremal metrics for these functionals are known. In the present paper a new countable family of extremal metrics on the torus is provided.

2.1.5 Dmitry Oganesyan

[1] Rational functions with two critical points of maximum multiplicity
J. Fundamental and Applied Mathematics, 2013, Vol. 18, No. 6, pp. 185-208.

In this paper we consider the functions on algebraic curves, whose divisors have the form $nA - nC$, especially in the context of the Dessin d'enfant theory, Belyi pairs, i.e. functions with a small number of critical values. Choice of this kind of divisors can be explained by the simplification of the computation of concrete examples of Belyi pairs and emerging links with other areas of mathematics.
http://mech.math.msu.su/ fpm/ps/k13/k136/k13611.pdf

[2] Abel pairs

to appear in Journal of Mathematical Sciences.

An *Abel pair* is a pair (X, α), where X is an complete algebraic curve and α is a rational function on it, whose divisor has the form $\text{div}(\alpha) = nA - nC$. Such pairs appear in connection with the calculation of quasi-elliptic integrals in Abel work [3].

Similarly to the case of Belyi pairs the embedded graph $\Gamma_{X,\alpha}$ is associated to an Abel pair; it provides a combinatorial-topological description of Abel pairs. This description will be applied to families of Abel pairs, essentially parametrized by modular curves. The Abel-Belyi pairs will be introduced and counted and their Galois orbits described.

In the current literature similar questions are studied; for example, ref. [1] is devoted to the p-adic reduction of Belyi-Abel pairs. In [2] similar considerations are undertaken.

References

[1] L. Zapponi, *Lame curves with bad reduction.* Preprint, (2006).

[2] F. B. Pakovitch, "Combinatoire des arbres planaires et arithmetique des courbes hyperelliptiques," *Ann. Inst. Fourier.* **48**, N2, 323–351, (1998).

[3] N. H. Abel, Üher die Integration der Differential-Formel $\rho dx/\sqrt{R}$, wenn R und ρ ganze Funktionen sind. J. für Math. **1** (1826), 185-221.

2.1.6 Alexei Okunev

[1] With S. Minkov; in russian

Omega-limit sets of generic points for partially hyperbolic diffeomorphisms with one-dimensional unstable foliation

prepared to submit in print.

We will prove that ω-limit set of a generic (with respect to the Lebesgue measure) point is saturated by unstable leaves for any partially hyperbolic diffeomorphism with one-dimensional unstable foliation. This gives a positive answer to a conjecture by Ilyashenko that Milnor attractor is saturated by unstable leaves under the same assumptions. Assuming this conjecture is true, Ilyashenko constructed an example of locally generic thick Milnor attractor in class of boundary preserving diffeomorphisms of $[0, 1] \times \mathbb{T}^2$.

[2] Milnor attractor of partially hyperbolic skew products with circle fiber

in preparation.

We prove that for a C^2-generic partially hyperbolic skew product with circle fiber over linear Anosov diffeomorphism of $\mathbb{T}^2$ its Milnor attractor is Lyapunov stable and is either of zero measure, or coincides with the whole phase space.

2.1.7 Daniil Rudenko

[1] Arithmetic of 3-valent graphs and equidissections of flat surfaces
arXiv:1411.0285*submitted to Discrete and Computational Geometry*

Our main object of study is a 3-valent graph with a vector function on its edges. The function assignes to an edge a pair of 2-adic integer numbers and satisfies additional condition: the sum of its values on three edges, terminating in the same vertex, is equal to 0. For each vertex of the graph three vectors corresponding to these edges generate a lattice over the ring of 2-adic integers. In this paper we study the restrictions, imposed on these lattices by the combinatorics of the graph. As an application we obtain the following fact: a rational balanced polygon cannot be cut into an odd number of triangles of equal areas. First result of this type was obtained by Paul Monsky in 1970. He proved that a square cannot be cut into an odd number of triangles of equal areas. In 2000 Sherman Stein conjectured that the same holds for any balanced polygon. We prove this conjecture in the case, when coordinates of all vertices of the cut are rational numbers.

[2] Hilbert Third Problem and reciprocity laws.
Will be on arxive till the end of December2014.

We formulate a tangential analog of A. Goncharov strong reciprocity law conjecture and prove it for projective line. We also explain the corollary of this: existence of a linear map, taking three meromorphic functions on a compact Riemann surface to a linear combinations of polytopes in Euclidean space, whose Dehn invariant and volume can be read directly from the curve and functions.

2.1.8 Ivan Shilin

[1] Locally topologically generic diffeomorphisms with Lyapunov unstable Milnor attractors
in preparation, preprint will be available in the middle of December

Whenever there is an open domain in $\mathrm{Diff}^r(M)$ (M being a smooth manifold, $r \geq 1$) exhibiting a persistent homoclinic tangency associated to a basic set with a sectionally dissipative periodic saddle, topologically generic diffeomorphisms in this domain have Lyapunov unstable Milnor attractors. This implies, in particular, that instability of Milnor attractors is locally topologically generic in C^1 if $\dim M \geq 3$, and in C^2 if $\dim M = 2$.

Moreover, it follows from the results of C. Bonatti, L. J. Díaz and E. R. Pujals that for a C^1 topologically generic diffeomorphism of a closed manifold either any homoclinic class admits some dominated splitting, or this diffeomorphism has an unstable Milnor attractor, or the inverse diffeomorphism has an unstable Milnor attractor.

2.2 Scientific conferences and seminar talks

2.2.1 Alexei Bufetov

[1] Workshop "From Macdonald processes to Hecke Algebras and Quantum Integrable Systems", IHP, Paris; May.

[2] Visit to Massachusetts Institute of Technology, Boston; February — April.

2.2.2 Boris Bychkov

[1] Workshop "Primitive forms and related objects", Tokio, Kavli IPMU, February, 2 – 5

Talk "'On the number of coverings of the sphere ramified over given points"

[2] Workshop "Graduate workshop on Moduli of curves", Stony Brook, SCGP, July, 7 – 18

Talk "On the geomtery of decomposition of the cyclic permutation into the product of a given number of permutations"

[3] Conference "Embedded Graphs", Saint-Petersburg, PDMI, October, 27 – 31

Talk "On the geometry of decomposition of the cyclic permutation into the product of a given number of permutations"

[4] Visit to Saint-Petersburg, March

Talk "On the number of coverings of the sphere ramified over given points" on the seminar "Low-Dimensional Mathematics" (PDMI)

[5] Conference "4th Workshop on combinatorics of moduli spaces, cluster algebras and topological recursion", Moscow, May, 26–31

2.2.3 Andrei Voynov

[1] Talk "Matrix semigroups with constant spectral radius" at Moscow State University.

[2] School "Geometrical methods in mathematical physics", Moscow, June 24–27.

2.2.4 Mikhail Karpukhin

[1] Conference "Lomonosov", Moscow, April 7 – 11.

Talk "Large Laplace and Steklov egenvalues"

[2] Talk "Regularity theorems for maximal metrics" at "Spectral theory seminar" (McGill University, Montreal), November 27.

2.2.5 Dmitry Oganesyan

[1] Conference "Embedded Graphs" October 27-31, 2014, St. Petersburg.

Talk "Abel pairs and modular curves"

[2] Fall 2014, Moscow

Talk "Dessin d'enfant and modular curves" at "Graphs on surfaces and curves over number fields" (Moscow state university)

Talk "Abel pair over fields of positive characteristic" "Graphs on surfaces and curves over number fields" (Moscow state university)

2.2.6 Alexei Okunev

[1] Visit to Lyon, October-December

Talk "Attractors of random dynamics on circle" at at the internal seminar (sem'in) of UMPA, ENS Lyon

[2] Summer school of the seminar "Dynamic systems" in Dubna (Russia), June, 28 - July, 8

[3] Talks on the seminar "Dynamic systems" (Moscow, MSU; in russian):

"Step skew products with circle fiber" (April, 4)

"Bounded distortion in partial hyperbolic situation" (September, 26)

2.2.7 Daniil Rudenko

[1] in HSE

Talk "Equidissections of flat surfaces" at "Seminar of the laboratory of algebraic geometry and its applications " (Higher School of Economics)

Talk "Scissor congruence and reciprocity laws" at "Seminar of the laboratory of algebraic geometry and its applications " (Higher School of Economics)

Talk "Scissor congruence and reciprocity laws" at "Cohomology of Moduli Spaces" (Higher School of Economics)

Talk "K theory and Goncharov Conjectures" at "Graduate student seminar" (Higher School of Economics)

2.2.8 Ivan Shilin

[1] "Dynamical systems" seminar at Moscow State University
Talk "Dominated splitting or sinks, sources and instability of Milnor attractors"

3 Program: Simons IUM fellowships

3.1 Research

3.1.1 Yurii Burman

[1] (with S.Lvovski) On projections of smooth and nodal plane curves
Moscow Math. Journal, 2014, to appear.

Suppose that $C \subset \mathbb{C}P^2$ is a general enough nodal plane curve of degree > 2, $\nu\colon \hat{C} \to C$ is its normalization, and $\pi\colon \hat{C} \to \mathbb{C}P^1$ is a finite morphism simply ramified over the same set of points as a projection $\mathrm{pr}_p \circ \nu\colon \hat{C} \to \mathbb{C}P^1$, where $p \in \mathbb{C}P^2 \setminus C$ (if $\deg C = 3$, one should assume in addition that $\deg \pi \neq 4$). We prove that the morphism π is equivalent to such a projection if and only if it extends to a finite morphism $X \to (\mathbb{C}P^2)^*$ ramified over C^*, where X is a smooth surface.

As a by-product, we prove the Chisini conjecture for mappings ramified over duals to general nodal curves of any degree ≥ 3 except for duals to smooth cubics; this strengthens one of Victor Kulikov's results.

[2] (with A.Ploskonosov and A.Trofimova) Matrix-tree theorems and discrete path integration.

Linear Algebra and Applications, 2015, to appear.

We calculate characteristic polynomials of operators explicitly presented as polynomials of rank 1 operators. Corollaries of the main result include a generalization of the Forman's formula for the determinant of the graph Laplacian, the celebrated Matrix-tree theorem by G.Kirchhoff, and some its extensions and analogs, both known (e.g. the Matrix-hypertree theorem by G.Masbaum and A.Vaintrob) and new.

3.1.2 Andrei Domrin

[1] On real-analytic solutions of the nonlinear Schroedinger equation (in Russian).
Published in Trudy Moskovskogo Matematicheskogo Obschestva, 2014, Vol. 75, No. 2, pp. 205–218. English translation to appear in Transactions of Moscow Mathematical Society.

In this paper we establish solubility of the Riemann problem on factorization of formal matrix-valued Laurent series with unitary symmetry conditions. As an application, we show that every local real-analytic (in x and t) solution of the focusing nonlinear Schroedinger equation extends to a real-analytic function in a strip parallel to the x-axis, and for every such strip there is a solution admitting no real-analytic extension to a larger domain.

3.1.3 Alexei Elagin

[1] "On equivariant triangulated categories", arXiv:1403.7027.
In this paper we present a construction of G-equivariant category for an action of a finite group G on a triangulated category, supposing that the action is induced by an action on a DG-enhancement.

3.1.4 Evgeny Feigin

[1] With G. Cerulli Irelli and M. Reineke
Homological approach to the Hernandez-Leclerc construction and quiver varieties,
Representation Theory 2014, no. 18, pp.1–14.

In a previous paper the authors have attached to each Dynkin quiver an associative algebra. The definition is categorical and the algebra is used to construct desingularizations of arbitrary quiver Grassmannians. In the present paper we prove that this algebra is isomorphic to an algebra constructed by Hernandez-Leclerc defined combinatorially and used to describe certain graded Nakajima quiver varieties. This approach is used to get an explicit realization of the orbit closures of representations of Dynkin quivers as affine quotients.

[2] With M. Finkelberg and P. Littelmann
Symplectic Degenerate Flag Varieties,
Canadian Journal of Mathematics. 2014. Vol.66. No. 6, P. 1250-1286

Let F^a_λ be the degenerate symplectic flag variety. These are projective singular irreducible $\mathbb{G}_a^M$ degenerations of the classical flag varieties for symplectic group Sp_{2n}. We give an explicit construction for the varieties F^a_λ and construct their desingularizations, similar to the Bott-Samelson resolutions in the classical case. We prove that F_λ^a are normal locally complete intersections with terminal and rational singularities. We also show that these varieties are Frobenius split. Using the above mentioned results, we prove an analogue of the Borel-Weil-Bott theorem and obtain a q-character formula for the characters of irreducible Sp_{2n}-modules via the Atiyah-Bott-Lefschetz fixed points formula.

[3] With M. Finkelberg and M. Reineke
Degenerate affine Grassmannians and loop quivers
arXiv:1410.0777, *submitted*

We study the connection between the affine degenerate Grassmannians in type A, quiver Grassmannians for one vertex loop quivers and affine Schubert varieties. We give an explicit description of the degenerate affine Grassmannian of type GL_n and identify it with semi-infinite orbit closure of type A_{2n-1}. We show that principal quiver Grassmannians for the one vertex loop quiver provide finite-dimensional approximations of the degenerate affine Grassmannian. Finally, we give an explicit description of the degenerate affine Grassmannian of type $A_1^{(1)}$, propose a conjectural description in the symplectic case and discuss the generalization to the case of the affine degenerate flag varieties.

[4] With I. Makedonskyi
Nonsymmetric Macdonald polynomials, Demazure modules and PBW filtration
arXiv:1407.6316 *submitted*

The Cherednik-Orr conjecture expresses the $t \to \infty$ limit of the nonsymmetric Macdonald polynomials in terms of the PBW twisted characters of the affine level one Demazure modules. We prove this conjecture in several special cases.

3.1.5 Anton Fonarev

None.

3.1.6 Alexei Gorodentsev

I have no scientific papers published during the period from January 01, 2014 until today.

3.1.7 Maxim Kazarian

[1] Maxim Kazarian, Peter Zograf
Virasoro constraints and topological recursion for Grothendieck's dessin counting
arXiv:1406.5976 *submitted to Communications in Mathematical Physics*

We compute the number of coverings of the Rieman sphere with a given monodromy type over infinity, a given numbers of preimages of 0 and 1, and no other critical values.

We show that the generating function for these numbers enjoys several remarkable integrability properties: it obeys the Virasoro constraints, an evolution equation, the Kadomtsev-Petviashvili hierarchy and satisfies a topological recursion in the sense of Eynard-Orantin.

3.1.8 Anton Khoroshkin

[1] with A. Berenstein, M. Bennet, V. Chari, S. Loktev
"Macdonald Polynomials and BGG reciprocity for current algebras"

Selecta Mathematica New series, April 2014, Volume 20, Issue 2, pp 585–607

In this paper We study the category of graded representations with finite–dimensional graded pieces for the current algebra associated to a simple Lie algebra. This category has many similarities with the category O of modules for $\mathfrak{g}$ and in this paper, we use the combinatorics of Macdonald polynomials to prove an analogue of the famous BGG duality in the case of $\mathfrak{sl}_{n+1}$.

[2] "Characteristic classes of flags of foliations and Lie algebra cohomology."

We prove the conjecture by Feigin, Fuchs and Gelfand describing the Lie algebra cohomology of formal vector fields on an n-dimensional space with coefficients in symmetric powers of the coadjoint representation. We also compute the cohomology of the Lie algebra of formal vector fields that preserve a given flag at the origin. The latter encodes characteristic classes of flags of foliations and was used in the formulation of the local Riemann-Roch Theorem by Feigin and Tsygan. Feigin, Fuchs and Gelfand described the first symmetric power and to do this they had to make use of a fearsomely complicated computation in invariant theory. By the application of degeneration theorems of appropriate Hochschild-Serre spectral sequences we avoid the need to use the methods of FFG, and moreover we are able to describe all the symmetric powers at once.
arxiv:1303.1889v2, *submitted to Transformation Groups*

[3] "Highest weight categories and Macdonald polynomials."

The aim of this paper is to introduce the categorical setup which helps us to relate the theory of Macdonald polynomials and the theory of Weyl modules for current Lie algebras discovered by V. Chari and collaborators.

We identify Macdonald pairing with the homological pairing on the ring of characters of the Lie algebra of currents $\mathfrak{g}\otimes\mathbb{C}[x,\xi]$. We use this description in order to define complexes of modules whose Euler characteristic of characters coincide with Macdonald polynomials. We generalize this result to the case of graded Lie algebras with anti-involution. We show that whenever the BGG reciprocity holds for the corresponding category of modules then these complexes collapse to the modules concentrated in homological degree 0. The latter modules generalizes the notion of Weyl modules for current Lie algebras and the notion of Verma modules in the BGG category O. We give different criterions of BGG reciprocity and apply them to the Lie algebra of currents $\mathfrak{g} \otimes \mathbb{C}[x, \xi]$ with $\mathfrak{g}$ semisimple.

arxiv:1312.7053 (submitted to Advances in Mathematics)

[4] with T. Willwacher, M. Živković
"Differentials on graph complexes I"

We study the cohomology of complexes of ordinary (non-decorated) graphs, introduced by M. Kontsevich. We construct spectral sequences converging to zero whose first page contains the graph cohomology. In particular, these series may be used to show the existence of an infinite series of previously unknown and provably non-trivial cohomology classes, and put constraints on the structure of the graph cohomology as a whole.

arxiv:1411.2369

3.1.9 Andrei Levin

[1] With A.Zotov and M.Olshanetsky Planck Constant as Spectral Parameter in Integrable Systems and KZB Equations

arXiv:1408.6246v3 [hep-th] 24 Oct 2014

We construct special rational Knizhnik-Zamolodchikov-Bernard (KZB) equations with N punctures by deformation of the corresponding quantum rational R-matrix. They have two parameters. The limit of the first one brings the model to the ordinary rational KZ equation. Another one is . At the level of classical mechanics the defor- mation parameter allows to extend the previously obtained modified Gaudin models to the modified Schlesinger systems. Next, we notice that the identities underlying generic (elliptic) KZB equations follow from some additional relat ions for the properly normalized R-matrices. The relations are noncommutative analogues of identities for (scalar) elliptic functions. The simplest one is the unitarity condition. The quadratic (in R matrices) relations are generated by noncommutative Fay identities. In particular, one can derive the quantum Yang-Baxter equations from the Fay identities. The cubic relations provide identities for the KZB equations as well as quadrati c relations for the classical rmatrices which can be halves of the classical Yang-Baxter e quation. At last we discuss the R-matrix valued linear problems which provide Calogero-Moser (CM) models and Painlev e equations via the above mentioned identities. Th e role of the spectral parameter plays the Planck constant of the quantum R-matrix. When the quantum R-matrix is scalar (N = 1) the linear problem reproduces the Krichevers ansatz for the Lax matrices with spectral parameter for the CM models. The linear problems for the quantum CM models generalize the KZ equations in the same way as the Lax pairs with spectral parameter generalize those without it.

3.1.10 Grigori Olshanski

[1] With A. Borodin and A. Bufetov
Limit shapes for growing extreme characters of $U(\infty)$.
To appear in Annals of Applied Probability.

We prove the existence of a limit shape and give its explicit description for certain probability distribution on signatures (or highest weights for unitary groups). The distributions have representation theoretic origin — they encode decomposition on irreducible characters of the restrictions of certain extreme characters of the infinite-dimensional unitary group $U(\infty)$ to growing finite-dimensional unitary subgroups $U(N)$. The characters of $U(\infty)$ are allowed to depend on N. In a special case, this describes the hydrodynamic behavior for a family of random growth models in $(2+1)$-dimensions with varied initial conditions.

[2] The Gelfand-Tsetlin graph and Markov processes
arXiv:1404.3646 *to appear in Proceedings of the International Congress of Mathematicians, Seoul 2014.*

The goal of the paper is to describe new connections between representation theory and algebraic combinatorics on one side, and probability theory on the other side.

The central result is a construction, by essentially algebraic tools, of a family of Markov processes. The common state space of these processes is an infinite dimensional (but locally compact) space Ω. It arises in representation theory as the space of indecomposable characters of the infinite-dimensional unitary group $U(\infty)$.

Alternatively, Ω can be defined in combinatorial terms as the boundary of the Gelfand-Tsetlin graph — an infinite graded graph that encodes the classical branching rule for characters of the compact unitary groups $U(N)$.

We also discuss two other topics concerning the Gelfand-Tsetlin graph:

(1) Computation of the number of trapezoidal Gelfand-Tsetlin schemes (one could also say, the number of integral points in a truncated Gelfand-Tsetlin polytope). The formula we obtain is well suited for asymptotic analysis.

(2) A degeneration procedure relating the Gelfand-Tsetlin graph to the Young graph by means of a new combinatorial object, the Young bouquet.

At the end we discuss a few related works and further developments.

This is an extended version of my invited sectional lecture at ICM 2014, Seoul

[3] The representation ring of the unitary groups and Markov processes of algebraic origin
65 pp, submitted

The paper consists of two parts. The first part introduces the representation ring for the family of compact unitary groups $U(1)$, $U(2)$, … . This novel object is a commutative graded algebra R with infinite-dimensional homogeneous components. It plays the role of the algebra of symmetric functions, which serves as the representation ring for the family of finite symmetric groups. The purpose of the first part is to elaborate on the basic definitions and prepare the ground for the construction of the second part of the paper.

The second part deals with a family of Markov processes on the dual object to the infinite-dimensional unitary group $U(\infty)$. These processes were defined in a joint work with Alexei Borodin (J. Funct. Anal. 2012). The main result of the present paper consists in the derivation of an explicit expression for their infinitesimal generators. It is shown that the generators are implemented by certain second order partial differential operators with countably many variables, initially defined as operators on R.

3.1.11 Alexei Penskoi

[1] Generalized Lawson tori and Klein bottles
Journal of Geometric Analysis, 2014, DOI 10.1007/s12220-014-9529-7.

Using Takahashi theorem we propose an approach to extend known families of minimal tori in spheres. As an example, the well-known two-parametric family of Lawson tau-surfaces including tori and Klein bottles is extended to a three-parametric family of tori and Klein bottles minimally immersed in spheres. Extremal spectral properties of the metrics on these surfaces are investigated. These metrics include i) both metrics extremal for the first non-trivial eigenvalue on the torus, i.e. the metric on the Clifford torus and the metric on the equilateral torus and ii) the metric maximal for the first non-trivial eigenvalue on the Klein bottle.

[2] with A. A. Gaifullin and S. V. Smirnov
Problems in Linear Algebra and Geometry, Moscow, MCCME, 2014. 152pp (in Russian).

This book contains elaborated solutions of typical problems in Linear Algebra and Geometry.

3.1.12 Yuri Prokhorov

Papers published in 2014

[1] With T. Kishimoto and M. Zaidenberg.

Unipotent group actions on del Pezzo cones. Algebraic Geometry, 1(1):46–56, 2014.

`http://www.algebraicgeometry.nl/2014-1/2014-1-003.pdf`

In a previous paper we established that for any del Pezzo surface Y of degree at least 4, the affine cone X over Y embedded via a pluri-anticanonical linear system admits an effective $\mathbf{G}_a$-action. In particular, the group $\mathrm{Aut}(X)$ is infinite dimensional. In contrast, we show in this note that for a del Pezzo surface Y of degree at most 2 the generalized cones X as above do not admit any non-trivial action of a unipotent algebraic group.

[2] With S. Mori.

Threefold extremal contractions of types (IC) and (IIB).

Proc. Edinburgh Math. Soc., 57(1):231–252, 2014.

http://dx.doi.org/10.1017/S0013091513000850

Let (X, C) be a germ of a threefold X with terminal singularities along an irreducible reduced complete curve C with a contraction $f : (X, C) \to (Z, o)$ such that $C = f^{-1}(o)_{\mathrm{red}}$ and $-K_X$ is ample. Assume that (X, C) contains a point of type (IC) or (IIB). We complete the classification of such germs in terms of a general member $H \in |-K_X|$ containing C.

[3] 2-elementary subgroups of the space Cremona group.

In I. Cheltsov, C. Ciliberto, H. Flenner, J. McKernan, Y. G. Prokhorov, and M. Zaidenberg, editors, Automorphisms in birational and affine geometry. Levico Terme, Italy, October 2012, volume 79 of Springer Proceedings in Mathematics & Statistics, pages 215–229. 2014.

http://link.springer.com/chapter/10.1007/978-3-319-05681-4_12

We give a sharp bound for orders of elementary abelian 2-groups of birational automorphisms of rationally connected threefolds.

[4] With Takashi Kishimoto and Mikhail Zaidenberg.

Affine cones over Fano threefolds and additive group actions.

Osaka Journal of Mathematics, 51(4):1093–1112, 2014.

http://projecteuclid.org/euclid.ojm/1414761913

We address the following question: When an affine cone over a smooth Fano threefold admits an effective action of the additive group? In this paper we deal with Fano threefolds of index 1 and Picard number 1. Our approach is based on a geometric criterion from our previous paper, which relates the existence of an additive group action on the cone over a smooth projective variety X with the existence of an open polar cylinder in X. Non-trivial families of Fano threefolds carrying a cylinder were found in loc. cit. Here we provide new such examples.

[5] Del Pezzo fibrations.

Appendix to "Five embeddings of one simple group" by I. Cheltsov and C. Shramov, Trans. Amer. Math. Soc. 366 (2014), 1289-1331, 2014.

Let X be a threefold with at worst terminal singularities such that the group $\mathrm{Aut}(X)$ has a subgroup $G \simeq A_6$, and let $\pi : X \to \mathbb{P}^1$ be a G-Mori fibration. The goal of this appendix is to prove that there exists a G-isomorphism $X \simeq \mathbb{P}^1 \times \mathbb{P}^2$ so that π is the projection to the first factor.

Papers accepted in 2014

[6] With C. Shramov.

Jordan property for Cremona groups.

ArXiv e-print, 1211.3563, 2012. to appear in Amer. J. Math.

Assuming Borisov-Alexeev-Borisov conjecture, we prove that there is a constant $J = J(n)$ such that for any rationally connected variety X of dimension n and any finite subgroup $G \subset \mathrm{Bir}(X)$ there exists a normal abelian subgroup $A \subset G$ of index at most J. In particular, we obtain that the Cremona group $\mathrm{Cr}_3 = \mathrm{Bir}(\mathbb{P}^3)$ enjoys the Jordan property.

[7] With C. Shramov.

Jordan property for groups of birational selfmaps.

ArXiv e-print, 1307.1784, 2013. to appear in Compositio Math.

`http://dx.doi.org/doi:10.1112/S0010437X14007581`

Assuming Borisov-Alexeev-Borisov conjecture, we prove that finite subgroups of the automorphism group of a finitely generated field over $\mathbb{Q}$ have bounded orders. Further, we investigate which algebraic varieties have groups of birational selfmaps satisfying the Jordan property.

[8] A note on degenerations of del Pezzo surfaces.

ArXiv e-print, 1108.5051, 2011. to appear in Annales de l'Institut Fourier, vol. 65 (2015).

We prove that for a $\mathbb{Q}$-Gorenstein degeneration X of del Pezzo surfaces, the number of non-Du Val singularities is at most $\rho(X)+2$. Degenerations with $\rho(X)+2$ and $\rho(X)+1$ non-Du Val points are investigated.

Papers written in 2014

[9] With M. Zaidenberg.

Examples of cylindrical Fano fourfolds.

ArXiv e-print, 1406.6339.

We construct 4 different families of smooth Fano fourfolds with Picard rank 1, which contain cylinders, i.e., Zariski open subsets of the form $Z \times \mathbb{A}^1$, where Z is a quasiprojective variety. The affine cones over such a fourfold admit effective $\mathbf{G}_a$-actions. Similar constructions of cylindrical Fano threefolds were done previously in our papers jointly with Takashi Kishimoto.

3.1.13 Petr Pushkar'

I am working on a book on Morse theory for manifolds with boundaries.

Main subject is a generalization of classical Morse inequalities.

3.1.14 Sergei Rybakov

[1] DG-modules over de Rham DG-algebra. Accepted to European Journal of Mathematics. arxiv.org/abs/1311.7503

For a morphism of smooth schemes over a regular affine base we define functors of derived direct image and extraordinary inverse image on coderived categories of DG-modules over de Rham DG-algebras. Positselski proved that for a smooth algebraic variety X over a field k of characteristic zero the coderived category of DG-modules over $\Omega^\bullet_{X/k}$ is equivalent to the unbounded derived category of quasi-coherent right D_X-modules. We prove that our functors correspond to the functors of the same name for D_X-modules under Positselski equivalence.

3.1.15 Anton Savin

[1] With B.Yu. Sternin
Index of elliptic operators for diffeomorphisms of manifolds
J. of Noncommutative Geometry, 2014, V. 8, No. 3, 695–734

In this paper we develop elliptic theory for operators associated with a diffeomorphism of a closed smooth manifold. The aim of the present paper is to obtain an index formula for such operators in terms of topological invariants of the manifold and the symbol of the operator. The symbol in this situation is an element of a certain crossed product. We express the index as the pairing of the class in K-theory defined by the symbol and the Todd class in periodic cyclic cohomology of the crossed product.

[2] With B.Yu. Sternin
On the index of nonlocal elliptic operators associated with fibrations
Doklady Mathematics, 2014, V. 89, No. 1, 61–64

In this paper we consider the algebra of nonlocal operators associated with the structure of a fibration with a marked section. Together with pseudodifferential operators, this class contains also some Fourier integral operators. For elliptic elements in this algebra we obtain finiteness theorem and obtain an index formula.

[3] With B.Yu. Sternin
Index of Sobolev problems on manifolds with many-dimensional singularities
Differ. Equations, 2014, V. 50, No. 2, 232–245

In this paper we consider Sobolev problems on manifolds with multi-dimensional singularities. We prove the Fredholm property of such problems and derive the corresponding index formula. The results are based on the theory of translators on manifolds with singularities.

[4] With B.Yu. Sternin and E. Schrohe
On the Index Formula for an Isometric Diffeomorphism
Journal of Mathematical Sciences, 2014, V. 201, No. 6, pp. 818–829

In this paper we give an elementary solution to the problem of the index of elliptic operators associated with shift operator along the trajectories of an isometric diffeomorphism of a smooth closed manifold. This solution is based on index-preserving reduction of the operator under consideration to some elliptic pseudo-differential operator on a higher-dimension manifold and on the application of the Atiyah–Singer formula. The final formula for the index is given in terms of the symbol of the operator on the original manifold.

[5] With B.Yu. Sternin

Index of nonlocal problems associated with a bundle
Differ. Equations, 2014, V. 50, No. 8, (2014), 1112–1121

In this paper we study operators associated with a bundle with compact base and fiber. We construct an algebra of such operators. For elliptic elements in the algebra, we prove the finiteness theorem and derive an index formula.

[6] With B.Yu. Sternin and E. Schrohe
Uniformization and an Index Theorem for Elliptic Operators Associated with Diffeomorphisms of a Manifold
arXiv:1111.1525 *to appear in Russian Journal of Mathematical Physics*

In this paper, we consider the index problem for a wide class of nonlocal elliptic operators on a smooth closed manifold, namely differential operators with shifts induced by the action of an isometric diffeomorphism. The key to the solution is the method of uniformization: to the nonlocal problem we assign a pseudodifferential operator with the same index, acting in sections of an infinite-dimensional vector bundle on a compact manifold. We then determine the index in terms of topological invariants of the symbol, using the Atiyah-Singer index theorem.

[7] With B. Yu. Sternin
Elliptic G-operators on manifolds with singularities
Preprint

In this paper, we study elliptic operators on manifolds with singularities in the situation, when the manifold is endowed with an action of a discrete group G. As usual in elliptic theory, the Fredholm property of an operator is governed by the properties of its principal symbol. We show that the principal symbol in our situation is a pair, consisting of the symbol on the main stratum (interior symbol) and the symbol at the conical point (conormal symbol). Fredholm property of elliptic elements is obtained.

3.1.16 Arkady Skopenkov

[1] D. Gonçalves and A. Skopenkov, A useful lemma on equivariant maps, Homology, Homotopy and Applications, 16:2 (2014), 307 - 309.

The purpose of this note is to present a short proof of the following known result.

Lemma. *Suppose X, Y are finite connected CW-complexes with free involutions, $f : X \to Y$ is an equivariant map and l is a non-negative integer. If $f^* : H^i(Y) \to H^i(X)$ is an isomorphism for each $i > l$ and is onto for $i = l$, then*

(a_l) $f^\sharp : \pi^i_{eq}(Y) \to \pi^i_{eq}(X)$ is a 1-1 correspondence for $i > l$ and is onto for $i = l$.

Here $H^i(X)$ is (non-equivariant) cohomology with integral coefficients and $\pi^i_{eq}(X)$ is the set of equivariant homotopy classes of equivariant maps from X to S^i with the antipodal involution.

We could not find a proof in standard textbooks and references. So we feel obliged to present a short proof of such a basic and useful result.

[2] D. Crowley and A. Skopenkov, Classification of smooth embeddings of non-simply-connected 4-manifolds into R^7, preprint, 2014 (work continued from 2013)

Let N be a closed connected orientable 4-manifold with torsion free integral homology. The main result is *a complete readily calculable classification of embeddings* $N \to R^7$, in the smooth and in the piecewise-linear (PL) categories. Such a classification was earlier known only for simply-connected N, in the PL case by Boéchat-Haefilger-Hudson 1970, in the smooth case by the authors 2008. In particular, for $N = S^1 \times S^3$ we define geometrically a 1–1 correspondence between the set of PL isotopy classes of PL embeddings $S^1 \times S^3 \to R^7$ and the quotient set of $Z \oplus Z_6$ up to equivalence $(l, b) \sim (l, b')$ for $b \equiv b' \mod 2GCD(3, l)$. This particular case allows us to disprove the conjecture on the completeness of the Multiple Haefliger-Wu invariant, as well as the Melikhov informal conjecture on the existence of a geometrically defined group structure on the set of PL isotopy classes of PL embeddings in codimension 3. For $N = S^1 \times S^3$ and the smooth case we identify the isotopy classes of embeddings with an explicitly defined quotient of $Z_{12} \oplus Z \oplus Z$.

[3] A. Skopenkov, Classification of knotted tori, preprint, 2014, (work continued from 2013)

We describe the group of (smooth isotopy classes of smooth) embeddings $S^p \times S^q \to R^m$ for $p \le q$ and $m \ge 2p+q+3$. Earlier such a description was known only for $2m \ge 3p+3q+4$. We use (and reprove) a recent exact sequence of M. Skopenkov.

[4] A. Skopenkov, How do autodiffeomorphisms act on embeddings, submitted, http://arxiv.org/abs/140

We work in the smooth category. For an n-manifold N denote by $E^m(N)$ the set of isotopy classes of embeddings $N \to R^m$. The following problem was suggested by E. Rees in 2002: describe the action of self-diffeomorphisms of $S^p \times S^{n-p}$ on $E^m(S^p \times S^{n-p})$.

Let $g : S^p \times S^{n-p} \to R^m$ be an embedding such that $g|_{a \times S^{n-p}} : a \times S^{n-p} \to R^m - g(b \times S^{n-p})$ is null-homotopic for some different points $a, b \in S^p$ and $m \ge n+2+\frac{1}{2}\max\{p, n-p\}$.

Theorem. *For a map $\varphi : S^p \to SO_{n-p}$ define an autodiffeomorphism φ' of $S^p \times D^{n-p}$ by $\overline{\varphi}(a, b) := (a, \varphi(a)b)$. Let φ'' be the S^{n-p-1}-symmetric extension of φ to an autodiffeomorphism of $S^p \times S^{n-p}$. Then for each map $\varphi : S^p \to SO_{n-p}$ embedding $g \circ \varphi''$ is isotopic to embedded connected sum $g\#u$ for some embedding $u : S^n \to S^m$.*

Let N be an oriented n-manifold and $f : N \to R^m$ an embedding. Denote by $E^m(N)/\#$ the quotient set of $E^m(N)$ by embedded connected sum with embeddings $S^n \to R^m$. As a corollary we obtain that under certain conditions *for orientation-preserving embeddings $s : S^p \times D^{n-p} \to N$ the class of S^p-parametric embedded connected sum $f\#_s g$ in $E^m(N)/\#$ depends only on f, g and the isotopy (the homotopy or the homology) class of $s|_{S^p \times 0}$.*

Expository publications for university students.

[5] A. Chernov, A. Daynyak, A. Glibichuk, M. Ilyinskiy, A. Kupavskiy, A. Raigorodskiy and A. Skopenkov, Elements of Discrete mathematics as a sequence of problems, Moscow, MCCME, to appear http://www.mccme.ru/circles/oim/discrbook.pdf

We present sequences of problems on combinatorics and graph theory (including random graphs).

[6] D. Ilyinskiy, A. Raigorodskiy and A. Skopenkov, Existence proofs in combinatorics using independence, Mat. Prosveschenie, 19 (2015), http://arxiv.org/abs/1411.3171

This note is purely expository and is in Russian. We show how to prove interesting combinatorial results using the *local Lovasz lemma*. The note is accessible for students having basic knowledge of combinatorics; the notion of independence is defined and the Lovasz lemma is stated and proved. Our exposition follows 'Probabilistic methods' of N. Alon and J. Spencer. The main difference is that we show how the proof could have been invented. The material is presented as a sequence of problems, which is peculiar not only to Zen monasteries but also to advanced mathematical education; most problems are presented with hints or solutions.

[7] V.V. Prasolov, A.B. Skopenkov, Some reflections on why Lobachevsky geometry was recognized, Mat. Prosveschenie, 19 (2015), http://arxiv.org/abs/1307.4902

Sometimes arguments that preceded recognition of non-Euclidean (Lobachevsky) geometry are represented in a simplified 'black and white' pattern: 'conservators made nonsense of genius'. Although there is something in this point of view, the real situation was more complicated, and up to some time there were decent grounds for not recognizing the importance of the new theory. We try to explain why non-Euclidean geometry was not recognized at once. We show how important for such recognition was discovery of applications of the new geometry. These reflections have practical importance for modern mathematics because they are related to the question: how a mathematician should choose directions for research?

[8] A. Skopenkov, Algebraic topology from geometric viewpoint, http://arxiv.org/abs/0808.1395 v2, http://www.mccme.ru/circles/oim/obstruct.pdf (a new version prepared) Moscow, MCCME, to appear

This book is purely expositional and is in Russian. It is shown how in the course of solution of interesting geometric problems (close to applications) naturally appear main notions of algebraic topology (homology groups, obstructions and invariants, characteristic classes). Thus main ideas of algebraic topology are presented with minimal technicalities. Familiarity of a reader with basic notions of topology (such as 2-dimensional manifolds and vector fields) is desirable, although definitions are given at the beginning. The book is accessible to undergraduates and could also be an interesting easy reading for mature mathematicians.

[9] A. Skopenkov, Some more proofs from the Book: solvability and insolvability of equations in radicals, in Russian, submitted, http://arxiv.org/abs/0804.4357 v6 (a new version prepared)

This paper is purely expository and is in Russian. We present short elementary proofs of

- the Gauss Theorem on constructibility of regular polygons;
- the existence of a cubic equation unsolvable in real radicals;
- the existence of a quintic equation unsolvable in complex radicals (Galois Theorem).

We do not use the terms 'Galois group' or even 'group'. However, our presentation is a good way to learn (or recall) starting idea of the Galois theory. The paper is accessible for students familiar with elementary algebra (including complex numbers), and could be an interesting easy reading for mature mathematicians. The material is presented as a sequence of problems, which is peculiar not only to Zen monasteries but also to serious mathematical education; most problems are presented with hints or solutions.

[10] A. Skopenkov and M. Skopenkov, Some short proofs of the unrealizability of hypergraphs, http://arxiv.org/abs/1402.0658, submitted.

We present short elementary proofs of van Kampen–Flores and Ummel's theorems on unrealizability of certain hypergraphs in four-dimensional space. The proofs are based on reduction to *Ramsey linking theory* resuls for graphs in three-dimensional space.

[11] A. Skopenkov and A. Zimin, Realizability of hypergraphs in Euclidean spaces, preprint, http://www.mccme.ru/circles/oim/exalong.pdf .

The following problem is well known: can a given graph be embedded in the plane, i.e., can the graph be drawn on the plane so that its edges have no pairwise intersections and no self intersections except at end points? The present cycle of problems is about the embedding of the two-dimensional analogs of graphs (called hypergraphs) in three-dimensional space and even in four-dimensional space. The most important results are simple proofs of the non-realizability in four-dimensional space of the complete hypergraph on seven vertices and (solution of the Menger Problem) of the product $K_5 \times K_5$. The proofs are based on reduction to *Ramsey linking theory* resuls for graphs in three-dimensional space.

[12] A. Skopenkov, Some notes on basic link and knot theory, lecture notes, http://www.mccme.ru/circles/oim/exalg_eng.pdf

We present a short exposition of some basic knot theory. In particular, we define Sato-Levine number and state the Vassiliev-Kontsevich Theorem in a way convenient for calculation of the invariants themselves, not only of the dimension of the space of the invariants.

3.1.17 Mikhail Skopenkov

[1] A. Bobenko, M. Skopenkov, Discrete Riemann surfaces: linear discretization and its convergence, J. für die reine und angewandte Mathematik, Published Online (2014). Available at:http://arxiv.org/abs/1210.0561

We develop linear discretization of complex analysis, originally introduced by R. Isaacs, J. Ferrand, R. Duffin, and C. Mercat. We prove convergence of discrete period matrices and discrete Abelian integrals to their continuous counterparts. We also prove a discrete counterpart of the Riemann–Roch theorem. The proofs use energy estimates inspired by electrical networks.

[2] D. Crowley, S. Ferry, M. Skopenkov, The rational classification of links in codimension > 2, Forum Math. 26:1 (2014), 239–269;

Let m and $p_1, \dots, p_r < m-2$ be positive integers. The set of links of codimension > 2, $E^m(\sqcup_{k=1}^r S^{p_k})$, is the set of smooth isotopy classes of smooth embeddings $\sqcup_{k=1}^r S^{p_k} \to S^m$. Haefliger showed that $E^m(\sqcup_{k=1}^r S^{p_k})$ is a finitely generated abelian group with respect to embedded connected summation and computed its rank in the case of knots, i.e. $r = 1$. For $r > 1$ and for restrictions on $p_1, \dots, p_r$ the rank of this group can be computed using results of Haefliger or Nezhinsky. Our main result determines the rank of the group $E^m(\sqcup_{k=1}^r S^{p_k})$ in general. In particular we determine precisely when $E^m(\sqcup_{k=1}^r S^{p_k})$ is finite. We also accomplish these tasks for framed links. Our proofs are based on the Haefliger exact sequence for groups of links and the theory of Lie algebras.

[3] A. Pakharev, M. Skopenkov, A. Ustinov, Through the resisting net, Mat. Prosv. 3rd ser. 18 (2014), 33–65;

This is a popular science paper devoted to an elementary proof of the following beautiful folklore result:

Theorem. (a) A man which is randomly walking in a 2-dimensional square lattice will hit the right neighbor of the initial point before returning to the initial point with probability 1/2.

(b) The resistance between neighboring nodes of an infinite 2-dimensional square lattice of unit resistances equals 1/2.

Parts (a) and (b) turn out to be equivalent to each other. The approach to the proof is based on a physical interpretation.

[4] M. Skopenkov, When the set of embeddings is finite?, submitted to Intern. J. Math (2013). http://arxiv.org/abs/1106.1878

Given a manifold N and a number m, we study the following question: *is the set of isotopy classes of embeddings $N \to S^m$ finite*? In case when the manifold N is a sphere the answer was given by A. Haefliger in 1966. In case when the manifold N is a disjoint union of spheres the answer was given by D. Crowley, S. Ferry and the author in 2011.

We consider the next natural case when N is a product of two spheres. In the following theorem, $FCS(i,j) \subset \mathbb{Z}^2$ is a concrete set depending only on the parity of i and j which is defined in the paper.

Theorem. Assume that $m > 2p+q+2$ and $m < p+3q/2+2$. Then the set of isotopy classes of smooth embeddings $S^p \times S^q \to S^m$ is infinite if and only if either $q+1$ or $p+q+1$ is divisible by 4, or there exists a point (x,y) in the set $FCS(m-p-q, m-q)$ such that $(m-p-q-2)x + (m-q-2)y = m-3$.

Our approach is based on a group structure on the set of embeddings and a new exact sequence, which in some sense reduces the classification of embeddings $S^p \times S^q \to S^m$ to the classification of embeddings $S^{p+q} \sqcup S^q \to S^m$ and $D^p \times S^q \to S^m$. The latter classification problems are reduced to homotopy ones, which are solved rationally.

[5] A. Skopenkov, M. Skopenkov, Some short proofs of the unrealizability of hypergraphs, submitted to Arnold Math. J.

We present short elementary proofs of van Kampen–Flores and Ummel's theorems on unrealizability of certain hypergraphs in four-dimensional space. The proofs are based on reduction to *Ramsey linking theory* resuls for graphs in three-dimensional space.

In addition to [1]–[5], several talk abstracts have been published in 2014.

3.1.18 Evgeni Smirnov

[1] Young diagrams, plane partitions and alternating sign matrices
Book (64 pp., in Russian), MCCME publishing house, 2014

These are extended notes of a four-lecture minicourse given at the summer school "Contemporary mathematics" (Dubna, Russia) in July 2013. These notes contain an introduction to the theory of Young diagrams and plane partitions. We also discuss relation between plane partitions and alternating sign matrices; this relation was the key ingredient in D. Zeilberger's proof of the alternating sign matrix conjecture.

[2] Three glances on the Aztec diamond
Book (48 pp., in Russian). MCCME publishing house, to appear, 2015. Preliminary version available at `http://www.mccme.ru/~smirnoff/papers/dubna15.pdf`.

These are notes of a three-lecture minicourse given at the summer school "Contemporary mathematics" (Dubna, Russia) in July 2014. In these notes we prove the Aztec diamond theorem, stating that the number of domino tilings of the Aztec diamond of order n is $2^{n(n+1)/2}$. We present three different proofs of this theorem. The first proof is based on counting a weighted sum over alternating sign matrices associated with domino tilings. In the second proof we compute the number of perfect matchings by using "urban renewal" of planar bipartite graphs. In the last proof we establish a bijection between domino tilings of the Aztec diamond and certain lattice path configurations; their number is then found using the Gessel–Viennot method.

[3] With G. Merzon
Determinantal identities for flagged Schur and Schubert polynomials
Submitted, preprint arXiv:1410.6857, 15 pages

We prove new determinantal identities for a family of flagged Schur polynomials. As a corollary of these formulas we obtain determinantal expressions of Schubert polynomials for certain vexillary permutations.

[4] With V. Kleptsyn
Plane curves and bialgebra of Lagrangian subspaces
Preprint arXiv:1401.6160, 25 pages

We study multicomponent plane curves with possible singularities of selftangency type. To each such curve we assign a so-called L-space, which is a Lagrangian subspace in an even-dimensional vector space with the standard symplectic form. This invariant generalizes the notion of the intersection matrix for the framed chord diagram of a one-component plane curve. Moreover, the actions of Morse perestroikas and Vassiliev moves are reinterpreted nicely in the language of L-spaces, becoming changes of bases in this vector space. Finally, we define a bialgebra structure on the span of L-spaces.

3.1.19 Stanislav Shaposhnikov

[1] Manita O.A., Shaposhnikov S.V. Fokker–Planck–Kolmogorov equations with potential terms on arbitrary domains. Dokl. Math. 2015. V. 460. N 2. P. 1–5.

We study the Cauchy problem for Fokker–Planck–Kolmogorov equations with unbounded and degenerate coefficients. Sufficient conditions for the existence and uniqueness of solutions are indicated.

[2] Manita O.A., Romanov M.S., Shaposhnikov S.V. On uniqueness of solutions to nonlinear Fokker–Planck–Kolmogorov equations. MathArxiv: arXiv:1407.8047, 2014.

We study uniqueness of flows of probability measures solving the Cauchy problem for nonlinear Fokker-Planck-Kolmogorov equation with unbounded coefficients. Sufficient conditions for uniqueness are indicated and examples of non-uniqueness are constructed.

[3] Bogachev V.I., Roeckner M., Shaposhnikov S.V. Uniqueness problems for degenerate Fokker–Planck–Kolmogorov equations. Journal of Mathematical Sciences, 2015 (in print)

We study the uniqueness of solutions to the Cauchy problem for the Fokker–Planck–Kolmogorov equation with a degenerate diffusion matrix in the class of probability measures. A survey of known result and methods is given. In addition, we obtain new sufficient conditions for the uniqueness in the case of unbounded coefficients and a partially degenerate diffusion matrix and also in the case where the diffusion matrix is the square of a Lipschitzian matrix.

3.1.20 Ilya Vyugin

[1] (With R.R. Gontsov) Solvability of linear differential systems in the Liouvillian sense *arXiv:1312.2518* , 2013, (submited to journal in 2014).

The paper concerns the solvability by quadratures of linear differential systems, which is one of the questions of differential Galois theory. We consider systems with regular singular points as well as those with (non-resonant) irregular ones and propose some criteria of solvability for systems whose (formal) exponents are sufficiently small.

[2] (With I.D. Shkredov, E.V. Solodkova) Intersections of multiplicative subgroups and Heilbronn's exponential sum
arXiv:1302.3839, 2014, (will be submited to journal).

The paper is devoted to some applications of Stepanov method. In the first part of the paper we obtain the estimate of the cardinality of the set, which is obtained as an intersection of additive shifts of some different subgroups of F_p^*. In the second part we prove a new upper bound for Heilbronn's exponential sum and obtain a series of applications of our result to distribution of Fermat quotients.

[3] On the number of solutions of discret algebraic curve
Proceedings of the young mathematician conference, 2014, p. 29-33 (an extended version will be submited to journal soon).

The number of solutions (x, y) of an equation

$$y = f(x), \qquad f(x) \in F_p[x]$$

such that $x, y \in G \times G$, where G is a subgroup of $\mathbb{F}_p^*$, was obtained in the paper.

3.2 Scientific conferences and seminar talks

3.2.1 Andrei Domrin

[1] Conference dedicated to the 100 th anniversary of the distinguished mathematician B.M.Levitan, Moscow, June 23–27.
Talk "Real-analytic solutions of the nonlinear Schroedinger equation".

[2] Conference "Complex analysis and approximation theory" dedicated to the 80 th birthday of Professor E.P.Dolzhenko, Moscow, September 29–30.

Talk "Holomorphic solutions of soliton equations".

[3] Visit to Hannover, August. Joint work on integrability of PT-invariant deformations of Calogero–Moser systems.

Participation (without talk) in the Workshop "Gauge theories in higher dimensions", Leibniz Universitaet Hannover, August 11–14.

3.2.2 Alexei Elagin

[1] International conference "Algebraic geometry and number theory" on the occasion of M. A. Tsfasman's and S. G. Vladut's 60th birthdays, Moscow, June 23–27.

[2] Memorial conference in honour of A. Tyurin, Moscow, October 28.

[3] Visit to Indiana University (Blumington, USA), November 3-16. Joint work with V. Lunts.

[4] Seminar at Algebra Section of Steklov Institute, February 25, talk "Equivariant triangulated categories".

[5] Seminar at Algebra Section of Steklov Institute, December (scheduled), talk "Varieties with full strong exceptional collection of line bundles".

3.2.3 Evgeny Feigin

[1] Summer school/PhD-workshop on PBW filtrations of modules for Lie algebras and their appearance/applications in Representation Theory, Glasgow, UK, May 19-23, 2014.

Talk "PBW filtration - representations and flag varieties open questions"

[2] Conference "Lie algebras, algebraic groups and invariant theory", Moscow, January, 27 – February, 1

Talk "Representations of nilpotent algebras, Vinbergs polytopes and generalized flag varieties"

[3] Visit to Glasgow and Edinburgh, UK, October 2014

Talk "Affine degenerate flag varieties and Sato Grassmannians" (University of Glasgow)

Talk "PBW filtration and nonsymmetric Macdonald polynomials" (University of Edinburgh)

3.2.4 Anton Fonarev

[1] Talk “Exceptional vector bundles on Grassmannians” at “Shafarevich Seminar” (Steklov Mathematical Institute), Moscow, September 23.

[2] Talk “Exceptional vector bundles on Grassmannians” at “Arithmetic, Geometry and Coding Theory” (Laboratoire J.-V.Poncelet), Moscow, September 25.

[3] Talk “Structure of derived categories for Grassmannians” at “Lie groups and invariant theory” (Moscow State University), Moscow, October 1.

3.2.5 Alexei Gorodentsev

[1] Conference “Science of the future”, Math. Section, St. Petersburg, September, 17 – September, 20 (`http://www.p220conf.ru/en/program`)

Talk “ Abelian Lagrangian Algebraic Geometry” (abstract is available at `http://www.p220conf.ru/abstracts/download/3-math/208-a-gorodentsev`)

[2] XIII school and conference in theoretical and mathematical physics, Dubna, May, 1 – May, 10 (`http://sevastopol.bogomolov-lab.ru/`)

I was an advisor of section “Geometric quantization” and co-advisor of section “Algebraic K-theory”

3.2.6 Maxim Kazarian

[1] School-Conference “Contemporary problems of mathematics”, IMM, Ekaterinburg, February 02 – 08, 2014

Mini-course “Tropical geometry”

[2] Workshop and minicourse ”Mathematical physics of Hurwitz numbers for beginners” IHES, France, February 13 - 15, 2014

[3] Conference “Mirror Symmetry and Spin Curves” Cortona, Italy, April 28 - 30, 2014

Talk “Topological recursion for enumeration of dessins d’enfant”

[4] International Conference “Embedded graphs”, Euler Institute, St.Petersburg, Russia, October 27 – 31, 2014

Talk “Virasoro constraints and topological recursion for Grothendieck’s dessin counting”

[5] Conference “The Legacy of Vladimir Arnold”, Toronto, Canada, November 24 – 28, 2014

Talk “On Salmon’s enumeration of tangential singularities”

[6] Talks at Moscow HSE seminars

Talk “Topological recursion for counting maps and hypermaps” (Math department of HSE)

Talk “Weil-Petersson volumes and the proof of Witten’s conjecture (after M.Mirzakhani)” (Math department of HSE)

3.2.7 Anton Khoroshkin

[1] Visit Israel, January
Talk "Highest weight categories and orthogonal polynomials"
at *Algebraic Geometry & Number Theory seminar,* Ben Gurion University, Beer Sheba; and at *Algebra seminar,* Bar Ilan, Ramat Gan;
Talk "Around avoidance problem"
at *Combinatorics seminar,* Bar Ilan, Ramat Gan;
[2] Visit to Japan, August
Talk "Hypercommutative operad as a homotopy quotient of BV"
at *String theory Seminar,* Kavli IPMU, 2014;

3.2.8 Grigori Olshanski

[1] Conference "Complex Systems of Interacting Particles".
Bielefeld, Germany, May 19 - 22, 2014
Talk "Markov processes of algebraic origin"
[2] Conference "Representations, Dynamics, Combinatorics: in the Limit and Beyond" in honor of Anatoly Vershik's 80th birthday
St. Petersburg, Russia, June 9–14 , 2014
Talk "New results in asymptotic representation theory"
[3] International Congress of Mathematicians
Seoul, South Corea, August 2014
Invited speaker, "The Gelfand-Tsetlin graph and Markov processes"

3.2.9 Alexei Penskoi

[1] Exact Solvability and Symmetry Avatars, Conference held on the occasion of Luc Vinet's 60th birthday, Centre de recherches mathématiques, Université de Montréal, Canada, August, 25 – 29, 2014.
Talk "Symmetry Reduction and Spectral Geometry"
[2] Lectures at IV Summer School on Geometric Methods in Mathematical Physics, Moscow, June 2014.
Lectures "Spectral Geometry"
[3] International Workshop "Probability, Analysis and Geometry" organized by Lomonosov Moscow State University and Ulm University, Moscow, September, 30 – October, 4, 2014
Talk "Symmetry Reduction and Spectral Geometry"
[3] International Conference "Geometry, Topology and Integrability", Skolkovo Institute of Science and Technology, Moscow, October, 20 – 25, 2014

Talk “Symmetry Reduction and Spectral Geometry”

[4] Saint Petersburg Mathematical Society, March, 25, 2014

Talk “Spectral Geometry: hear the shape, see the sound”

[5] Geometry Seminar of Saint Petersburg Department of V.A.Steklov Institute of Mathematics of the Russian Academy of Sciences, March, 27, 2014

Talk “Metrics extremal for the eigenvalues of the Laplace-Beltrami operator”

[6] Seminar on Multidimensional Complex Analysis (Vitushkin Seminar), Moscow State University, October, 1, 2014

Talk “Geometric optimization of the Laplace-Beltrami operator eigenvalues”

3.2.10 Yuri Prokhorov

[1] *Frontiers of rationality*, Longyearbyen (Spitsbergen) 14–18 July, 2014

Talk: “On stable conjugacy of finite subgroups of the Cremona group”

`http://www.maths.ed.ac.uk/cheltsov/spitsbergen/schedule.html`

[2] *Landau-Ginzburg theory and Fano varieties*, Gyeongju, Korea, May 26–30, 2014

Talk: “Automorphisms of Fano varieties and Jordan properties of Cremona groups”

`http://cgp.ibs.re.kr/conferences/fano/`

[3] *Algebraic varieties and moduli spaces (with emphasis on self maps)*, RIMS, Kyoto University, 4-7 March, 2014

Talk: “On birational involutions of $\mathbb{P}^3$”

`http://www.kurims.kyoto-u.ac.jp/~pioggia/2014Mar.html`

[4] *Complex manifolds, dynamics and birational geometry*, Laboratory of Algebraic Geometry, SU-HSE, Moscow, 10-14 November, 2014

Talk: “Finite groups of birational automorphisms of algebraic varieties”

`http://bogomolov-lab.ru/Bir2014/`

[5] Visit to Institut Fourier, Université de Grenoble I, France (May 2014, Visiting Professor)

Talk: “Jordan properties of groups of birational automorphisms”, Séminaire ”Algèbre et géométries”, Institut Fourier, Grenoble, Lundi, 12 Mai, 2014

`http://www-fourier.ujf-grenoble.fr/?q=fr/content/yuri-prokhorov`

[6] Visit to Institute of Mathematics University of Basel (May 19-21 2014)

Talk: "Jordan properties of groups of birational automorphisms", Seminar "Algebra and Geometry", Institute of Mathematics University of Basel, 20.05.2014, 11:00 bis 12:00

https://math.unibas.ch/aktuelles/veranstaltungsdetails/article/seminar-algebra-geom
?tx_ttnews[backPid]=20911&cHash=8c966c8d8d5a112cf8c60c265d619064

[7] Visit to RIMS, Kyoto (February-March 2014)

[8] Traditional winter session MIAN–POMI, 18-20 December, 2014, St. Petersburg Department of Steklov Mathematical Institute, St. Petersburg

Talk: "Cylinders on Fano manifolds"

[9] Steklov Mathematical Institute Seminar, November 27, 2014

Talk: "Birational geometry and effective theory of minimal models "

http://www.mathnet.ru/php/seminars.phtml?presentid=10484&option_lang=eng

[10] Scientific session of the Steklov Mathematical Institute dedicated to the results of 2013 November 20, 2013

Talk (with C. Shramov): "Jordan properties of groups of birational automorphisms"

http://www.mathnet.ru/php/presentation.phtml?option_lang=eng&presentid=10425

3.2.11 Petr Pushkar'

[1] Talk "Morse theory on manifolds with boundaries" Annual memorial conference dedicated to the memory of Andrei Nikolaevich Tyurin (Fall 2014)

3.2.12 Sergei Rybakov

[1] Conference "Frontiers of rationality", Longyearbyen (Spitsbergen), July, 14 – 18, 2014

[2] Conference "Algebraic Geometry and Number Theory" on the occasion of M.A. Tsfasman's and S.G. Vladuts' 60th birthday, Moscow, June, 23 – 27, 2014

3.2.13 Anton Savin

[1] Conference “XXXIII Workshop on Geometric Methods in Physics”, Bialowieza, Poland, June 29 – July 5, 2014

Talk “Differential equations on complex manifolds”

[2] Conference “Complex Analysis and Related Topics”, St. Petersburg, Russia, April 14–18, 2014

Talk “Differential equations on complex manifolds”

[3] Conference “Modern methods and problems of operator theory and harmonic analysis”, Rostov on Don, Russia, April 27 – May 1, 2014

Talk “On the index of nonlocal elliptic problems”

[4] Conference “Voronezh spring mathematical school — Pontryagin readings”, Voronezh, Russia, May 3–9, 2014

Talk “Differential equations on complex manifolds”

[5] Conference “Seventh International Conference on Differential and Functional Differential Equations”, Moscow, Russia, August 22–29, 2014

Talk “Differential equations on complex manifolds”

[6] Visit to Hannover (Institute of Analysis) 6 July–August 26

[7] Talk ”Differential equations on complex manifolds” at the seminar on Differential and functional-differential equations (Peoples’ Friendship University of Russia), September 16.

3.2.14 Arkady Skopenkov

[1] Conference of Moscow Institute of Physics and Technology, November, 2014
Talk “Basic embeddings and Hilbert’s 13th problem”

[2] International conference on embeddeed graphs, October, 2014
Talk “Classification of link maps”

[3] Topology Seminar, Faculty of Mathematics, Higher School of Economics,
Talk “How do autodiffeomorphisms act on embeddings”

[4] Topology Seminar, Steklov Mathematical Institute,
Talks “Classification of knotted tori”

[5] Postnikov memorial seminar, Moscow State University,
Talk “How do autodiffeomorphisms act on embeddings”

[6] Seminar of N.P. Dolbilin and N.M. Moschevitin, Moscow State University,

Talk “Embedding and knotting of manifolds in Euclidean spaces”

[7] Seminar of A. M. Raigorodskiy, Moscow State University,
Talk “Embedding and knotting of manifolds in Euclidean spaces”

3.2.15 Mikhail Skopenkov

[1] International congress of mathematicians, Seoul, Korea, 12-21.08.2014
Talk “Discrete complex analysis: convergence results”
[2] International conference ”Embedded graphs” Sankt-Petersburg, Russia 27-31.10.2014
Talk: Discrete complex analysis: convergence results
[3] Talks at several seminars and other conferences in Moscow and Sankt-Petersburg.

3.2.16 Evgeni Smirnov

[1] Workshop “Convex Bodies and Representation Theory ”, Banff, Canada, February 2–7, 2014
Talk “Schubert calculus and Gelfand–Zetlin polytopes”
[2] Seoul ICM 2014 Satellite Conference on Topology of Torus Actions and Applications to Geometry and Combinatorics, Daejeon, Republic of Korea, August 7–11, 2014
Talk: “Schubert polynomials and pipe dreams”
[3] School-conference “Lie Algebras, Algebraic Groups and Invariant Theory”, Moscow, Russia, January 27 – February 1, 2014
Talk “Determinantal formulas for flagged Schur polynomials”
[4] Singularity Day, University of Warwick, England, October 30, 2014
Talk: “Plane curves and bialgebra of Lagrangian subspaces”
[5] Research stay at the University of Warwick, England, September 7 – November 2, 2014
[6] M.M.Postnikov Algebraic Topology Seminar, Lomonossov Moscow State University, Russia, April 15, 2014
Talk: Schubert polynomials and pipe dream complexes
[7] Colloquium, Durham University, England, September 29, 2014
Talk: “Enumerative geometry and Gelfand–Zetlin polytopes”
[8] Algebraic Geometry Seminar, University of Warwick, England, October 8, 2014
Talk: “Determinantal formulas for flagged Schur and Schubert polynomials”
[9] Algebra Seminar, University of Birmingham, England, October 14, 2014
Talk: “Schubert calculus and Gelfand–Zetlin polytopes”
[10] EDGE Geometry Seminar, University of Edinburgh, Scotland, October 23, 2014
Talk: “Spherical double flag varieties”

[11] Lie Groups and Invariant Theory seminar, Lomonossov Moscow State University, Russia, November 12, 2014

Talk: "Determinantal formulas for flagged Schur and Schubert polynomials"

[12] Colloquium, Nizhny Novgorod Mathematical Society, November 14, 2014

Talk: "Enumerative geometry and Gelfand–Zetlin polytopes"

3.2.17 Stanislav Shaposhnikov

[1]International Scientific Conference on "Modern Problems of Mathematics and Mechanics" devoted to the Academician V.A. Sadovnichys 75th Anniversary, talk "On Fokker–Planck–Kolmogorov equations", Moscow, April, 2014.

[2]International conference Spectral Theory and Differential Equations devoted to the 100th anniversary of B.M. Levitan, talk "On uniqueness of solutions to the cauchy problem for degenerate Fokker-Planck-Kolmogorov equations", Moscow, June, 2014.

3.2.18 Ilya Vyugin

[1] Workshop "Unlikely intersections", CIRM, Luminy, France, 3.02.2014-7.02.2014,

talk "On the intersection of additive shifts of subgroups of F_p^*".

[2] International conference on Differential and Functional Differential equations, Moscow, 22.08.2014-29.08.2014,

talk "On the Continous Limit for Systems of Diference Equations".

[3] The International Conference on Differential Equations and Dynamical Systems, Suzdal, Russia, 4.07.2014-9-07.2014,

Talk "Some estimates over finite fields and their continous analogs".

[4] Seminar "Analitic theory of differential equations" (D.V Anosov, V.P. Lexin), MIAN RAS

Talk "On the Continous limit for difference equations".

[5] Seminar "Modern Problems of Number theory" (S.V. Konyagin, I.D. Shkredov), MIAN RAS

Talk "On the number of solutions of an equation over finite field".

[6] Seminar of the group of complex analysis (B. Lamel, I. Kossovskiy, D. Dela Sala), University of Vienna

Talk "Linear differential equations and holomorphic vector bundles".

3.3 Teaching

3.3.1 Yurii Burman

[1] Classical mechanics and foundations of quantum mechanics, Independent University of Moscow, special course, September-December 2014, 2 hours of lectures + 2 hours of exercises per week.

Program

1. Kinematics.
   (a) Configuration spaces and phase spaces.
   (b) Building new configuration spaces out of old ones.
   (c) Reference frames.
2. Lagrangian mechanics.
   (a) Principle of least action.
   (b) How to find a Lagrangian for a physical system.
3. Hamiltonial mechanics
   (a) Hamilton's equations.
   (b) Symmetries.
4. Foundations of quantum mechanics.
   (a) Asymptotics of integrals.
   (b) Path integration.

[2] Introduction to topology, Higher School of Economics, II year students, April–December 2014, 6 hour per week.

Program.

1. Metric and topological spaces.
2. Separability, connectedness, arcwise connectedness, compactness.
3. Homotopies.
4. Fundamental group.

5. Coverings.
6. Covering homotopy property.
7. Vam Kampen's theorem.
8. Fundamental group of a simplicial space.
9. Group of a link.
10. Singular complex.
11. Singular homology and homotopy invariance.
12. Exact sequences and Bokstein's construction.
13. Meyer–Vietoris sequence. Homology of spheres.
14. Brouwer's theorem.
15. Degree of a map.
16. Fundamental group vs. first homology.
17. Homology with coefficients; cohomology.
18. Cup product and smash product.
19. Top homology of a manifold.
20. Poincaré duality.
21. Intersection index.

3.3.2 Andrei Domrin

[1] Complex Analysis. Independent University of Moscow, II year students, February-May 2014, 2 hours per week.

Program.

1. Differentiation (definition of a holomorphic function, Cauchy–Riemann equations, the exponential function and the logarithm, conformal maps, the point at infinity).

2. Integration (the Newton–Leibniz formula, the lemma of Goursat, antiderivative in a disk, the Cauchy integral theorem, the residue theorem, the Cauchy integral formula).

3. Power series (the Taylor expansion of a holomorphic function in a disk, Cauchy inequalities, Liouville theorem, infinite differentiability of holomorphic functions, the uniqueness theorem).

4. Laurent series (the Laurent expansion of a holomorphic function in an annulus, classification of isolated singular points, formulae for residues at poles, Sokhotskii's [Casorati–Weierstrass] theorem).

5. Local properties (the argument principle, Rouché's theorem, the open mapping principle, the inverse function theorem, the maximum modulus principle).

6. Sequences of holomorphic functions (the theorems of Weierstrass and Hurwitz, the compactness principle).

7. The theorems of Riemann and Carathéodory (they say which domains can be conformally mapped onto the unit disk and what may happen at the boundary).

8. Analytic continuation (definitions and examples, the sheaf of germs of holomorphic functions, the monodormy theorem, branches of analytic functions, classification of ramification points).

9. The symmetry principle, the modular function, and the Picard's theorems.

10. Infinite products (conditions for their convergence, construction of holomorphic functions with prescribed zeros, description of zero sets of bounded holomorphic functions in the unit disk).

11. The Hardy–Ramanujan–Uspensky asymptotic formula for the partition function.

[2] Complex Analysis, Moscow State University, III year students, September-December 2014, 2 hours per week.

Program.

1. Continuity and limits of functions of a complex variable. Real and complex differentiability. Cauchy–Riemann equations. Definition and properties of holomorphic functions. The function e^z.

2. Conformal maps. Their relation to holomorphic functions. The extended complex plane and linear fractional maps. Preservation of (generalized) circles and symmetry with respect to a circle. Description of all linear fractional maps of the unit disk onto itself.

3. Definition and properties of the integrals $\int_\gamma f(z)\,dz$. Examples of their calculation by the Newton–Leibniz formula. The lemma of Goursat. The existence of an antiderivative in a disk. The Cauchy theorem on the integral over the boundary of a domain.

4. The notion of residue. The Cauchy residue theorem. Calculation of residues. Example: calculation of the Fourier transform of $(1+x^2)^{-1}$. Another example: the Cauchy integral formula.

5. Taylor series expansions of holomorphic functions. The Cauchy inequalities. The Liouville theorem. Its corollaries.

6. The set of convergence of a power series. The Cauchy–Hadamard formula. Infinite differentiability of holomorphic functions. The Cauchy integral formula for derivatives. The Morera theorem. Three equivalent definitions of a holomorphic function.

7. Zeros of holomorphic functions. The notion of order. The notion of a domain. The uniqueness theorem.

8. Laurent series expansions of hoomorphic functions. The Cauchy inequalities. Calssification of isolated singular points of single-valued functions. The notion of order of a pole. Formulae for the residues at poles. Sokhotskii's theorem.

9. Infinity as an isolated singular point. Description of all conformal maps of the plane and the extended plane onto themselves. Description of functions meromorphic on the extended plane. The partial fraction decompostion of rational functions.

10. Calculation of the sums of series by means of residues. The partial fraction expansion of the cotangent function. The Mittag–Leffler theorem.

11. The Weierstrass theorem on the expansion of an entire function into a product over its zeros. The Euler infinite product for the sine function. Construction of holomorphic functions with prescribed zeros in an arbitrary domain. An interpolation theorem.

12. Holomorphic dependence of integrals on parameters. The definition and analytic extension of the Euler Γ-function. A proof of the reflection formula for the Γ-function by means of residues. The definition and analytic extension of the Riemann ζ-function.

13. The notions of a function element, analytic continuation along a path, complete analytic functions in the sense of Weierstrass. Definitions and examples of analytic functions in a domain.

14. The theorem on analytic continuation along homotopic paths. The notion of a simply connected domain. Examples. The mondoremy theorem.

15. The notion of number of sheets of an analytic function in a domain. Description of analytic functions with number of sheets equal to 1. Corollaries of the monodromy theorem (the existence of branches, antiderivatives, roots and logarithms of holomorphic functions in simply connected domains).

16. Operations over analytic functions. Classification of ramification points. Examples. The difference between the notions of a complete analytic function and an analytic function in a domain. Puiseaux series.

17. The notions of one-dimensional complex manifolds and holomorphic maps. Covering maps and the path lifting property. The Riemann surface of a function analytic in a domain.

3.3.3 Alexei Elagin

[1] Algebraic geometry - 1. Independent University of Moscow & MIPT, II year students/master students, September-December 2014, 2 hours per week.

Program:

1. Affine algebraic sets
2. Affine varieties
3. Local properties of plane curves

4. Projective varieties
5. Projective plane curves
6. Varieties, morphisms, and rational maps
7. Resolution of singularities
8. Riemann-Roch theorem

Topics of exercise sheets:

1. The first one
2. Lines and conics
3. Infinite points
4. Varieties and regular maps
5. Quadric surfaces
6. Images, preimages and poles
7. Plane curve singularities
8. Curve singularities — 2
9. Hilbert polynomial
10. Cubic curves
11. Divisors

See *http://ium.mccme.ru/f14/elagin-f14.html* for the lecture drafts and exercise sheets.

3.3.4 Evgeny Feigin

[1] Infinite dimensional Lie algebras and vertex operator algebras, from 2nd year to PhD students, September-December 2014, 2 hours per week.

Program

The theory of Lie groups and Lie algebras is one of the central areas of modern mathematics. It has various interrelations with algebraic geometry, combinatorics, theory of symmetric functions, integrable systems, classical and quantum field theories. Lie groups and Lie algebras usually show up as the sets of symmetries of objects of a theory. For example, infinite-dimensional Lie algebras (such as affine Kac-Moody algebras) turn out to be very important for the description of many quantum field theories: namely, they are realized as symmetries of the spaces of states. Infinite-dimensional Lie algebras also play an important role in the theory of integrable systems and in algebraic geometry. It turns out that it is very natural in this context to consider more general algebraic objects, the vertex operator algebras. VOAs capture the main properties of the Lie algebras and have rich additional structure. Vertex operator algebras proved to be very useful in many situations; the classical example is the KP integrable hierarchy. They are also extensively used in modern algebraic geometry. Our goal is to give an introduction to the theory of infinite-dimensional Lie algebras and vertex operator algebras. We describe the main definitions, constructions and applications of the theory.

3.3.5 Anton Fonarev

[1] Coherent sheaves on algebraic varieties. Independent University of Moscow, III–IV year students, September–December 2014, 2 hours per week.

- Where coherent sheaves come from, what one can do with them, and why there are not scary.
- Cohomology and classical derived functors. How to compute.
- Spectral sequences as a tool for a modern mathematician.
- Nice classes of morphisms and how to understand flatness.
- Solving equations using the functor of points.
- Hilbert polynomial and Hilbert schemes.
- Chern classes and Rieman–Roch.
- Derived categories for a working mathematician.

- Exceptional collections and semiorthogonal decompositions.
- Serre duality as seen by a functor.
- How Bondal and Orlov find varieties in their derived categories.

3.3.6 Alexei Gorodentsev

[1] Algebra-2. Independent University of Moscow, I year students (this is 2nd term of 1 year course), February–May 2014, 4 hours per week.
(`http://gorod.bogomolov-lab.ru/ps/stud/algebra-1/1314/list.html`)
Program.

1. Groups actions, orbit length, the Polia-Bernside enumeration of orbits, regular and adjoint actions. Normal subgroups and quotients, p-groups and the Silov theorems.
2. The Jordan–Hölder theorem for groups. Free groups, generators and relations. Presentation of symmetric groups.
3. Non-degenerated bilinear forms, orthogonal projections. Skew symmetric forms.
4. Non degenerated symmetric bilinear functions, Witt theory.
5. Hermitean spaces, distances, angles. Orthogonal diagonalization of normal operators. Polar decomposition.
6. Complex and real structures. Hermitean extensions of a symplectic structure form the Siegel domain.
7. Quaternions, the covering $\mathrm{SU}_2 \to \mathrm{SO}_3$.

[2] Algebra-3, Independent University of Moscow, II year students, September-December 2014, 4 hours per week.
(`http://gorod.bogomolov-lab.ru/ps/stud/algebra-3/1415/list.html`)
Program.

1. Tensor products of modules over a commutative ring.
2. Tensor algebra, symmetric algebra, and exterior algebra of a vector space. Linear support of a tensor, Plücker, Segre, and Veronese varieties.
3. Symmetric and skew symmetric tensors in zero characteristic. Polarization of polynomials.

4. Symmetric functions, transitions between standard bases in $\mathbb{Z}$-module of symmetric functions.

5. Young tableaux calculus, Schur polynomials, Littlwood–Richardson rule.

6. Semisimple modules over associative algebras, Schur lemma, double commutator theorem, surjectivity of irreducible representations.

7. Finite dimensional $\mathfrak{sl}_2$-modules.

8. Representations of finite groups: structure of the group algebra, character theory, ring of representations, induced representations and Frobenius reciprocity.

9. Representations of symmetric groups. Shur-Wayl correspondence.

10. Categories and functors, Yoneda lemma, adjoint functors, (co)limits of diagrams.

[3] Algebraic Geometry: Start Up Course, Independent University of Moscow, Math. In Moscow program for foreign students, September-December 2014, 4 hours per week. (`http://gorod.bogomolov-lab.ru/ps/stud/projgeom/1415/list.html`)

Program.

1. Projective spaces and projective geometry: projective groups, projections, cross-ratio, the Veronese curve.

2. Projective quadrics: singular points and tangent spaces, linear subspaces on a quadric, polar mapping, pencils of quadrics.

3. Plane curves. Non-rationality of a smooth cubic. Bézout theorem. Dual curve and Plücker relations.

4. Grassmannians and the Plücker embedding. Quadric $Gr(2, 4) \subset \mathbb{P}_5$ in details.

5. Commutative algebra draught: Noetherian rings, integral extetsions, Nullstellensatz, transcendence degree, resultants.

6. Antiequivalence between affine algebraic varieties over algebraically closed field $\Bbbk$ and finitely generated reduced $\Bbbk$-algebras. Maximal spectrum and Zariski topology. Properties of finite morphisms.

7. The structure sheaf and its sections over the principal open sets. Definition of an algebraic manifold. Separability, morphisms from projective to separable varieties are closed.

8. Blow up of a point, projection of projective variety from an external point is finite. Normalization theorems, normalization in algebraic families.

9. Dimension of algebraic variety. Dimensions of fibers, semi-continuity and constructibility. Generic plane section of a projective variety, geometric definition of the dimension of a projective variety.

10. 27 lines on a smooth cubic surface.

11. Locally trivial vector bundles and locally free $\mathcal{O}_X$-modules. Line bundles and Picard group, Pic(Spec$A$) = 0 for factorial A, working examples: $\mathrm{Pic}(Gr(k,n)) = \mathbf{Z}$. Splitting of locally trivial vector bundles on a line.

12. Cotangent sheaf and tangent bundle. Local properties of smooth points.

[4] Geometric Introduction to Algebraic Geometry (this is 2-nd term of 1 year course), Faculty of Mathematics, National Research University "Higher School of Economics", February–May 2014, 4 hours per week.
(http://gorod.bogomolov-lab.ru/ps/stud/giag_ru/list.html)
Program.

1. Sheaves of $\mathcal{O}_X$-modules and $\mathcal{O}_X$-algebras. Relative affine and projective manifolds over X. Families of vector spaces.

2. Relative cotangent sheaf and tangent bundle. Properties of smooth morphisms. Degree of a locally free finite morphism. Étale morphisms.

3. Examples of tangent bundles: Grassmannians, flags, other incidence varieties. The second fundamental form and the Gauss mapping.

4. Blow up of subschemes.

5. Local properties of smooth points: factoriality, Cohen-Macolay, etc.

6. Working example: algebraic toric varieties. Normality and criteria of smoothness, resolution of singularities. Picard group and Cox ring, description of ample line bundles. Example of a non-projective algebraic variety.

7. Degree of projective variety and the Bézout theorem.

8. Algebraic cycles: rational equivalence, pull back and push forward, intersection with divisors. Introduction to the intersection theory, chow ring of a smooth variety.

9. Localized Chern classes of vector bundles. Splitting principle. Degeneration loci of morphisms of vector bundles.

10. Chow varieties of effective cycles on $\mathbb{P}_n$.

3.3.7 Maxim Kazarian

[1] Calculus. Independent University of Moscow, I year students (2nd term), January-May 2014, 2 hours per week.

Program

Representation of curves given parametrically and implicitely.*Singular and other special points, behaviour near the special points and at infinity, asymptotes.*

Differentiation of functions in several variables. *Directional derivative, examples of non-differentiable functions possessing partial derivatives, equality of mixed partial derivatives, chain rule for the derivative of a composition.*

Taylor theorem for functions in several variables.

Critical points. *Maxima, minima, and saddle points of functions in two variables, second differential, Hessian, Morse index, Morse lemma.*

Inversion function theorem. *Invertibility of a mapping and non-degeneracy of the Jacobian, derivative of the inverse function, curvilinear coordinates.*

Contraction mapping principle. *The proof of the inversion mapping theorem.*

Implicit function theorem. *The derivative of an implicit function, smooth multidimensional surfaces in Euclidean spaces, local coordinates.*

Conditional extremum. *The derivative of the restriction function, Lagrange multipliers, Morse index of the restriction function.*

Typical singularities of maps of surfaces. *Foldings and pleats, developable of a family of curves, evolute of a plane curves and rigidity of its singular points.*

[2] Differential Geometry, Moscow State University, III year students, September-December 2014, 4 hours per week.

Program.

Vector fields and differential forms. *Lie bracket, exterior differential and wedge product, distributions, Frobenius criterium of integrability.*

Plane and space curves. *Length, curvature, focal set of a plane curve, normal and geodesic curvature of a space curve on a surface.*

Surface geometry. *Riemann structure, IInd quadratic form, principal curvatures, Gaussian curvature.*

Gauss' Theorema Egregium. *Connection and curvature forms of a metric on a surface, Euclidean coordinates of a flat metric.*

Topological connection. *Fiber bundles, trivializations, parallel transport, curvatur as an infinitesimal holonomy.*

Covariant derivative. *Vector bundles, sections, connection matrix, structure Cartan equation, curvature tensor.*

Riemann manifolds. *Levi-Civita connection, Riemann tensor, geodesics.*

3.3.8 Anton Khoroshkin

[1] Homological Algebra. Independent University of Moscow, II–III year students, September-December 2014, 2 hours per week.

Program

The goal of the course is to give the introduction to the language of derived functors and their applications. In particular, we will emphasize on the following subjects:

- Comlexes and homology;
- Basic category theory;
- Projective modules;
- Exact functors and projective resolutions;
- Derived functors;
- Functors Tor and Ext;
- Ext and extensions;
- Homological dimension;
- Spectral sequences;
- Group cohomology;
- Derived and Triangulated categories;

[2] Basic representation theory, III year students, Independent University of Moscow, Math in Moscow, Higher School of Economics, 4 hours per week.

Program.

This is an introduction to representation theory. Upon completion of this course student will know enough to understand classical applications and will be ready to take more advanced courses leading to modern problems and applications of representation theory. In more detail, the aims are to learn

- common notions and problems of representation theory;
- various instruments for dealing with finite-dimensional representations of finite groups: intertwining operators, characters, Maschke's and Burnside's theorems;
- representations of ring and algebra in a particular case of group ring and algebra.

- representations of symmetric group and related algebraic and combinatorial constructions: Young diagrams and tableau, Young symmetrizers;
- basics of Lie algebras and their representations;
- representations of $\mathsf{sl}_2(\mathbb{C})$.

3.3.9 Andrei Levin

[1] Algebra Independent University of Moscow, I year students, September-December 2014, 2 hours per week.

Program

1. Rings and fields. Polynoms. Euclid algorithm. Prime decomposition.

2. Systems of linear equations Gauss algorithm.

3. Vector spacees. Linear independence, basis, dimension.

4 Volume form. Determinant and linear independence.

5. Symmetric group, sign of permutation. ealisation of the determinant as sum over symmetric group.

6. Resultant of polynoms. Solution of the system of the polynomial equations.

7. Modules over the polynomial ring. Jordan and Frobenius normal form of matrix.

3.3.10 Grigori Olshanski

[1] Representations and Probability. Independent University of Moscow. January–April and September-December 2014, 2 hours per week.

Program

Lectures at the research seminar on combinatorial, algebraic, and probabilistic aspects of representation theory and related topics: Characters of classical groups. Radial part of the Haar measure on a compact classical groups. Second H. Weyl's character formula. Karlin-McGregor theorem for Markov chains. Birth-death processes related to discrete orthogonal polynomials. Orthogonal polynomial ensembles. Orbital integrals. Orthogonal polynomials of hypergeometric type. Roots of orthogonal polynomials. Kerov's theorem on asymptotics of interlacing sequences arising from roots of orthogonal polynomials.

[2] Geometry and Dynamics. National Research University Higher School of Economics, September-December 2014, 2 hours per week.

Program

A seminar for 1-2 year students, joint with A. Bufetov, A. Klimenko, and V. Timorin. I gave a series of introductory lectures "Geometry of classical groups" (generalities on group actions; Grassmanians; Jordan angles between linear subspaces; flag varieties; Bruhat decomposition; Schubert cells).

3.3.11 Alexei Penskoi

[1] Lectures on Spectral Geometry II, Independent University of Moscow, $\geq$3rd year students, February-June 2014, 2 hours per week.

Program.

1. Nadirashvili-El Soufi-Ilias Theorem about extremal metrics and minimal submanifolds and spheres, after the paper A. El Soufi, S. Ilias "Laplacian eigenvalue functionals and metric deformations on compact manifolds", Journal of Geometry and Physics 58 (2008) 89-104.

2. Estimates of multplicities of eigenvalues of Laplace-Beltrami operator, after the papers N. Nadirashvili, Math. Sb., 133(175):2(6) (1987), 223-237 (Russian); T. Hoffmann-Ostenhof, P.W. Michor, N. Nadirashvili "Bounds on the Multiplicity of Eigenvalues for Fixed Membranes", Geom. Funct. Anal. Vol. 9 (1999) 1169-1188.

3. Steklov problem, after the paper A. Girouard, Iosif Polterovich, "Shape optimization for low Neumann and Steklov eigenvalues", Math. Meth. Appl. Sci. 2010, (33) 501-516.

4. Bounds on the second non-zero Steklov eigenvalue, after the paper A. Girouard, Iosif Polterovich, "Shape optimization for low Neumann and Steklov eigenvalues", Math. Meth. Appl. Sci. 2010, (33) 501-516.

5. Hersh-Payne-Schiffer Theorem, after the paper J. Hersch, L. E. Payne, M. M. Schiffer, "Some inequalities for Stekloff eigenvalues", Arch. Ration. Mech. Anal. 1974 Vol 57, 99-114.

6. Recent results from papers by Fraser and Schoen.

7. Isospectrality, after the chapter V of the book M. Levitin, L. Boulton, "Trends and tricks in spectral theory" and after the papers P. Buser, J. Conway, P. Doyle, K.-D. Semmler, "Some planar isospectral domains", Internat. Math. Res. Notices 1994, no. 9, 391-400.

8. Length of nodal curves, after the paper Alessandro Savo, "Eigenvalue estimates and nodal length of eigenfunctions", Steps in Differential Geometry, Proceedings of the Colloquium on Differential Geometry, 25-30 July, 2000, Debrecen, Hungary.

9. Estimates of multplicities of Steklov eigenvalues, after the papers M. Karpukhin, G. Kokarev, I. Polterovich, "Multiplicity bounds for Steklov eigenvalues on Riemannian surfaces", Preprint arXiv:1209.4869.

10. Filonov proof of the Friendlander inequality, after the paper N. Filonov, Algebra i analiz, 16:2 (2004), 172-176 (Russian).

[2] Introduction to Spectral Geometry, Independent University of Moscow, ≥2rd year students, September-December 2014, 2 hours per week.

Program.

1. Laplace-Beltrami operator on Riemannian manifolds
2. Eigenvalues of Laplace-Beltrami operator (Dirichlet problem, Neumann problem, problem on manifolds without boundary)
3. Variational desription of eigenvalues, Rayleigh quotient
4. Weyl function and its asymptotics
5. Inequalities for eigenvalues, Dirichlet-Neumann bracketing
6. Nodal domains, Courant nodal domain theorem
7. Isoperimetric inequalities, symmetrization
8. Cheeger isoperimetric constant, Cheeger inequality
9. Conformal volume, Yang-Yau inequality
10. Extremal metrics and minimal submanifolds of spheres

[3-4] Differential Geometry. Math in Moscow program of the Independent University of Moscow for undergraduate students from the U.S. and Canada, February-May & September-December 2014, 4 hours per week (lecture 2 hours + exersise class 2 hours).

1. Plane and space curves. Curvature, torsion, Frenet frame.
2. Surfaces in 3-space. Metrics and the second quadratic form. Curvature.
3. Connections in tangent and normal bundles to a k-surfaces in $\mathbf{R}^n$.
4. Parallel translations.
5. Geodesics.
6. Gauß and Codazzi formulas. "Theorema egregium" of Gauß.
7. Gauß-Bonnet theorem.
8. Extremal properties of geodesics. Minimal surfaces.

9. Vector bundles, connection in vector bundles.
10. Levi-Civita connection.
11. Connection curvature. Riemann curvature tensor.

[5] Geometry-I, National Research University "High School of Econmics", September-December 2014, 4 hours per week (lecture 2 hours + exersise class 2 hours).

1. Conical sections
2. Descartes approach to conics
3. Vector spaces, Affine spaces
4. Lines, planes, k-planes
5. Scalar product, Vector product, Oriented area and volume
6. Orthogonal transformations, Euler angles and Cayley-Klein theorem.
7. Quadrics and their orthogonal invariants.
8. Projective space, duality.
9. Felix Klein's Erlangen program.
10. Euclidean, affine and projective geometry of quadrics.
11. Bilinear and quadratic forms.

[6] Exercise classes for various courses at Moscow State University: Classical Differential Geometry, February-May 2014, 2 hours per week; Analytic Geometry, September-December 2014, 4 hours per week; Differential Geometry, September-December 2014, 2 hours per week.

3.3.12 Yuri Prokhorov

[1] Automorphisms of algebraic varieties, Independent University of Moscow and Steklov Institute, graduate students, September-December 2014, 2 hours per week.

Program: Introduction. Automorphisms of algebraic curves. Elliptic curves. Hurwitz bound. Klein quartic. The structure of the Picard group. The action of the automorphism group on $\mathrm{Pic}(X)$. Examples. A variety with nef canonical class is not uniruled. Automorphisms of hypersurfaces and complete intersections. Cubic surfaces. Their automorphisms. K3 surfaces. Basic properties. Examples: complete intersections and the double covers. Further examples of K3 surfaces. Automorphisms. Finite groups of automorphisms. Automorphisms without fixed points. Enriques surface. Examples. Symplectic automorphisms. Actions on the lattice of transcendental cycles. Action on the automorphism group on the cohomology is faithful. Holomorphic Lefschetz fixed-point formula. Orders of symplectic automorphisms. Notion of Du Val singularities. Their resolutions. Quotients of K3 surfaces for different types of automorphisms. Mori cone of a surface. (-2)-curves on K3 surfaces. Cone of ample divisors. Description cones $\mathrm{NE}(X)$ and $\mathrm{Amp}(X)$ for K3 surfaces and abelian surfaces. The Fano-Severi example of K3 surfaces with an infinite automorphism group. Other examples. Minimal models. Relative version. Minimal resolution of singularities of surfaces. Kummer surface. Examples (the product of elliptic curves and Jacobians). More about automorphisms of K3 surfaces with Picard number 2 (two examples). Picard group of Kummer surfaces. Examples of K3 surface with a maximal Picard rank. Elliptic pencils on K3 surfaces. Basic facts. Examples: Kummer and quartic surfaces with lines. Mordell-Weil group of an elliptic K3 surface. Mordell-Weil group and automorphisms. Minimal models of surfaces. Kodaira dimension and canonical ring. The classification theorem. Elliptic pencils. Zariski lemma . Classification of degenerate fibers. Multiple fibers. Examples. The formula for the canonical divisor of elliptic fibrations. Bielliptic surfaces. Description. Examples. Automorphisms of abelian varieties. Enriques surfaces. Properties. The number of parameters. Examples. Properties of an elliptic pencil on Enriques surfaces. Picard lattice of an Enriques surface. Properties. Reflections in the Picard lattice. The existence of an elliptic pencil on Enriques surfaces. Representation of the general Enriques surface in the form of "double quadric." Automorphisms of Enriques surfaces. Surfaces of general type. Examples. Bounding the automorphism group of surfaces of general type.

[2] Algebraic curves, part 1 (introduction), Moscow State University, 3 year students, September-December 2014, 2 hours per week.

Program: Introduction. Algebraic varieties. Rings. Ideals. Examples. Zariski topology. Noetherian rings. Examples. Basic theorems of commutative algebra. Prime and maximal ideals. Decomposition into irreducible components. Dimension. Ring of regular functions $\daleth[X]$ of an affine algebraic variety. Field of rational functions $\daleth(X)$. Local ring and its maximal ideal. Transcendence degree of $\daleth(X)$ and dimension. Differentials. The tangent space. and its various interpretations. The set of singular points. Complex manifolds. Local parameters. Formal power series. Factoriality of the local ring of a smooth point. Discrete valuations. Examples. Local ring of a smooth point on the curve (different characterizations). Homogeneous rings and ideals. Projective varieties. Algebraic curves. Differentials.

[3] Algebraic geometry (teaching seminar, with D. Orlov and C. Shramov), Steklov Institute, 3-5 year students, September-December 2014, 2 hours per week.

Program: Schemes. First Properties. Separated and Proper Morphisms. Sheaves of Modules. Cohomology. Derived Functors. Cohomology of Sheaves. Cohomology of a Noetherian Affine Scheme. Cech Cohomology. The Cohomology of Projective Space.

[4] Explicit birational geometry and Fano varieties, European Mathematical Society School "New Perspectives on the classification of Fano Manifolds", September 29 - October 3, 2014, Udine (Italy), 5 lectures and 3 seminars, 15 hours.

Program: The object of this course is to reflect the present state of the classification theory of Fano varieties. This classification is closely related to the one of the finite subgroups of the group $\mathrm{Bir}(X)$ of birational automorphisms of a Fano variety X and these subgroups can be studied by a method coming from the Mori theory. In the course we will present many recent applications of this method obtained by many mathematicians (Beauville, Blanc, Dolgachev, de Fernex, Iskovskikh, Lamy, Cantat, Cheltsov, Shramov and others). The following topics will be covered:

basic properties, lower-dimensional Fano varieties; equivariant Mori theory: arithmetic and geometric cases; singular Fano varieties and "good" degenerations; applications: rationality problems, Cremona groups, etc.

`http://fano.dimi.uniud.it/index.html`

3.3.13 Petr Pushkar'

[1] Topology-3. Independent University of Moscow, 2 year students, February-May 2014, 4 hours per week.

We tried to cover following topics in Topology

Program

Obstruction theory

Vector bundles and classyfing spaces

Characteristic classes

Spectral sequence

[2] Differential Geometry-2, High School of Economics, Dept of Math, 3-4 year students January-June 2014, 4 hours per week.

The following topics were considered

1. Integrability of distributions 2. Elements of Symplectic Jeometry 3. Elements of Contact geometry 4. Connection, curvature 5. Riemanian Geometry, geodesics, Morse theory

[3] Calculus on manifolds. Independent University of Moscow, 2 year students, September-December 2014, 4 hours per week.

Program

1. Manifolds and submanifolds, implicit function theorem

2. Tangent space, vector fields, diffeomorfisms and phase flows
3. Transversality, Sard's theorem, Whitney embedding theorem
4. Tensors on manifolds, differential forms, exterior and Lie derivative
5. Orientation, manifold with the boundary, Stokes theorem
6. Frobenius integrability theorem
7. Moser trick - Morse lemma and Darboux theorem

[4] Special course in Morse theory on manifolds with boundary

The course is devoted to Morse theory on manifolds with the boundary.

[5] Basics in Contact and Symplectic geometry

The main purpose of the course is to give an introduction to Symplectic and Contact topology and geometry. I will try to cover following topics.

1. Symplectic and contact structures Darboux theorems. Examples of symplectic and contact manifolds.
2. Hamiltonian vector fields and Arnold conjecture.
3. Generating families, lagrangian and legendrian mappings.
4. Symplectic and contact reductions, examples.
5. Maslov class.
6. Chekanov theorem.
7. Examples and applications.

3.3.14 Sergei Rybakov

[1] Algebraic geometry codes, September-December 2014, 1 hour per week.

The three-term course "Algebraic geometry and coding theory" is tought by Alexey Elagin, Sergey Galkin and Sergey Rybakov in turn, one term each. This is the third term devoted to the coding theory. Students who did not attend previous two terms on algebraic geometry are also accepted into the class. For this reason a breaf overview of algebraic curves over finite fields is included into the program.

Program:

1. Linear codes. Examples. $[n, k, d]_q$-systems.

2. Algebraic curves over finite fields. Riemann-Roch theorem. Zeta functions and Weil conjectures.

3. Definitions of algebraic geometry codes. The decoding problem.

4. Codes of low genus. Elliptic curves and elliptic codes.

5. Asymptotic bounds. Drinfeld-Vladut theorem.

6. Curves with many points and modular curves.

3.3.15 Anton Savin

[1] Differential equations on complex manifolds (with B.Yu. Sternin), Peoples' Friendship university of Russia, PhD students, January–May 2014, 2 hours per week.

Program.

1. Leray residues. Definitions. Leray exact sequences and residue theorems.

2. Ramified integrals. Why do integrals ramify? General theory. Landau manifolds. Integrals over relative cycles.

3. Asymptotics of ramified integrals. Ramification of cycles around Landau manifolds (Picard–Lefschetz theorem). Leray theorem on asymptotic of integrals.

4. Main integral transform. Definition of the integral transform. Ramified homology classes.

5. Properties of the integral transform.

6. Cauchy problem for equations with constant coefficients.

7. Singularities of the solution of Cauchy problem.

8. Cauchy problem for equations with variable coefficients. Leray uniformization.

9. Application. Balayage inwards problem.

10. Application. Motherbody problem.

[2] Mathematics, Peoples' Friendship University of Russia, I year students of engineering faculty, February-December 2014, 5 hours per week.

Program: linear algebra, analytic geometry, differential and integral calculus in one variable, differential calculus in several variables, ordinary differential equations.

[3] Applied mathematics, Peoples' Friendship University of Russia, II year students of engineering faculty, September-December 2014, 3 hours per week.

Program: classical probability, discrete random variables, continuous random variables, point estimates, statistical intervals, tests of hypotheses.

[4] Mathematics (additional chapters), Peoples' Friendship University of Russia, I-II year students of engineering faculty, February-December 2014, 4 hours per week.

Program: multiple integrals, series, ordinary differential equations, equations of mathematical physics.

3.3.16 Arkady Skopenkov

[1] Combinatorial topology, II year students, February-May 2014, 2 hours per week. Moscow Institute of Physics and Technology (DIHT)

Program. It is shown how in the course of solution of interesting geometric problems (close to discrete mathematics and computer science) naturally appear main notions of algebraic topology (homology groups, obstructions and invariants). Thus main ideas of algebraic topology are presented with minimal technicalities.

Detailed information in Russian: http://www.mccme.ru/circles/oim/home/combtop13.htm

[2] Modern topological methods in physics, II year students, February-May 2014, 2 hours per week. Moscow Institute of Physics and Technology (DGAP)

Program. It is shown how in the course of solution of interesting geometric problems (close to physics) naturally appear main notions of algebraic topology (homology groups, obstructions and invariants). Thus main ideas of algebraic topology are presented with minimal technicalities.

Detailed information in Russian: http://www.mccme.ru/circles/oim/home/combtop13.htm

[3] Knot Theory, IUM, MiM, February-May 2014, 4 hours per week.

Program. Some basic knot theory is studied. In particular, Sato-Levine number is defined and the Vassiliev-Kontsevich Theorem is studied in terms of calculation of the invariants themselves, not of the dimension of the space of the invariants.

Detailed information: http://www.mccme.ru/circles/oim/home/combtop13.htm#mim14knot

[4] Topology-I, I year students, Independent University of Moscow, February-May 2014, 4 hours per week (joint with M. Skopenkov).

Program. It is shown how in the course of solution of interesting geometric problems (close to applications) naturally appear main notions of algebraic topology (homology groups, obstructions and invariants, characteristic classes). Thus main ideas of algebraic topology are presented with minimal technicalities.

Detailed information in Russian: http://www.mccme.ru/circles/oim/home/combtop13.htm

[5] Topology-II, II year students, Independent University of Moscow, September-December 2014, 4 hours per week (joint with M. Skopenkov).

Program. It is shown how in the course of solution of interesting geometric problems (close to applications) naturally appear main notions of algebraic topology (homology groups, obstructions and invariants, characteristic classes). Thus main ideas of algebraic topology are presented with minimal technicalities.

Detailed information in Russian: http://www.mccme.ru/circles/oim/home/combtop13.htm

[6] Discrete analysis (exercises), II year students, September-December 2014, 2 hours per week. Moscow Institute of Physics and Technology

Program. We study certain topics in combinatorics and graph theory (including random graphs).

Detailed information in Russian: http://www.mccme.ru/circles/oim/home/discran1314.htm

[7] Vector fields on 2- and 3-manifolds (minicourse), Summer School 'Modern Mathematics', July 2014, 8 hours.

Program. It is shown how in the course of solution of interesting geometric problems on vector fields on manifolds naturally appear main notions of algebraic topology (homology groups and characteristic classes). Thus main ideas of algebraic topology are presented with minimal technicalities.

Detailed information in Russian: http://www.mccme.ru/dubna/2014/courses/askopenkov.htm

3.3.17 Mikhail Skopenkov

[1] Topology-1. Independent University of Moscow, I year students, February- May 2014, 4 hours per week + distant excercise class http://dist-math.ru.

Program (short version).

1. Visual topological problems.
2. Graphs and maps on surfaces.
3. Knots and links.
4. Classification of 2-manifolds.
5. Vector fields in the plane and their homotopies.
6. Main theorem of topology.
7. Two-dimensional simplicial complexes.
8. Fundamental group.
9. Vector fields on surfaces and the Euler–Poincare theorem.
10. Homologies of 2-manifolds.

[2] Topology-2. Independent University of Moscow, II year students, September- December 2014, 4 hours per week + distant excercise class http://dist-math.ru.

Program (short version).

1. Immersions. Classification of immersions of a circle into the plane.
2. All maps $S^1 \to S^2$ are homotopic. A square and a cube are not homeomorphic.
3. Homotopy classification of maps $S^n \to S^n$. Degree of a map.
4. Submanifolds in $\mathbb{R}^n$. Vector fields. The Euler–Poincare and Hopf theorems on the existence of a vector field on a manifold.
5. Three-dimensional simplicial complexes and manifolds. Homologies of 2- and 3-manifolds.
6. The Hopf map and the Hopf invariant of maps $S^3 \to S^2$.
7. Fundamental group and coverings.

[3] Visual potential theory, Higher School of Economics, I-III year students, September-December 2014, 2 hours per week.

Program.

1. Definition of an electric network.

2. The existence and uniqueness of a potential in an electric network. Conductance.

3. Physical interpretation of dissections of a rectangle into squares. The Dehn theorem on tiling of a rectangle.

4. Alternating-current networks. The Laszkovich–Freiling–Rinne-Szekeres theorem on dissections into similar rectangles.

[4] Distant courses for mathematical olympiads winners (http://school.dist-math.ru/moodle), 2007–now. Since 2014 supported by Independent University of Moscow. Program in Russian available at the website

http://school.dist-math.ru/moodle

3.3.18 Evgeni Smirnov

[1] Representation Theory of Classical Groups, Independent University of Moscow and Higher School of Economics, course for 2nd–4th year students, February–May 2014, 2 hours per week

Course outline:

1. Lie groups. Matrix Lie groups, tangent space as a Lie algebra, exponential map.

2. Representations of a Lie group and its Lie algebra. Compact Lie groups. Reductive Lie groups, Weyl's unitary trick, complete reducibility. Lie Theorem and Engel Theorem. Solvable Lie groups.

3. Representations of $sl(2)$. Casimir operator. Characters of representations. Decomposition of tensor products, the Klebsch–Gordan rule.

4. Representations of $sl(n)$ and $gl(n)$. Roots and weights. Relation to Schur polynomials. Decomposition of tensor products. Schur–Weyl duality.

5. Classification of complex (semi)simple Lie algebras. Weyl groups, Dynkin diagram. Classical and exceptional semisimple Lie algebras.

6. Construction of a representation with the given highest weight. Verma modules, BGG resolution. Weyl character and dimension formulas.

3.3.19 Stanislav Shaposhnikov

Mathematical calculus. Independent University of Moscow, 1 year students, September – December 2014, 4 hours per week.

Program:

1. Sets. Functions. Equivalence relations. Partially or linearly ordered sets. Mathematical induction. Axiom of choice and Zorn's lemma.

2. Real numbers: an axiomatic approach. Complex numbers.

3. Sequences and series. The Cauchy sequences.

4. Real numbers are the completion of the rational numbers. p-adic numbers.

5. Topology of the real line. Compact sets. The Cantor set.
6. Continuous functions.
7. Pointwise and uniform convergence.
8. Differentiable functions. The Weierstrass function.
9. Taylor series.

3.3.20 Ilya Vyugin

[1] Differential Geometry (lectures and seminars). Independent University of Moscow, II year students, Feburary-May 2013, 2+2 hours per week (joint with Yu. M. Burman).

Program

Rimannian Geometry

1. Curves. Ortogonal transformations classifications (Frenet formulas).
2. Surfacies. Bonne classification theorem.
3. An existence of Riemannian metric.
4. Levy-Civita connection.
5. Gauss-Bonne formula.
6. Euler–Lagrange equation.

Symplectic geometry

7. Darboux theorem.
8. Jacobi equality for invertible Poisson brackets.
9. Symplectic lies.
10. Moment map.

[2] Analitic theory of differential equations (lectures) Special course in HSE and IUM (joint with V.A. Poberezhny).

Program

Linear differential equations: asymptotics, geometry and Riemann–Hilbert problem

1. Fuchs conditions.
2. Riccati equation.
3. Linear differential equations: monodromy, singular points. Levelt decomposition.
4. Hypergeometric equation and hypergeometric functions.
5. Riemann–Hilbert problem