The IUM report to the Simons foundation, 2015

Contents

1 Introduction: list of awardees

The Simons foundation supported two programs launched by the IUM:
Simons stipends for students and graduate students;
Simons IUM fellowships.

19 applications were received for the Simons stipends contest. The selection committee consisting of *Yu.Ilyashenko (Chair), G.Dobrushina, G.Kabatyanski, S.Lando, I.Paramonova (Academic Secretary), A.Sossinsky, M.Tsfasman* awarded Simons stipends for 2015 year to the following students and graduate students:

1. Balitsky, Alexei Mikhailovich
2. Berdnikov, Alexander Sergeevich
3. Goncharuk, Nataliya Borisovna
4. Kruglik, Stanislav Alexandrovich
5. Makhlin, Igor Yurevich
6. Minkov, Stanislav Sergeevich
7. Oganesyan, Vardan Spartakovich
8. Soukhanov, Lev Alexandrovich.

12 applications were received for the Simons IUM fellowships contest for the first half year of 2015 and 13 applications were received for the second half year. The selection committee consisting of *Yu.Ilyashenko (Chair), G.Dobrushina, B.Feigin, I.Paramonova (Academic Secretary), A.Sossinsky, M.Tsfasman, V.Vassiliev* awarded

Simons IUM-fellowships for the first half year of 2015 to the following researches:

1. Belavin, Alexander Iosifovich
2. Bershtein, Mikhail Alexandrovich
3. Chekanov, Yuri Vitalevich
4. Fedorovsky, Konstantin Yurevich
5. Gorodentsev, Alexei Lvovich
6. Levin, Andrei Mikhailovich

7. Panov, Taras Evgenevich
8. Penskoi, Alexei Victorovich
9. Pushkar, Petr Evgenevich
10. Shaposhnikov, Stanislav Valerevich
11. Smirnov, Evgeni Yurevich
12. Verbitsky, Mikhail Sergeevich

Simons IUM-fellowships for the second half year of 2015 to the following researches:

1. Beloshapka, Valery Konstantinovich
2. Feigin, Evgeny Borisovich
3. Galkin, Sergey Sergeevich
4. Kazarian, Maxim Eduardovich
5. Khoroshkin, Anton Sergeevich
6. Krasilshchik, Iosif Semenovich
7. Kustarev, Andrei Alexandrovich
8. Panov, Taras Evgenevich
9. Penskoi, Alexei Victorovich
10. Pushkar, Petr Evgenevich
11. Shabat, George Borisovich
12. Skopenkov, Mikhail Borisovich
13. Smirnov, Evgeni Yurevich

The report below is split in two sections corresponding to the two programs above. The first subsection in each section is a report on the research activities. It consists of the titles of the papers published or submitted in the year of 2015, together with the corresponding abstracts. The second subsection of each section is devoted to conference and some most important seminar talks. The last subsection of the second section is devoted to the syllabi of the courses given by the winners of the Simons IUM fellowships. Most of these courses are innovative, as required by the rules of the contest for the Simons IUM fellowships.

Andrei Levin was grievously ill a large part of the year. For this reason he was not teaching at the IUM this year. For the same reason the Simons-IUM fellowship was doubly important for him.

The support of the Simons foundation have drastically improved the financial situation at the IUM, and the whole atmosphere as well. On behalf of the IUM, I send my deep gratitude and the best New year wishes to Jim Simons, Yuri Tschinkel, and the whole team of the Simons foundation.

Yulij Ilyashenko

President of the Independent University of Moscow

2 Program: Simons stipends for students and graduate students

2.1 Research

2.1.1 Alexei Balitsky

[1] With Arseniy Akopyan
Billiards in convex bodies with acute angles
arXiv:1506.06014 *submitted to Israel Journal of Mathematics*

In this paper we investigate the existence of closed billiard trajectories in not necessarily smooth convex bodies. In particular, we show that if a body $K \in \mathbb{R}^n$ has the property that the tangent cone of every non-smooth point $q \in \partial K$ is acute (in a certain sense) then there is a closed billiard trajectory in K.

[2] Equality cases in Viterbo's conjecture related to permutohedra
arXiv:1512.01657

In this note we show, using the billiard technique, that the product of a regular permutohedron and a regular simplex delivers an equality in Viterbo's conjecture.

2.1.2 Alexander Berdnikov

[1] Bounds on Multiplicities of Laplace Operator Eigenvalues on Surfaces.
arxiv:1511.09043, submitted to Journal of Spectral Theory

In the present paper several bounds on multiplicities of eigenvalues of the Laplacian operator on surfaces are generalized from the case of either closed surface or simply-connected planar domain to the case of a surface of positive genus with holes.

2.1.3 Nataliya Goncharuk

[1] Complex rotation numbers
Journal of Modern Dynamics, 2015, Vol. 9, pp. 169 - 190.

We investigate the notion of complex rotation number which was introduced by V. I. Arnold in 1978. Let $f\colon \mathbb{R}/\mathbb{Z} \to \mathbb{R}/\mathbb{Z}$ be a (real) analytic orientation preserving circle diffeomorphism and let $\omega \in \mathbb{C}/\mathbb{Z}$ be a parameter with positive imaginary part. Construct a complex torus by glueing the two boundary components of the annulus $\{z \in \mathbb{C}/\mathbb{Z} \mid 0 < \Im(z) < \Im(\omega)\}$ via the map $f + \omega$. This complex torus is isomorphic to $\mathbb{C}/(\mathbb{Z} + \tau\mathbb{Z})$ for some appropriate $\tau \in \mathbb{C}/\mathbb{Z}$. According to Moldavskis, if the ordinary rotation number $rot(f + \omega_0)$ is Diophantine and if ω tends to ω_0 non tangentially to the real axis, then τ tends to $rot(f + \omega_0)$. We show that the Diophantine and non tangential assumptions are unnecessary: If $rot(f + \omega_0)$ is irrational, then τ tends to $rot(f + \omega_0)$ as ω tends to ω_0. This, together with results of N. Goncharuk, motivates us to introduce a new fractal set ("bubbles") given by the limit values of τ as ω tends to the real axis. For the rational values of $rot(f + \omega_0)$, these limits do not necessarily coincide with $rot(f + \omega_0)$ and form a countable number of analytic loops in the upper half-plane.

[2] With Yu. Kudriashov
Bounded limit cycles of polynomial foliations in $\mathbb{C}^2$
Accepted to the Bulletin of the Brazilian Mathematical Society, arXiv:1504.03313

In this article we prove in a new way that a generic polynomial vector field in $\mathbb{C}^2$ possesses countably many homologically independent limit cycles. The new proof needs no estimates on integrals, provides thinner exceptional set for quadratic vector fields, and provides limit cycles that stay in a bounded domain.

[3] With Yu. Kudriashov
Genera of non-algebraic leaves of polynomial foliations of $\mathbb{C}P^2$
arXiv:1407.7878, *submitted to Annales de l'Institut Fourier.*

In this article, we prove two results. First, we construct a dense subset in the space of polynomial foliations of degree n such that each foliation from this subset has a leaf with at least $(n + 1)(n + 2)/2 - 4$ handles. Next, we prove that for a generic foliation invariant under the map $(x, y) \to (x, -y)$ all leaves have infinitely many handles.

2.1.4 Stanislav Kruglik

[1] With G. Kabatiansky
On codes correcting constant number of errors in l1 metric
Proceeding of ITaS 2015, 152-157
`itas2015.iitp.ru/pdf/1570161805.pdf`

We give a number-theoretical construction of codes in l1 (or modular) metric which for the case of fixed number of corrected errors have asymptotically minimal possible redundancy (the same as the corresponding Hamming bound). This construction is based on Bose-Chowla theorem from additive number theory.

[2] With G. Kabatiansky
On separating redundancy of linear codes (in russian)
Proceeding of 58th MIPT scientific conference
`conf58.mipt.ru/static/reports_pdf/1042.pdf`

In this work we reformulate task about finding minimal redundancy of linear codes in the language of boolean function and solve its' simplified version for small number of possible erasures.

2.1.5 Igor Makhlin

[1] Characters of Feigin-Stoyanovsky Subspaces and Brion's Theorem
Functional Analysis and Its Applications, 2015, Vol. 49, No. 1, P. 15-24.

We give an alternative proof of the main result of [B. Feigin, M. Jimbo, S. Loktev, T. Miwa, E. Mukhin, The Ramanujan J., 7:3 (2003), 519530]; the proof relies on Brion's theorem about convex polyhedra. The result itself can be viewed as a formula for the character of the Feigin-Stoyanovsky subspace of an integrable irreducible representation of the affine Lie algebra $\widehat{\mathfrak{sl}}_n(\mathbb{C})$. Our approach is to assign integer points of a certain polytope to the vectors comprising a monomial basis of the subspace and then compute the character via (a variation of) Brion's theorem.

[2] With B.Feigin
A Combinatorial Formula for Affine Hall-Littlewood Functions via a Weighted Brion Theorem
arXiv:1505.04269, *submitted to Selecta Mathematica*

We present a new combinatorial formula for Hall-Littlewood functions associated with the affine root system of type $\tilde{A}_{n-1}$, i.e. corresponding to the affine Lie algebra $\widehat{\mathfrak{sl}}_n$. Our formula has the form of a sum over the elements of a basis constructed by Feigin, Jimbo, Loktev, Miwa and Mukhin in the corresponding irreducible representation.

Our formula can be viewed as a weighted sum of exponentials of integer points in a certain infinite-dimensional convex polyhedron. We derive a weighted version of Brion's theorem and then apply it to our polyhedron to prove the formula.

[3] Brion's Theorem for Gelfand-Tsetlin Polytopes (*in Russian*)
submitted to Functional Analysis and Its Applications

This paper is inspired by the observation that the character of an irreducible $\mathfrak{gl}_n$-module (a Schur polynomial) is equal to the sum of certain exponentials of integer points within the Gelfand-Tsetlin polytope and, consequently, may be computed via Brion's theorem. It is shown that in the case of a regular highest weight the Brion theorem contributions of all nonsimplicial vertices are zero, while the number of simplicial ones is $n!$ and their contributions are the precisely the summands in Weyl's character formula.

2.1.6 Stanislav Minkov

[1] With A. Okunev.
Omega-limit sets of typical points for partially hyperbolic diffeomorphisms (in Russian).
to appear in Functional Analysis and Its Applications.

We proved that omega-limit set of any metrically typical point consists of unstable leaves for C^2 partially hyperbolic diffeomorphisms. Consequently Ilyashenko Conjecture on the structure of Milnor attractors was proved. This give to us locally typical existence of positive measure Milnor attractors in the class of boundary-preserving skew products.

2.1.7 Vardan Oganesyan

[1] AKNS hierarchy and finite gap Schrodinger potentials, arXiv:1512.03981

In this paper we study AKNS hierarchy and Schrodinger finite-gap potentials. We find necessary and sufficient condition for functions p and q to be solution of some equation of AKNS hierarchy. Then we construct Schrodinger finite-gap potentials using functions p and q.

2.1.8 Lev Soukhanov

[1] An operad from the secondary polytope
http://arxiv.org/abs/1505.08157 *preprint*

In this paper we study the enumerative geometry of orbits of multidimensional toric action on projective algebraic varieties and develop a new cyclic differential-graded operad, conjecturally governing the real version of the enumerative geometry of these toric orbits. The work turned out to be related to the recent work of Gaiotto-Moore-Witten (algebra of the infrared).

2.2 Scientific conferences and seminar talks

2.2.1 Alexei Balitsky

[1] Conference "Meeting of generations", Moscow, June 9 – 11
Talk "Billiards in convex bodies in asymmetric norm" (in Russian)
[2] Conference "Intuitive Geometry, Laszlo Fejes Toth Centennial", Budapest, June 22 - 28
Talk "Billiards with almost all trajectories of equal lengths"
[3] Conference "Geometry and Symmetry", Veszprem, June 29 – July 3
Talk "Existence of closed billiard trajectories in "acute-angle" bodies"

2.2.2 Alexander Berdnikov

[1] Mini-course "Introduction into h-principle", Moscow State University, March, 15 – April, 7
[2] Seminar "Geometric Structures on Manifolds" (Higher School of Economics, Moscow)
Talk "Spectral Geometry"
Seminar "Topological Field Theories" (Higher School of Economics, Moscow)
Talk "ChernSimons Theory"
[3] Graduate Topology Seminar (MIT, USA):
Talk "On Milnor's Exotic Spheres"
Talk "On Vector Fields on Spheres"
Talk "Introduction to h-principle"

2.2.3 Nataliya Goncharuk

[1] AMS - EMS - SPM joint meeting (Porto, Portugal), June 2015
Talk "Genera of non-algebraic leaves of polynomial foliations of $\mathbb{C}P^2$"
[2] Oliver Mathematical club (Cornell university, Ithaca, USA), October 2015
Talk "Bubbles"

[3] Weekly seminar of Laboratory of Algebraic Geometry and its Applications (Moscow, Higher School of Economics), November 2015

Talk "Complex rotation numbers"

[4] Dobrushin Mathematical Laboratory Seminar (Moscow, Institute for Information Transmission Problems), November 2015

Talk "Complex rotation numbers"

2.2.4 Stanislav Kruglik

[1] Conference "2015 European School of Information Theory", Zandvoort, April 20-24

Poster "On codes in L1 metric, correcting fixed number of errors"

[2] Conference "Information Technology and Systems 2015", Sochi, September 6-10

Talk "On codes correcting constant number of errors in l1 metric"

[3] Conference "58th MIPT scientific conference", Dolgoprudny, November 23-28

Talk "On separating redundancy of linear codes"

2.2.5 Igor Makhlin

[1] Conference "25th British Combinatorial Conference", University of Warwick, July, 6 – July, 10

Talk "A combinatorial formula for affine Hall-Littlewood functions via a weighted Brion Theorem"

[2] Conference "Fifth School-Conference on Lie Algebras, Algebraic Groups and Invariant Theory", Samara, June, 22 – June, 27

Talk "Character Formulas and Brion's Theorem" (*in Russian*)

2.2.6 Stanislav Minkov

[1] Conference "Summer School "Dynamical Systems"", Dubna, July,3-July, 13.

2.2.7 Vardan Oganesyan

[1] International Conference "Integrability in algebra, geometry and physics: new trends", Switzerland, July 13-17, 2015

Poster "New commuting differential operators of rank 2 and arbitrary genus".

[2] Conference "Lomonosov"-2015, Moscow State University, Moscow, April 13-17, 2015

Talk "Commuting differential operators".

[3] Several seminar talks at Moscow State University.

2.2.8 Lev Soukhanov

Talk “Quantum cohomologies of orbifolds” at “Geometric structures on manifolds” (HSE)

Talk “2-analogue of the commutativity equation” at “Geometric structures on manifolds” (HSE)

3 Program: Simons IUM fellowships

3.1 Research

3.1.1 Valery Beloshapka

[1] With I. G. Kossovskii, The Sphere in $\mathbb{C}^2$ as a Model Surface for Degenerate Hypersurfaces in $\mathbb{C}^3$, Russian Journal of Mathematical Physics, Vol. 22, No. 4, 2015, pp. 437 - 443.

In the paper, an unexpected correspondence between the automorphisms of 5D real uniformly 2-nondegenerate hypersurfaces of the space $\mathbb{C}^3$ and the automorphisms of the 3D hypersphere in $\mathbb{C}^2$ is constructed. In a certain sense, the 3D hypersphere, which is, as is known, a model surface for the class of nondegenerate 3D hypersurfaces in $\mathbb{C}^2$, has this status also with respect to the above class of 5D hypersurfaces in $\mathbb{C}^3$.

[2] A Seven-Dimensional Family of Simple Harmonic Functions, Mathematical Notes, 2015, Vol. 98, No. 6, pp. 867 871. Original Russian Text V. K. Beloshapka, 2015, published in Matematicheskie Zametki, 2015, Vol. 98, No. 6, pp. 803 - 808.

From the point of view of analytic complexity theory, all harmonic functions of two variables split into three classes: functions of complexity zero, one, and two. Only linear functions of one variable have complexity zero. This paper contains a complete description of simple harmonic functions, i.e., of functions of analytic complexity one. These functions constitute a seven-dimensional family expressible as integrals of elliptic functions. All other harmonic functions have complexity two and are, in this sense, of higher complexity. Solutions of the wave equation, the heat equation, and the Hopf equation are also studied.

[3] Complex Analisys: the algebraic functions and the functons of several variables, 2015, pp.1 - 60, http://new.math.msu.su/tffa/index.html.

Some studying materials for post graduate students.

3.1.2 Alexander Belavin

[1] Minimal Liouville gravity from Douglas String equation.
Moscow Mathematical Journal , 2015, Vol. 15 , No.2 , pp.1-14 .

In this paper we conjecture that the Douglas equation is applicable to the Minimal Liouville gravity as well as to Matrix Models of 2D gravity. This conjecture requires the following two questions to be answered: how to choose the desired solution of the Douglas string equation and an appropriate form of theso called resonance transformation from the KdV times to the Liouville coupling constants to satisfy the needed constraints which have to be fulfilled in MLG. In our study, using the connection of the approach to MLG based on the connection of the String equation with the Frobenius manifold structure, we find the necessary solution of the String equation. We also show that this solution together with the suitable choosen resonance transformation lead to the results which are consistent with the main requirements of (p, q) models of MLG . It is remarkable that the needed solution of the Douglas equation has a very simple form in the flat coordinates on the Frobenious manifold in the general case of (p,q) Minimal Liouville .

3.1.3 Mikhail Bershtein

[1]with A. Tsymbaliuk
Homomorphisms between different quantum toroidal and affine Yangian algebras
to appear

This paper concerns the relation between the quantum toroidal algebras and the affine Yangians of $\mathfrak{sl}_n$, denoted by $\mathcal{U}^{(n)}_{q_1,q_2,q_3}$ and $\mathcal{Y}^{(n)}_{h_1,h_2,h_3}$, respectively. Our motivation arises from the milestone work Gautam and Toledano Laredo, where a similar relation between the quantum loop algebra $U_q(L$
$mathfrakg)$ and the Yangian $Y_h(\mathfrak{g})$ has been established by constructing an isomorphism of $\mathbb{C}[[\hbar]]$-algebras $\Phi : \widehat{U}_{\exp(\hbar)}(L\mathfrak{g}) \to \widehat{Y}_\hbar(\mathfrak{g})$ (with $\widehat{\ }$ standing for the appropriate completions). These two completions model the behavior of the algebras in the formal neighborhood of $h = 0$. The same construction can be applied to the toroidal setting with $q_i = \exp(\hbar_i)$ for $i = 1, 2, 3$. In the current paper, we are interested in the more general relation: $\mathrm{q}_1 = \omega_{mn} e^{h_1/m}, \mathrm{q}_2 = e^{h_2/m}, \mathrm{q}_3 = \omega_{mn}^{-1} e^{h_3/m}$, where $m, n \in \mathbb{N}$ and ω_{mn} is an mnth root of 1. For any such choice of m, n, ω_{mn} and the corresponding values $\mathrm{q}_1, \mathrm{q}_2, \mathrm{q}_3$, we construct a homomorphism $\Phi^{\omega_{mn}}_{m,n}$ from the completion of the formal version of $\mathcal{U}^{(m)}_{\mathrm{q}_1,\mathrm{q}_2,\mathrm{q}_3}$ to the completion of the formal version of $\mathcal{Y}^{(mn)}_{h_1/mn,h_2/mn,h_3/mn}$. We also construct homomorphisms $\Psi^{\omega',\omega}_{m,n}$ between the completions of the formal versions of $\mathcal{U}^{(m)}_{\mathrm{q}_1,\mathrm{q}_2,\mathrm{q}_3}$ with different parameters m and ω_{mn}. The existence of the homomorphisms $\Phi^{\omega_{mn}}_{m,n}$ and $\Psi^{\omega',\omega}_{m,n}$ explains the similarity between the representation theories of $\mathcal{U}^{(n)}_{q_1,q_2,q_3}$ and $\mathcal{Y}^{(n)}_{h_1,h_2,h_3}$ with different parameters n.

[2] with B. Feigin, G. Merzon
Plane partitions with a "pit": generating functions and representation theory
to appear.

We study plane partitions satisfying condition $a_{n+1,m+1} = 0$ (this condition is called "pit") and asymptotic conditions along three coordinate axes. We find the formulas for generating function of such plane partitions.

Such plane partitions label the basic vectors in certain representations of quantum toroidal $\mathfrak{gl}_1$ algebra, therefore our formulas can be interpreted as the characters of these representations. The resulting formulas resemble formulas for characters of tensor representations of super Lie algebra $\mathfrak{gl}_{m|n}$.

3.1.4 Yuri Chekanov

[1] With F. Schlenk
Lagrangian product tori in tame symplectic manifolds
arXiv:1502.00180 *submitted to Commentarii Mathematici Helvetici*

Product Lagrangian tori in the standard symplectic space R^{2n} were classified up to symplectomorphism by the first named author in 1996. We extend this classification to tame symplectically aspherical symplectic manifolds. We show by examples that omitting the asphericity assumption alters the classification.

3.1.5 Konstantin Fedorovsky

[1] With P. Paramonov
Tverberg's proof of the Jordan close curve theorem
Algebra i Analiz, 2015, Vol. 27, No. 5, pp. 207–220, in Russian; Engl. transl: St. Petersburg Math. J., 2016, Vol. 27, No. 5.

In this paper we discussed one little known proof of the Jordan closed curve theorem (Jordan theorem) which was obtained by Norwegian mathematician H. Tverberg. This proof is of metric nature and it allowed us to obtain one metric refinement of the Jordan theorem which is of independent interest.

[2] With A. Baranov
On L^1-estimates of derivatives of univalent rational functions
arXiv: 1312.3312v4, *to appear in J. Anal. Math.*

In this paper we studied the growth of the quantity

$$\int_{\mathbb{T}} |R'(z)|\, dm(z)$$

for rational functions R of degree n, which are bounded and univalent in the unit disk, and prove that this quantity may grow as n^τ, $\tau > 0$, when $n \to \infty$. Some applications of this result to problems of regularity of boundaries of Nevanlinna domains are considered. We also discussed a related result by Dolzhenko which applies to general (non-univalent) rational functions.

3.1.6 Evgeny Feigin

[1] With with I.Makedonskyi
Nonsymmetric Macdonald polynomials, Demazure modules and PBW filtration,
Journal of Combinatorial Theory, Series A, pp. 60–84, 2015.

The Cherednik-Orr conjecture expresses the $t \to \infty$ limit of the nonsymmetric Macdonald polynomials in terms of the PBW twisted characters of the affine level one Demazure modules. We prove this conjecture in several special cases.

[2] With I. Cherednik
Extremal part of the PBW-filtration and E-polynomials,
Advances in Mathematics, Volume 282, Pages 220–264.

Given a reduced irreducible root system, the corresponding nil-DAHA is used to calculate the extremal coefficients of nonsymmetric Macdonald polynomials in the limit $t \to \infty$ and for antidominant weights, which is an important ingredient of the new theory of nonsymmetric q-Whittaker function. These coefficients are pure q-powers and their degrees are expected to coincide in the untwisted setting with the extremal degrees of the so-called PBW-filtration in the corresponding finite-dimensional irreducible representations of the simple Lie algebras for any root systems. This is a particular case of a general conjecture in terms of the level-one Demazure modules. We prove this coincidence for all Lie algebras of classical type and for G_2, and also establish the relations of our extremal degrees to minimal q-degrees of the extremal terms of the Kostant q-partition function; they coincide with the latter only for some root systems.

[3] With G. Cerulli Irelli and M. Reineke
Schubert Quiver Grassmannians
arXiv:1508.00264, *submitted*

Quiver Grassmannians are projective varieties parametrizing subrepresentations of given dimension in a quiver representation. We define a class of quiver Grassmannians generalizing those which realize degenerate flag varieties. We show that each irreducible component of the quiver Grassmannians in question is isomorphic to a Schubert variety. We give an explicit description of the set of irreducible components, identify all the Schubert varieties arising, and compute the Poincaré polynomials of these quiver Grassmannians.

[4] With I. Makedonskyi
Weyl modules for $\mathfrak{osp}(1,2)$ and nonsymmetric Macdonald polynomials
arXiv:1507.01362 *submitted*

The main goal of our paper is to establish a connection between the Weyl modules of the current Lie superalgebras (twisted and untwisted) attached to $\mathfrak{osp}(1,2)$ and the nonsymmetric Macdonald polynomials of types $A_2^{(2)}$ and $A_2^{(2)\dagger}$. We compute the dimensions and construct bases of the Weyl modules. We also derive explicit formulas for the $t = 0$ and $t = \infty$ specializations of the nonsymmetric Macdonald polynomials. We show that the specializations can be described in terms of the Lie superalgebras action on the Weyl modules.

3.1.7 Sergey Galkin

[1] With L. Katzarkov, A. Mellit, E. Shinder
Derived categories of Keum's fake projective planes
Advances in Mathematics 278 (2015) 238–253.

In this paper we conjecture that derived categories of coherent sheaves on fake projective n-spaces have a semi-orthogonal decomposition into a collection of $n+1$ exceptional objects and a category with vanishing Hochschild homology. We prove this for fake projective planes with non-abelian automorphism group (such as Keum's surface). Then by passing to equivariant categories we construct new examples of phantom categories with both Hochschild homology and Grothendieck group vanishing.

[2] With A. Mellit and M. Smirnov
Dubrovin's conjecture for $IG(2,6)$
International Mathematics Research Notices 2015 (18): 8847–8859.

We show that the big quantum cohomology of the symplectic isotropic Grassmanian $IG(2,6)$ is generically semisimple, whereas its small quantum cohomology is known to be non-semisimple. This gives yet another case where Dubrovins conjecture holds and stresses the need to consider the big quantum cohomology in its formulation.

[3] With V. Golyshev and H. Iritani
Gamma classes and quantum cohomology of Fano manifolds: Gamma conjectures
arXiv:1404.6407 and IPMU 10-0200, *to appear in Duke Mathematical Journal.*

We propose Gamma Conjectures for Fano manifolds which can be thought of as a square root of the index theorem. Studying the exponential asymptotics of solutions to the quantum differential equation, we associate a principal asymptotic class A_F to a Fano manifold F. We say that F satisfies Gamma Conjecture I if A_F equals the Gamma class

$\hat{\Gamma}_F$. When the quantum cohomology of F is semisimple, we say that F satisfies Gamma Conjecture II if the columns of the central connection matrix of the quantum cohomology are formed by $\hat{\Gamma}_F Ch(E_i)$ for an exceptional collection E_i in the derived category of coherent sheaves $D^b_{coh}(F)$. Gamma Conjecture II refines part (3) of Dubrovin's conjecture. We prove Gamma Conjectures for projective spaces and Grassmannians.

[4] With T. Coates, A. Corti, A. Kasprzyk
Quantum periods for 3-dimensional Fano manifolds
arXiv:1303.3288 and IPMU 13-0113, *to appear in Geometry and Topology.*

The quantum period of a variety X is a generating function for certain Gromov–Witten invariants of X which plays an important role in mirror symmetry. In this paper we compute the quantum periods of all 3-dimensional Fano manifolds. In particular we show that 3-dimensional Fano manifolds with very ample anticanonical bundle have mirrors given by a collection of Laurent polynomials called Minkowski polynomials. This was conjectured in joint work with Golyshev. It suggests a new approach to the classification of Fano manifolds: by proving an appropriate mirror theorem and then classifying Fano mirrors. Our methods are likely to be of independent interest. We rework the Mori–Mukai classification of 3-dimensional Fano manifolds, showing that each of them can be expressed as the zero locus of a section of a homogeneous vector bundle over a GIT quotient V/G, where G is a product of groups of the form $GL_n(\mathbb{C})$ and V is a representation of G. When $G = GL_1(\mathbb{C})^r$, this expresses the Fano 3-fold as a toric complete intersection; in the remaining cases, it expresses the Fano 3-fold as a tautological subvariety of a Grassmannian, partial flag manifold, or projective bundle thereon. We then compute the quantum periods using the Quantum Lefschetz Hyperplane Theorem of Coates–Givental and the Abelian/non-Abelian correspondence of Bertram–Ciocan-Fontanine–Kim–Sabbah.

[5] Degenerations, transitions and quantum cohomology
to appear in Tropical Aspects in Geometry, Topology and Physics, Mathematisches Forschungsinstitut Oberwolfach, Report No. 23/2015, DOI: 10.4171/OWR/2015/23

Given a singular variety I discuss the relations between quantum cohomology of its resolution and smoothing. In particular, I explain how toric degenerations helps with computing Gromov-Witten invariants, and the role of this story in "Fanosearch" program.

[6] With T. Coates, A. Kasprzyk, A. Strangeway
Quantum Periods For Certain Four-Dimensional Fano Manifolds
arXiv:1406.4891, *submitted to Documenta Mathematica*

We collect a list of known four-dimensional Fano manifolds and compute their quantum periods. This list includes all four-dimensional Fano manifolds of index greater than one, all four-dimensional toric Fano manifolds, all four-dimensional products of lower-dimensional Fano manifolds, and certain complete intersections in projective bundles.

[7] With H. Iritani
Gamma conjecture via mirror symmetry
arXiv:1508.00719, *submitted to the proceedings of the conference "Primitive Forms and Related Subjects" at IPMU (Feb 2014), to be published in a volume of Advanced Studies in Pure Mathematics*

[8] With E. Shinder
On a zeta-function of a dg-category
arXiv:1506.05831

We define a zeta-function of a pre-triangulated dg-category and investigate its relationship with the motivic zeta-function in the geometric case.

[9] With I. Karzhemanov and E. Shinder
Acyclicity of non-linearizable line bundles on fake projective planes
IPMU 15-0202

On the projective plane there is a unique cubic root of the canonical bundle and this root is acyclic. On fake projective planes such root exists and is unique if there are no 3-torsion divisors (and usually exists but not unique otherwise). Earlier we conjectured that any such cubic root (assuming it exists) must be acyclic. In the present note we give a new short proof of this statement and show acyclicity of some other line bundles on those fake projective planes with at least 9 automorphisms. Similarly to our earlier work we employ simple representation theory for non-abelian finite groups. The novelty stems from the idea that if some line bundle is non-linearizable with respect to a finite abelian group, then it should be linearized by a finite (non-abelian) Heisenberg group. Our argument also exploits J. Rogawski's vanishing theorem and the linearization of an auxiliary line bundle.

3.1.8 Alexei Gorodentsev

I have no scientific papers published during the period from January 01, 2015 until today.

Among teaching materials, I have prepard the 2-nd part of my 'Algebra for mathematicians' textbook (281 p. with figures). It was given to the MCCME publishing house in June 2015. Available on-line at
`http://gorod.bogomolov-lab.ru/ps/stud/algebra-3/1415/algebra-2_2015.VI.15.pdf`
Currently I'm preparing the English versions of both parts. They will be published next year by Springer Verlag.

3.1.9 Maxim Kazarian

[1]Dunin-Barkowski, P.; Kazarian, M.; Orantin, N.; Shadrin, S.; Spitz, L. Polynomiality of Hurwitz numbers, Bouchard-Marino conjecture, and a new proof of the ELSV formula.

Adv. Math. 279 (2015), 67103.

In this paper we give a new proof of the ELSV formula. First, we refine an argument of Okounkov and Pandharipande in order to prove (quasi-)polynomiality of Hurwitz numbers without using the ELSV formula (the only way to do that before used the ELSV formula). Then, using this polynomiality we give a new proof of the Bouchard-Mari ?no conjecture. After that, using the correspondence between the Givental group action and the topological recursion coming from matrix models, we prove the equivalence of the Bouchard-Mari ?no conjecture and the ELSV formula (it is a refinement of an argument by Eynard).

[2]Kazarian, Maxim; Zograf, Peter Virasoro constraints and topological recursion for Grothendieck's dessin counting. Lett. Math. Phys. 105 (2015), no. 8, 10571084.

We compute the number of coverings of the Rieman sphere with a given monodromy type over infinity, a given numbers of preimages of 0 and 1, and no other critical values. We show that the generating function for these numbers enjoys several remarkable integrability properties: it obeys the Virasoro constraints, an evolution equation, the Kadomtsev-Petviashvili hierarchy and satisfies a topological recursion in the sense of Eynard-Orantin.

[3]Kazarian, M.; Lando, S.; Combinatorial solutions to integrable hierarchies. (Russian) Uspekhi Mat. Nauk 70 (2015), no. 3(423), 77106. English tramslation: 2015 Russ. Math. Surv. 70 453-482

We give a review of modern approaches to constructing formal solutions to integrable hierarchies of mathematical physics, whose coefficients are answers to various enumerative problems. The relationship between these approaches and combinatorics of symmetric groups and their representations is explained. Applications of the results to constructing efficient computations in problems related to models of quantum field theories are given.

[4] Maxim Kazarian, Sergey Lando, Dimitri Zvonkine, Universal cohomological expressions for singularities in families of genus 0 stable maps, arXiv:1512.03285

We consider families of curve-to-curve maps that have no singularities except those of genus 0 stable maps and that satisfy a versality condition at each singularity. We provide a universal expression for the cohomology class Poincaré dual to the locus of any given singularity. Our expressions hold for any family of curve-to-curve maps satisfying the above properties.

3.1.10 Anton Khoroshkin

[1] "Characteristic classes of flags of foliations and Lie algebra cohomology."

Transformation Groups (2015) published online

We prove the conjecture by Feigin, Fuchs, and Gelfand describing the Lie algebra cohomology of formal vector fields on an n-dimensional space with coefficients in symmetric powers of the coadjoint representation. We also compute the cohomology of the Lie algebra of formal vector fields that preserve a given ag at the origin. The latter encodes characteristic classes of ags of foliations and was used in the formulation of the local Riemann-Roch Theorem by Feigin and Tsygan.

Feigin, Fuchs, and Gelfand described the first symmetric power and to do this they had to make use of a fearsomely complicated computation in invariant theory. By the application of degeneration theorems of appropriate Hochschild-Serre spectral sequences, we avoid the need to use the methods of FFG, and moreover, we are able to describe all the symmetric powers at once.

[2] with Dmitri Piontkovski
"On generating series of finitely presented operads."
Journal of Algebra 426 (2015) pp.377–429

Given an operad $\mathcal{P}$ with a finite Groebner basis of relations, we study the generating functions for the dimensions of its graded components $\mathcal{P}(n)$. Under moderate assumptions on the relations we prove that the exponential generating function for the sequence $\dim \mathcal{P}(n)$ is differential algebraic, and in fact algebraic if $\mathcal{P}$ is a symmetrization of a non-symmetric operad. If, in addition, the growth of the dimensions of $\mathcal{P}(n)$ is bounded by an exponent of n (or a polynomial of n, in the non-symmetric case) then, moreover, the ordinary generating function for the above sequence $\dim \mathcal{P}(n)$ is rational. We give a number of examples of calculations and discuss conjectures about the above generating functions for more general classes of operads.

[3] with T. Willwacher, M. Živković
"Differentials on graph complexes II – Hairy graphs"
preprint arxiv:1508.01281

We study the cohomology of the hairy graph complexes which compute the rational homotopy of embedding spaces, generalizing the Vassiliev invariants of knot theory. We provide spectral sequences converging to zero whose first pages contain the hairy graph cohomology. Our results yield a way to construct many hairy graph cohomology classes out of non-hairy classes by a mechanism which we call the waterfall mechanism. By this mechanism we can construct many previously unknown classes and provide a first glimpse at the tentative global structure of the hairy graph cohomology.

3.1.11 Iosif Krasilshchik

[1] With H. Baran, O.I. Morozov, P. Vojčák

Five-dimensional Lax-integrable equation, its reductions and recursion operator, Lobachevskii J. of Mathematics, 2015, Vol. 36, Issue 3, pp. 225–233

We consider a five-dimensional nonlinear PDE associated to the five-dimensional equation introduced by Martínez Alonso and Shabat. For our equation we find differential coverings with non-removable parameters and list its reductions to known 4D and 3D integrable equations. One of the coverings produces a new family of integrable 5D equations. We show that each pair of these equations is related by a Bäcklund transformation, including the Bäcklund auto-transformation for each equation from the family. Also we find a recursion operator for symmetries of the equation and study its action.

[2] Integrability in differential coverings, J. Geometry and Physics, Vol. 87, 2015, pp. 296–304.

Let $\tau\colon \tilde{\mathcal{E}} \to \mathcal{E}$ be a differential covering of a PDE $\tilde{\mathcal{E}}$ over $\mathcal{E}$. We prove that if $\mathcal{E}$ possesses infinite number of symmetries and/or conservation laws then $\tilde{\mathcal{E}}$ has similar properties.

[3] With A. Sergyeyev
Integrability of S-deformable surfaces: Conservation laws, Hamiltonian structures and more
J. Geometry and Physics, 2015, Vol. 97, pp. 266–278.

In this paper we present infinitely many nonlocal conservation laws, a pair of compatible local Hamiltonian structures and a recursion operator for the equations describing surfaces in three-dimensional space that admit nontrivial deformations which preserve both principal directions and principal curvatures (or, equivalently, the shape operator).

[4] With H. Baran, O.I. Morozov, P. Vojčák
Coverings over Lax integrable equations and their nonlocal symmetries
arXiv:1507.00897, *to appear in Theoretical and Mathematical Physics*

Using the Lax representation with non-removable parameter, we construct two hierarchies of nonlocal conservation laws for the 3D rdDym equation $u_{ty} = u_x u_{xy} - u_y u_{xx}$ and describe the algebras of nonlocal symmetries in the corresponding coverings.

[5] With A. Sergyeyev, O.I. Morozov
Infinitely many nonlocal conservation laws for the ABC equation with $A + B + C \neq 0$
arXiv:1511.09430, *submitted to J. of Differential Equations*

We construct an infinite hierarchy of nonlocal conservation laws for the ABC equation $Au_t u_{xy} + Bu_x u_{ty} + Cu_y u_{tx} = 0$, where A, B, C are constants and $A + B + C \neq 0$, using a novel nonisospectral Lax pair. As a byproduct, we present new coverings for the ABC equation. The method of proof of nontriviality of the conservation laws under study is quite general and can be applied to many other integrable multidimensional systems.

3.1.12 Andrei Kustarev

[1] Chern numbers of manifolds with torus action. arXiv:1506.05355

We show that every set of numbers that occurs as the set of Chern numbers of an almost complex manifold M^{2n}, $n > 2$, may be realized as the set of Chern numbers of a connected almost complex manifold with an almost complex action of two-dimensional compact torus.

3.1.13 Andrei Levin

[1] with G. Aminov, M. Olshanetsky and A. Zotov

Classical integrable systems and Knizhnik-Zamolodchikov-Bernard equations

Journal of Experimental and Theoretical Physics Letters (JETP Letters). 2015. Vol. 101. No. 9. P. 648-655.

We mainly focus on interrelations between classical integrable systems, PainlevSchlesinger equations and related algebraic structures such as classical and quantum R-matrices. The constructions are explained in terms of simplest examples.

3.1.14 Taras Panov

[1] With V. M. Buchstaber

Toric Topology

Mathematical Surveys and Monographs, vol. 204, American Mathematical Society, Providence, RI, 2015, 518 pages.

This book is about toric topology, a new area of mathematics that emerged at the end of the 1990s on the border of equivariant topology, algebraic and symplectic geometry, combinatorics, and commutative algebra. It has quickly grown into a very active area with many links to other areas of mathematics, and continues to attract experts from different fields.

The key players in toric topology are moment-angle manifolds, a class of manifolds with torus actions defined in combinatorial terms. Construction of moment-angle manifolds relates to combinatorial geometry and algebraic geometry of toric varieties via the notion of a quasitoric manifold. Discovery of remarkable geometric structures on moment-angle manifolds led to important connections with classical and modern areas of symplectic, Lagrangian, and non-Kaehler complex geometry. A related categorical construction of moment-angle complexes and polyhedral products provides for a universal framework for many fundamental constructions of homotopical topology. The study of polyhedral products is now evolving into a separate subject of homotopy theory. A new perspective on torus actions has also contributed to the development of classical areas of algebraic topology, such as complex cobordism.

This book includes many open problems and is addressed to experts interested in new ideas linking all the subjects involved, as well as to graduate students and young researchers ready to enter this beautiful new area.

[2] On the cohomology of quotients of moment-angle complexes
Russian Math. Surveys, 2015, Vol. 70, No. 4, pp. 779–781

We describe the cohomology of the quotient Z_K/H of a moment-angle complex Z_K by a freely acting subtorus H in T^m by establishing a ring isomorphism of $H^*(Z_K/H, R)$ with an appropriate Tor-algebra of the face ring $R[K]$, with coefficients in an arbitrary commutative ring R with unit. The quotients Z_K/H include moment-angle manifolds themselves, projective toric manifolds (the result was known for both these cases), and also 'projective' moment-angle manifolds. The latter admit non-Kaehler complex-analytic structures as LVM-manifolds. We prove the collapse of the corresponding Eilenberg-Moore spectral sequence using the extended functoriality of Tor with respect to 'strongly homotopy multiplicative' maps in the category DASH, following Gugenheim-May and Munkholm.

[3] With Jelena Grbić, Stephen Theriault and Jie Wu
The homotopy types of moment-angle complexes for flag complexes
Transactions of the Amer. Math. Soc., 2015, published electronically: November 12, 2015

We study the homotopy types of moment-angle complexes, or equivalently, of complements of coordinate subspace arrangements. The overall aim is to identify the simplicial complexes K for which the corresponding moment-angle complex Z_K has the homotopy type of a wedge of spheres or a connected sum of sphere products. When K is flag, we identify in algebraic and combinatorial terms those K for which Z_K is homotopy equivalent to a wedge of spheres, and give a combinatorial formula for the number of spheres in the wedge. This extends results of Berglund and Jöllenbeck on Golod rings and homotopy theoretical results of the first and third authors. We also establish a connection between minimally non-Golod rings and moment-angle complexes Z_K which are homotopy equivalent to a connected sum of sphere products. We go on to show that for any flag complex K the loop space ΩZ_K is homotopy equivalent to a product of spheres and loops on spheres when localised rationally or at any prime $p \neq 2$.

[4] With Zhi Lu
On toric generators in the unitary and special unitary bordism rings
arXiv:1412.5084. *submitted to Algebraic and Geometric Topology*

We construct a new family of toric manifolds generating the unitary bordism ring. Each manifold in the family is the complex projectivisation of the sum of a line bundle and a trivial bundle over a complex projective space. We also construct a family of special unitary quasitoric manifolds which contains polynomial generators of the special unitary bordism ring with 2 inverted in dimensions > 8. Each manifold in the latter family is obtained from an iterated complex projectivisation of a sum of line bundles by amending the complex structure to make the first Chern class vanish.

3.1.15 Alexei Penskoi

[1] Generalized Lawson Tori and Klein Bottles
J. of Geometric Analysis, 2015, Vol. 25, No. 4, pp. 2645-2666

Using Takahashis theorem, we propose in this paper an approach to extend known families of minimal tori in spheres. As an example, the well-known two-parametric family of Lawson tau-surfaces including tori and Klein bottles is extended to a three-parametric family of tori and Klein bottles minimally immersed in spheres. Extremal spectral properties of the metrics on these surfaces are investigated. These metrics include (i) both metrics extremal for the first non-trivial eigenvalue on the torus, i.e., the metric on the Clifford torus and the metric on the equilateral torus and (ii) the metric maximal for the first non-trivial eigenvalue on the Klein bottle.

3.1.16 Petr Pushkar'

[1] I continue to work on a book on Morse theory for manifolds with boundaries.
Main subject is a generalization of classical Morse inequalities.

[2] A paper Positive isotopies of Legendrian submanifolds and applications,. preprint arXiv:1004.5263 was corrected and resubmitted to International Mathematics Research Notices

3.1.17 George Shabat

[1] With P. Dunin-Barkovsky, A. Popolitov and A. Sleptsov.

On the Homology of Certain Smooth Covers of Moduli Spaces of Algebraic Curves
Differential Geometry and its Applications, June 2015, 86-102.

In this paper we suggest a general method of computation of the homology of certain smooth covers of moduli spaces of pointed curves of. Namely, we consider moduli spaces of algebraic curves with a level structure. The method is based on the lifting of the Strebel-Penner stratification. It is applied to genera 1 and 2, and the corresponding Betti numbers modulo 2 are obtained.

[2] On the Mordell and Weil conjectures (in russian)
Materials of the 5-th summer school-conference on the algebraic geometry and complex analysis for the young mathematicians of Russia (Koriajma, August 2015), 19-31. Moscow, Steklov Institute of Mathematics, 2015.

These lectures contain a brief overview of the proofs of two famous conjectures that became theorems in XX-th century. The perspectives of further development are outlined.

[3]. With G.E. Kreidlin (in russian)

The space in the natural languages and in the languages of geometry.
Moscow Journal of Linguistics Vol. 17(1), 116-130.

The objective of this paper written by the linguist and the mathematician is to consider the languages of geometry in the written form. A special attention is paid to lexical units by means of which some basic space structures are described.

[4] Calculating and drawing Belyi pairs (43 p.).
In: Proceedings of the International Conference "Embedded graphs", S-Pb, 2014.

This is a survey article on some aspects of theory of dessin d'enfants. Some original results are contained in the last section, where the families of generically 4-branched coverings of the sphere are used to compute dessins d'enfants by specializing to the case in which a pair of branching values coalesce.

[5]. With G.E. Kreidlin (in russian)
The natural language and the language of geometrical sketches: the points of juxtaposition (18 p.). Warszawa, 2016.

In the joint paper of a linguist and of a mathematician the multilateral relations between the two semiotic systems – the natural language and the language of geometrical sketches – are discussed. The considerations are centered around the mechanisms of generation and comprehension of the texts in both languages.

[6] Several glimpses of Catalan numbers (in russian)
In the collection "Math in Problems", oscow Center of Continuous Mathematical Education, 2016 (3 p.).

In this note the equivalence of several definitions of the Catalan numbers is presented in the form of a chain of problems of several levels of difficulty. Some suggestions for the generalizations are included.

[7] Is a mathematician's intuition formalizable? (in russian)
Proceedings of the conference "The perspectives of the cognitive studies: interdisciplinarity and integrativity" , December 3, Russian State University for the Humanities (5 p.).

It is shown using the known results of mathematical logic, that the intuition of a mathematician is non-formalizable. Several related interdisciplinary directions of research are suggested.

3.1.18 Stanislav Shaposhnikov

1. Bogachev V.I., Da Prato G., Roeckner M., Shaposhnikov S.V. On the uniqueness of solutions to continuity equations. Journal of Differential Equations. 2015. V. 259, no. 8. P. 3854-3873. DOI:10.1016/j.jde.2015.05.003

We obtain sufficient conditions for the uniqueness of solutions to the Cauchy problem for the continuity equation in classes of measures that need not be absolutely continuous.

2. Manita O.A., Shaposhnikov S.V. On the Cauchy problem for Fokker-Planck-Kolmogorov equations with potential terms on arbitrary domains. Journal of Dynamics and Differential Equations online 2015, P. 1-26, DOI: 10.1007/s10884-015-9453-y

We study the Cauchy problem for FokkerPlanckKolmogorov equations for finite measures with unbounded and degenerate coefficients. Sufficient conditions for the existence and uniqueness of solutions are given.

3.1.19 Mikhail Skopenkov

[1] M. Skopenkov, When is the set of embeddings finite up to isotopy?, Intern. J. Math 26:7 (2015), 28 pp. http://arxiv.org/abs/1106.1878

Given a manifold N and a number m, we study the following question: *is the set of isotopy classes of embeddings* $N \to S^m$ *finite*? In case when the manifold N is a sphere the answer was given by A. Haefliger in 1966. In case when the manifold N is a disjoint union of spheres the answer was given by D. Crowley, S. Ferry and the author in 2011.

We consider the next natural case when N is a product of two spheres. In the following theorem, $FCS(i,j) \subset \mathbb{Z}^2$ is a concrete set depending only on the parity of i and j which is defined in the paper.

Theorem. Assume that $m > 2p+q+2$ and $m < p+3q/2+2$. Then the set of isotopy classes of smooth embeddings $S^p \times S^q \to S^m$ is infinite if and only if either $q+1$ or $p+q+1$ is divisible by 4, or there exists a point (x,y) in the set $FCS(m-p-q, m-q)$ such that $(m-p-q-2)x + (m-q-2)y = m-3$.

Our approach is based on a group structure on the set of embeddings and a new exact sequence, which in some sense reduces the classification of embeddings $S^p \times S^q \to S^m$ to the classification of embeddings $S^{p+q} \sqcup S^q \to S^m$ and $D^p \times S^q \to S^m$. The latter classification problems are reduced to homotopy ones, which are solved rationally.

[2] M. Skopenkov, O. Malinovskaya, S. Dorichenko, Compose a square, Kvant 2 (2015), 6-11. This paper does not contain any reference to the support of Simons–IUM fellowship because the format of the popular-science journal does not support this.

This is a popular science paper devoted to an elementary proof of the following beautiful folklore result:

Theorem. Let $x = a + b\sqrt{2} > 0$, where a and b are rational. Then a square can be composed from rectangles of side ratio x if and only if $a - b\sqrt{2} > 0$.

The approach to the proof is based on a physical interpretation.

[3] Skopenkov M., Krasauskas R., Surfaces containing two circles through each point and Pythagorean 6-tuples, submitted, http://arxiv.org/abs/1503.06481

We study analytic surfaces in 3-dimensional Euclidean space containing two circular arcs through each point. The problem of finding such surfaces traces back to the works of Darboux from XIXth century. We reduce finding all such surfaces to the algebraic problem of finding all Pythagorean 6-tuples of polynomials. The reduction is based on the Schicho parametrization of surfaces containing two conics through each point and a new approach using quaternionic rational parametrization.

[4] Pakharev A., Skopenkov M., Surfaces containing two circles through each point and decomposition of quaternionic matrices, submitted, http://arxiv.org/abs/1510.06510. This paper does not contain any reference to the support of Simons–IUM fellowship because of the requirement of the other funding organization.

We find all analytic surfaces in space $\mathbb{R}^3$ such that through each point of the surface one can draw two circular arcs fully contained in the surface. This paper announces the result and gives the idea of proof using a new decomposition technique for quaternionic matrices.

[5] Skopenkov M., Surfaces containing two circles through each point, preprint (planed to put into arxiv by the end of 2015).

We find all analytic surfaces in space $\mathbb{R}^3$ such that through each point of the surface one can draw two transversal circular arcs fully contained in the surface. The problem of finding such surfaces traces back to the works of Darboux from XIXth century. We prove that such a surface is an image of a subset of one of the following sets under some composition of inversions:

- the set $\{\, p + q : p \in \alpha, q \in \beta \,\}$, where α, β are two circles in $\mathbb{R}^3$;
- the stereographic projection of the set $\{\, p \cdot q : p \in \alpha, q \in \beta \,\}$, where α, β are two circles in the sphere S^3 identified with the set of unit quaternions;

- the stereographic projection of the intersection of S^3 with some other 3-dimensional quadric.

The proof uses a new factorization technique for quaternionic polynomials and matrices.

In addition to [1]–[5], several talk abstracts have been published in 2015.

3.1.20 Evgeni Smirnov

[1] With G. Merzon

Determinantal identities for flagged Schur and Schubert polynomials

European Journal of Mathematics, 2015 (to appear).

OnlineFirst: DOI 10.1007/s40879-015-0078-9.

We prove new determinantal identities for a family of flagged Schur polynomials. As a corollary of these formulas we obtain determinantal expressions of Schubert polynomials for certain vexillary permutations.

[2] Grassmannians, flag varieties, and Gelfand-Zetlin polytopes

More Expository Lectures on Representation Theory, Contemporary Mathematics series, AMS, 2016 (to appear).

The aim of these notes is to give an introduction into Schubert calculus on Grassmannians and flag varieties. We discuss various aspects of Schubert calculus, such as applications to enumerative geometry, structure of the cohomology rings of Grassmannians and flag varieties, Schur and Schubert polynomials. We conclude with a survey of results of V. Kiritchenko, V. Timorin and the author on a new approach to Schubert calculus on full flag varieties via combinatorics of Gelfand–Zetlin polytopes.

[3] Three glances on the Aztec diamond

Book (48 pp., in Russian). MCCME publishing house, 2015.

These are notes of a three-lecture minicourse given at the summer school "Contemporary mathematics" (Dubna, Russia) in July 2014. In these notes we prove the Aztec diamond theorem, stating that the number of domino tilings of the Aztec diamond of order n is $2^{n(n+1)/2}$. We present three different proofs of this theorem. The first proof is based on counting a weighted sum over alternating sign matrices associated with domino tilings. In the second proof we compute the number of perfect matchings by using "urban renewal" of planar bipartite graphs. In the last proof we establish a bijection between domino tilings of the Aztec diamond and certain lattice path configurations; their number is then found using the Gessel–Viennot method.

[4] With V. Kleptsyn

Ribbon graphs and bialgebra of Lagrangian spaces

Submitted for publication, 21 pages, available on arXiv:1401.6160v2 (major revisions compared to the previous version)

To each ribbon graph we assign a so-called L-space, which is a Lagrangian subspace in an even-dimensional vector space with the standard symplectic form. This invariant generalizes the notion of the intersection matrix of a chord diagram. Moreover, the actions of Morse perestroikas (or taking a partial dual) and Vassiliev moves on ribbon graphs are reinterpreted nicely in the language of L-spaces, becoming changes of bases in this vector space. Finally, we define a bialgebra structure on the span of L-spaces, which is analogous to the 4-bialgebra structure on chord diagrams.

3.1.21 Mikhail Verbitsky

[1] Ekaterina Amerik, Misha Verbitsky, Teichmüller space for hyperkahler and symplectic structures J. Geom. Phys. 97 (2015), 44-50

Let S be an infinite-dimensional manifold of all symplectic, or hyperkähler, structures on a compact manifold M, and $Diff_0$ the connected component of its diffeomorphism group. The quotient $S/Diff_0$ is called the Teichmüller space of symplectic (or hyperkähler) structures on M. MBM classes on a hyperkähler manifold M are cohomology classes which can be represented by a minimal rational curve on a deformation of M. We determine the Teichmüller space of hyperkähler structures on a hyperkähler manifold, identifying any of its connected components with an open subset of the Grassmannian variety $SO(b_2-3,3)/SO(3)\times SO(b_2-3)$ consisting of all Beauville-Bogomolov positive 3-planes in $H^2(M,\mathbb{R})$ which are not orthogonal to any of the MBM classes. This is used to determine the Teichmüller space of symplectic structures of Kähler type on a hyperkähler manifold of maximal holonomy. We show that any connected component of this space is naturally identified with the space of cohomology classes $v \in H^2(M,\mathbb{R})$ with $q(v,v) > 0$, where q is the Bogomolov-Beauville-Fujiki form on $H^2(M,\mathbb{R})$.

[2] Andrey Soldatenkov, Misha Verbitsky k-symplectic structures and absolutely trianalytic subvarieties in hyperkahler manifolds, J. Geom. Phys. 92 (2015), 147-156

Let (M, I, J, K) be a hyperkähler manifold, and $Z \subset (M, I)$ a complex subvariety in (M, I). We say that Z is trianalytic if it is complex analytic with respect to J and K, and absolutely trianalytic if it is trianalytic with respect to any hyperkähler triple of complex structures (M, I, J', K') containing I. For a generic complex structure I on M, all complex subvarieties of (M, I) are absolutely trianalytic. It is known that the normalization Z' of a trianalytic subvariety is smooth; we prove that $b_2(Z') \geq b_2(M)$, when M has maximal holonomy (that is, M is IHS). To study absolutely trianalytic subvarieties further, we define a new geometric structure, called k-symplectic structure; this structure is a generalization of hypersymplectic structure. A k-symplectic structure on a $2d$-dimensional manifold X is a k-dimensional space R of closed 2-forms on X which all have rank $2d$ or d. It is called non-degenerate if the set of all degenerate forms in R is a smooth, non-degenerate quadric hypersurface in R. We consider absolutely trianalytic tori in a hyperkähler manifold M of maximal holonomy. We prove that any such torus is equipped with a non-degenerate

k-symplectic structure, where $k = b_2(M)$. We show that the tangent bundle TX of a k-symplectic manifold is a Clifford module over a Clifford algebra $Cl(k-1)$. Then an absolutely trianalytic torus in a hyperkähler manifold M with $b_2(M) \geq 2r+1$ is at least 2^{r-1}-dimensional.

[3] Misha Verbitsky Degenerate twistor spaces for hyperkahler manifolds, Journal of Geometry and Physics Volume 91, Pages 2-11, 2015

Let M be a hyperkähler manifold, and η a closed, positive (1,1)-form with $rk\eta < \dim M$. We associate to η a family of complex structures on M, called a degenerate twistor family, and parametrized by a complex line. When η is a pullback of a Kähler form under a Lagrangian fibration L, all the fibers of degenerate twistor family also admit a Lagrangian fibration, with the fibers isomorphic to that of L. Degenerate twistor families can be obtained by taking limits of twistor families, as one of the Kähler forms in the hyperkähler triple goes to η.

[4] Misha Verbitsky Ergodic complex structures on hyperkahler manifolds Acta Mathematica September 2015, Volume 215, Issue 1, pp 161-182

Let M be a compact complex manifold. The corresponding Teichmüller space $Teich$ is a space of all complex structures on M up to the action of the group $Diff_0(M)$ if isotopies. The mapping class group $\Gamma := Diff(M)/Diff_0(M)$ acts on $Teich$ in a natural way. An **ergodic complex structure** is the one with a Γ-orbit dense in $Teich$. Let M be a complex torus of complex dimension ≥ 2 or a hyperkähler manifold with $b_2 > 3$. We prove that M is ergodic, unless M has maximal Picard rank (there is a countable number of such M). This is used to show that all hyperkähler manifolds are Kobayashi non-hyperbolic.

[5] Andrey Soldatenkov, Misha Verbitsky Holomorphic Lagrangian fibrations on hypercomplex manifolds Int Math Res Notices (2015) 2015 (4): 981-994

A hypercomplex manifold is a manifold equipped with a triple of complex structures satisfying the quaternionic relations. A holomorphic Lagrangian variety on a hypercomplex manifold with trivial canonical bundle is a holomorphic subvariety which is calibrated by a form associated with the holomorphic volume form; this notion is a generalization of the usual holomorphic Lagrangian subvarieties known in hyperkaehler geometry. An HKT (hyperkaehler with torsion) metric on a hypercomplex manifold is a metric determined by a local potential, in a similar way to the Kaehler metric. We prove that a base of a holomorphic Lagrangian fibration is always Kaehler, if its total space is HKT. This is used to construct new examples of hypercomplex manifolds which do not admit an HKT structure.

[6] Michael Entov, Misha Verbitsky Unobstructed symplectic packing for tori and hyperkahler manifold, arXiv:1412.7183, Journal of Topology and Analysis, DOI: 10.1142/S1793525316500229.

Let M be a closed symplectic manifold of volume V. We say that M admits an unobstructed symplectic packing by balls if any collection of symplectic balls (of possibly different radii) of total volume less than V admits a symplectic embedding to M. In 1994 McDuff and Polterovich proved that symplectic packings of Kahler manifolds can be characterized in terms of the Kahler cones of their blow-ups. When M is a Kahler manifold which is not a union of its proper subvarieties (such a manifold is called Campana simple) these Kahler cones can be described explicitly using the Demailly and Paun structure theorem. We prove that any Campana simple Kahler manifold, as well as any manifold which is a limit of Campana simple manifolds in a smooth deformation, admits an unobstructed symplectic packing by balls. This is used to show that all even-dimensional tori equipped with Kahler symplectic forms and all hyperkahler manifolds of maximal holonomy admit unobstructed symplectic packings by balls. This generalizes a previous result by Latschev-McDuff-Schlenk. We also consider symplectic packings by other shapes and show, using Ratner's orbit closure theorem, that any even-dimensional torus equipped with a Kahler form whose cohomology class is not proportional to a rational one admits a full symplectic packing by any number of equal polydisks (and, in particular, by any number of equal cubes).

[7] E. Amerik and M. Verbitsky, Rational Curves on Hyperkähler Manifolds, Int. Math. Res. Not. Advance Access, `http://imrn.oxfordjournals.org/content/early/2015/05/19/imrn.rnv133.refs`

Let M be an irreducible holomorphically symplectic manifold. We show that all faces of the Kähler cone of M are hyperplanes H_i orthogonal to certain homology classes, called monodromy birationally minimal (MBM) classes. Moreover, the Kähler cone is a connected component of a complement of the positive cone to the union of all H_i. We provide several characterizations of the MBM-classes. We show the invariance of MBM property by deformations, as long as the class in question stays of type $(1,1)$. For hyperkähler manifolds with Picard group generated by a negative class z, we prove that $\pm z$ is $\mathbb{Q}$-effective if and only if it is an MBM class. We also prove some results towards the Morrison-Kawamata cone conjecture for hyperkähler manifolds.

3.2 Scientific conferences and seminar talks

3.2.1 Valery Beloshapka

[1] One-day conference "Complex Analysis and Geometry" dedicated to the memory of A. G. Vitushkin (October 6, 2015, Steklov Mathematical Institute of RAS, Moscow)
Talk "The Elliptic Functions and Analytical Complexity of The Harmonic Functions".

3.2.2 Alexander Belavin

[1] Gribov Memorial Workshop ,June, 17-20,Landau Institute for Theoretical Physics, Chernogolovka, Russia.

Talk “Minimal Liouville gravity and Frobenius manifolds. ”

3.2.3 Mikhail Bershtein

[1] Visit to Japan March 2-12, participation in workshops *Representation theory, special function and Painlev equation* March 3-6 Kyoto and *Integrable Systems and Representation Theory* Tokyo March 9-11, 2015,

Two talks: “Conformal field theory of Painleve VI (after Gamayun, Iorgov, Lisovyy)” and

“Bilinear equations on Painleve tau functions from CFT”.

[2] Visit of SISSA, Trieste, June 27 – July 8.

Talk “Painleve III(D_8): tau functions, Kiev formula, Backlund transformations, bilinear equations and q-deformation” at Integrable Systems Seminar SISSA, Trieste Italy July 1 2015.

[3] IAS/Park City Mathematics Institute, USA, Summer Session Geometry of moduli spaces and representation theory July 12-18.

[4] Summer school “Lie algebras, algebraic groups and invariant theory”, Samara June 22- 27

Course of three lectures “Bases in representations of Lie algebras, integrable systems and moduli spaces”.

[5] Workshop and School ”Quantum Geometry, Duality and Matrix Models” 2015, August, 24 - August, 30 Moscow, http://wwwth.itep.ru/mathphys/conf/moscow-2015/

Talk “Painleve III (D8): tau -functions, representation theory (and q deformations)”.

[6] Workshop classical and quantum integrable systems, Protvino, July 8-10

Talk “Liouville reflection operators, Yangians and Bethe ansatz 2”.

3.2.4 Yuri Chekanov

[1] Conference “Symplectic Techniques in Topology and Dynamics”, Cologne, August 24 – 28

Talk “Homotopy type of the space of tight contact structures on certain 3-manifolds”

3.2.5 Konstantin Fedorovsky

[1] "Conference on Harmonic Analysis, Function Theory, Operator Theory and Applications in honor of Jean Esterle", Bordeaux, France, June, 1 – June, 4

Talk "Nevanlinna domains and density of certain polynomial modules"

[2] Conference "24th St.Petersburg Summer Meeting in Mathematical Analysis and a Summer School for Young Scientists", St. Petersburg, Russia, June, 25 – June, 30

Talk "Caratheodory domains and Rudin's converse of the maximum modulus principle"

[3] Conference "Meeting of Generations", Moscow, Russia, June, 9 – June, 11

Talk "Approximation by polyanalytic functions"

[4] Visit to Barcelona, July

Talk "Carathéodory sets and their properties" at "Seminari d'Anàlisi de Barcelona" (Universitat Autónoma de Barcelona & Universitat de Barcelona)

[5] Conference "Workshop and Autumn School "Spaces of Analytic Functions and Singular Integrals (SAFSI2015)", St. Petersburg, Russia, October, 12 – October, 15

Talk "Carathéodory sets and their conformal mappings"

[6] Conference "International conference Journees du GDR AFHP", CIRM, Marseille, France, November, 30 - December, 4

Talk "Approximative properties of polyanalytic polynomial modules".

3.2.6 Evgeny Feigin

[1] Conference "Enveloping Algebras and Geometric Representation Theory", Oberwolfach, 10 May – 16 May, 2015.

[2] Workshop "On the Interaction of Representation Theory with Geometry and Combinatorics", Bonn, 22 May – 02 April, 2015.

Talk "Abelianization and affine Grassmannians".

[3] Workshop "Aspects of Lie theory", Rome, Italy, January 7–10, 2015.

[4] International summer school "Theoretical problems of physics of fundamental interactions", Zelenogorsk, 19 July – 31 July, 2015.

Talk "Solitons, vertex operators and symmetries".

[5] Summer school "Lie algebras, algebraic groups and invariants theory", Samara, 22 June – 27 June, 2015.

Talk "Poincare-Birkhoff-Witt filtration and flag varieties"

3.2.7 Sergey Galkin

[1] Conference NoGAGS, Berlin, December 21–22,

Talk "Hyperkähler manifolds and modular forms"

[2] Conference Magadan Algebraic Geometry International Conference, Magadan, December 6–12,

Talk “Counting rational curves on abelian varieties”

[3] Conference Categorical and analytic invariants in Algebraic geometry II, Kashiwa, November 16 – 20,

Talk “Counting rational curves on abelian varieties”

[4] Conference Categorical and analytic invariants in Algebraic geometry 1, Moscow, September 14 – 18

Talk “Joins and Hadamard products”

[5] Conference Second SwissMAP Geometry and Topology Conference, Les Diablerets, June 23–26

Talk “Calculus of algebraic dynamics”

[6] Conference Amplitudes, Motives and beyond, Mainz, June 01–12

Talk “Exceptional minuscule graphs and motives”

[7] Conference Twenty-second Gökova Geometry/Topology Conference, Gökova, May 25–30

Talk “Homotopy quantization”

[8] Conference Topics in Geometry, Istanbul, May 18–22

Talk “Branched covers and columns of Golyshev–Mendeleev’s table”

Talk “Joins and Hadamard products”

[9] Conference Tropical Aspects in Geometry, Topology and Physics, MF, Oberwolfach, April 26–May 02

Talk “Degenerations, transitions and quantum cohomology”

[10] Conference New techniques in birational geometry, Stony Brook, March 07–11

Talk “A tentative integer-valued birational invariant of threefolds”

[11] First SwissMAP Geometry and Topology conference, Engelberg, January 18–23

Talk “Refined counting of holomorphic discs bounded on Lagrangian torus on a surface”

[12] Conference Algebraic geometry and complex analysis for young Russian mathematicians , Koryazhma, Russia, August 17 – 22

Talk “Class of a cubic in the Grothendieck–Levine ring”

[13] Conference ”Baikal readings”, Irkutsk, March 16–27

Talk “On the origin of groups”

[14] Talk Quantum indices of real curves and non-commutative Ginzburg-Landau potentia at “Riemannian surfaces, Lie algebras and mathematical physics seminar, Moscow, December 18 (Independent University of Moscow)

[15] Talk Markov triples and exotic tori on a plane at “Homotopy seminar”, Moscow, December 14 (HSE)

[16] Talk Homotopy quantization at “Homotopy seminar”, Moscow, December 7 (HSE)

[17] Talk Quantum, refined, tropical, real, and homotopic at ”Algebraic Geometry seminar”, Moscow, December 4 (Laboratory of Algebraic Geometry at HSE)

[18] Talk Mirror Moonshine at Automorphic Forms and their applications, Moscow, December 1 (HSE)

[19] Talk Measurements of varieties at University of Chicago Algebraic Geometry Seminar, Chicago, October 20 (University of Chicago)

[20] Talk What is moonshine? at Séminaire "Fables Géometriques", villa Batelle, Carouge, September 28 (Université de Genève)

[21] Talk Geometric moonshines at Automorphic Forms and their applications, Moscow, September 15 (HSE)

[22] Talk Gamma function in symplectic topology at Geometric structures on manifolds, Moscow, September 3 (HSE)

[23] Talk A zeta-function of a dg-category, Trieste, July 13 (ICTP)

[24] Talk The conifold point, conjecture $\mathcal{O}$, and related problems at "Summer Tropical Seminar", Bonn, June 9

[25] Talk Artin-Mumford, Ingalls-Kuznetsov and Hosono-Takagi at "Algebraic geometry seminar", Vienna, May 8 (University of Vienna)

[26] Talk "Branched covers and the explanation for miraculous "Golyshev–Mendeleevs table"" at Séminaire de Géométrie Tropicale, Paris, April 6 (Université Pierre et Marie Curie)

[27] Talk "On zeta-function of a category" at "Geometric structures on manifolds", Moscow, March 15 (Laboratory of Algebraic Geometry at HSE)

[28] Talk "An explicit construction of Miura's varieties" at "Algebraic Geometry seminar", Moscow, February 13 (Laboratory of Algebraic Geometry at HSE)

[29] Talk "An explicit construction of Miura's varieties" at Algebraic Geometry seminar, Stony Brook, February 4 (Stony Brook University)

3.2.8 Alexei Gorodentsev

[1] International Workshop "Particles, Fields and Strings", September 29 – October 02, 2015, Baku, Azerbaijan
`https://sites.google.com/site/bakupfs2015/home`
Mini-course "Algebraic geometry for physicists" (4 lectures)

3.2.9 Maxim Kazarian

[1] Invited professor to Banach Center, January 15 -February 28, 2015, Warsaw, Poland

[2] Algebraic Geometry conference IMPANGA April 12–18, 2015, Bedlewo, Poland, the talk "Gysin homomorphysm and degeneracy loci"

[3] Workshop on Geometric invariants and quantum spectral curves, June 4–9, 2015, Leiden, the Netherland, the talk "Symplectic geometry of topological recursion"

[4] Colloquium on topological recursion, October 19–20, 2015, St. Petersburg, Russia, the talk "Polynomiality property in the enumeration of maps and hypermaps"

3.2.10 Anton Khoroshkin

[1] Conference "GRT, MZV and associators" August 20–29 in Les Diablerets (Switzerland)
Talk "Group actions, framed little balls operads and graph complexes"
[2] Visit Switzerland, February 2015,
Talks "On equivariant Deligne Conjecture" "Macdonald polynomials and Highest weight categories"
at *Séminaire Groupes de Lie et espace de modules*, Université de Génève;
Talk "Around categorification of Macdonald polynomial"
at *Talks in mathematical physics,* ETH Zurich;

3.2.11 Iosif Krasilshchik

[1] International Conference "Physics and Mathematics of Nonlinear Phenomena (PMNP2015)", Gallipoli (Italy), June, 20 – 27
Talk "Symmetry reductions of Lax integrable 3D systems"

[2] International Workshop "Integrable Nonlinear Equations", Mikulov (Czech Republic), October, 18 – 24
Talk "Nonlocal symmetries of the 3D rdDym equation"

3.2.12 Andrei Kustarev

[1] Conference "Symplectic Techniques in Topology and Dynamics 2015", University of Cologne, August 24 - 28, 2015.
Discussing the paper on Chern numbers of manifolds with torus actions.

3.2.13 Taras Panov

[1] Conference " Integrability in Algebra, Geometry and Physics: New Trends", Congressi Stefano Franscini: ETH, Ascona, Switzerland, July, 12 – 17, 2015
Invited Talk "Geometric optimization of the eigenvalues of the Laplace operator and mathematical physics"
[2] Visit for scintific collaboration to Institut de Mathématiques de Marseille (UMR 7373 CNRS), Marseille, France, May, 1 – July, 31, 2015

3.2.14 Alexei Penskoi

[1] International Chinese-Russian Conference “Torus Actions: Topology, Geometry and Number Theory”, Beijing, China, October, 26 – 29
Talk “Cohomology of quotients of moment-angle manifolds”

[2] International Conference “Toric Topology, Number Theory and Applications”, Khabarovsk, Russia, September, 2 – 6
Talk “On toric generators in the unitary and special unitary bordism rings”

[3] International Conference Combinatorial and Toric Homotopy, on the occasion of Professor Frederick Cohen’s 70th Birthday, National University of Singapore, August 24 – 28
Talk “On the cohomology of partial quotients of moment-angle manifolds”

[4] International Meeting “Toric Topology 2015 in Osaka”, Osaka, Japan, June 16 – 19
Talk “On toric generators in the unitary and special unitary bordism rings”
[5] “Geometry and Topology”, a conference in honor of Martin Bendersky’s seventieth birthday and in commemoration of Sam Gitler, Princeton University, USA, March 18 – 21

Talk “On toric generators in the unitary and special unitary bordism rings”

[6] Visit to Osaka, June
Talk “Pontryagin algebras of moment-angle manifolds” (Osaka City University)

3.2.15 Petr Pushkar’

[1] Talk “Symplectic and contact reduction, isotopy lifting and Chekanov type theorem on spherization of cotangent bundle” – seminar on geometric structures on manifolds, HSE (Fall 2015)
[2] Talk “Generating families in contact topology” – Verboveckii-Krasilchik seminar on cohomological aspects of geometry of differential equations at IUM (23 Dec 2015)

3.2.16 George Shabat

[1] Workshop ”Riemann surfaces and related topics ”. Universidad Autonoma de Madrid, Spain, July 2-3. Talk ”Restoring a complex algebraic surface by its universal cover: old and new results”.
[2] International Conference ”p-adic mathematical physics and its applications”. Belgrade, September 7-12. Talk: ”On the elliptic time in the Newton-Kepler adelic dynamic”.
[3] International Conference ”Economy in the language and communication”. Moscow,

Russian State University for the Humanities October, 27-28. Talk ”Economy in the languages of science and in the natural language” (in russian, joint with G.E. Kreidlin).

[3] Visit to Lissabon, January.
Talk ”On the geometry of Tod-Hitchin solutions of Einstein equations” at Geometria em Lisboa Seminar (Instituto Superior Tecnico).

[4] The joint seminar of the Sector 4.1 (IITP RAS) and Poncelet French-Russian laboratory ”Arithmetic, Geometry and Coding Theory”, January 22. Talk ”Dessins d'enfants and the arithmetic of moduli spaces of curves.”
[5] Seminar of the theoretical Department of the Institute of Theoretical and Experimental Physics, October 12. Talk ”On the elliptic ”time” in the Kepler-Newton Dynamics” (in russian).
[6] Conference ”Topics of the research of E.V. Paducheva”, The Institute of Linguistic, Russian Academy of Sciences, October 30. Talk ”Languages of geometry in their relations with natural languages” (joint with G.E. Kreidlin, in russian).
[7] Seminar of the algebra Department of mechanico-mathematical faculty of Lomonosov Moscow State University, November2. Talk ”Fullerenes and dessins d'enfants”.
[8] Extended Seminar on the occasion of 75-th birthday of A.V. Mikhalev, Lomonosov Moscow State University, November 10. Talk ”Grothendieck's dessins d'enfants” (joint with N.M. Adrianov and E.M. Kreines, in russian).

3.2.17 Stanislav Shaposhnikov

1)“Bielefeld Stochastic Summer School” (Bielefeld University, Germany, August 2015), invited speaker, talk: “On uniqueness problems for nonlinear Fokker-Planck-Kolmogorov equations”

3.2.18 Mikhail Skopenkov

[1] Talks at several seminars in Moscow, Nizhniy Novgorod, and Berlin.

3.2.19 Evgeni Smirnov

[1] Conference “Torus Actions in Geometry, Topology, and Applications”, Skolkovo Institute of Science and Technology, Moscow 16–21 February 2015.
Talk: “Spherical multiple flag varieties”
[2] Conference “Geometry and Quantization”, ICMAT, Madrid, 14–18 September 2015.
Talk: “Spherical multiple flag varieties”

[3] Magadan Conference on Algebraic Geometry, Magadan, Russia, 6–12 December 2015.

Talk: "Spherical multiple flag varieties"

[4] 5th Conference and School "Lie Algebras, Algebraic Groups, and Invariant Theory", Samara, Russia, 22-27 June 2015.

Talk: "Cominuscule double flag varieties"

[5] Research stay at Cornell University, Ithaca, NY, October 25 – November 5, 2015. Host person: Prof. Allen Knutson.

[6] Discrete Geometry and Combinatorics Seminar, Cornell University, Ithaca, NY, November 2, 2015

Talk: "Spherical multiple flag varieties"

[7] Combinatorics Seminar, University of Minnesota, MN, November 6, 2015

Talk: "Schubert calculus and Gelfand–Zetlin polytopes"

3.2.20 Mikhail Verbitsky

[1] Conference "Hyperbolicity in algebraic geometry" 5.01.2015 - 15.01.2015 Brasil, Ilhabela, a talk "Symplectic packing and hyperbolicity"

[2] Conference and a school "Holomorphic dynamics" 5.01.2015 - 15.01.2015 Brasil, Ilhabela, a mini-course "Mapping class group and moduli spaces"

[3] Workshop on Hyperkaehler Geometry 2.03.2015 - 6.02.2015, KIAS, Seoul, Korea a talk "Symplectic packing for Campana simple manifolds"

[4] Conference Distribution of Rational and Holomorphic Curves in Algebraic Varieties 15.03.2015 - 20.03.2015 Banff, Canada, BIRS, a talk "MBM classes on hyperkahler manifolds"

[5] Simons Symposium on Geometry Over Nonclosed Fields: Geometry and Arithmetic of Holomorphic Symplectic Varieties, 22.03.2015 - 28.03.2015, Puerto-Rico, a talk "Proof of Morrison-Kawamata cone conjecture for holomorphically symplectic manifolds"

[6] Mathematische Arbeitstagung 2015 26.06.2015 - 3.07.2015, MPIM, Bonn, talk "Ergodic actions on the moduli of complex structures"

[7] Workshop in Dynamics and Geometry 17.08.2015 - 21.08.2015, KIAS, Seoul, Korea, talk "Gromov-Hausdorff metric on the space of hyperkahler structures"

[8] Workshop "Collapsing Calabi-Yau Manifolds" 31.08.2015 - 4.09.2015, Stony Brook, SCGP, talk "Limits of hyperkahler metrics"

[9] Conference Journees Complexes Lorraines 2015, 28.09-2.10.2015, l'Institut Élie Cartan (IECL), Nancy, France, "Transcendental Hodge algebra"

[10] Conference Hyper-Kähler Manifolds and Related Structures in Algebraic and Differential Geometry CIRM, Levico Terme (Trento), 3-6 November 2015, talk "Construction of automorphisms of hyperkähler manifolds."

[11] Belgian-Dutch Algebraic Geometry Seminar 11.12.2015, Leiden University, talk "Transcendental Hodge algebra".

[12] Seminaire de geometrie et dynamique, ENS Lyon, 08.10.2015, talk “Ergodic complex structures and Kobayashi metric”.

[13] GT Operateur de Dirac, Orsay, December 2 2015, talk “Ergodic complex structures”.

3.3 Teaching

3.3.1 Valery Beloshapka

[1] With A.I.Bufetov, Geometry-1. Independent University of Moscow, I year students, September-December 2015, 2 hours per week.

Course program

First unit: Elementary Lobachevsky geometry.
1. Introductory lection. History of geometry. Euclid’s elements. The fifth postulate. Spherical geometry. Lobachevsky geometry. Erlagen Klein’s program. Geometry of an infinite tree - geodesics, oricycles, a boundary.
2. Complex numbers in geometry. The plane’s completion - complex sphere, stereographic projection. Linear and fractional-linear transformations and its properties.
3. Classification of the Lobachevsky plane’s motions.

The second unit: Affine and projective geometry in the dimensions one, two and three.
4. $d = 1$: Real line $\mathbb{R}^1$, projective line $\mathbb{RP}^1$. Motions of $T(2, \mathbb{R})$, linear $GL(1, \mathbb{R}),\ SL(1, \mathbb{R}),\ O(1)$, affine $Aff(1, \mathbb{R})$ and projective $PSL(1, \mathbb{R})$ transformations and its dimensions, orbits, invariants and discrete subgroups. Unit circle S^1: three realizations ($\mathbb{R}/\mathbb{Z},\ |z| = 1, \mathbb{RP}^1$), transition functions.
5. $d = 2$: Real plane $\mathbb{R}^2$. Motions of $T(2, \mathbb{R})$, linear $GL(2, R), SL(2, R), O(2)$, affine $Aff(2, R)$ transformations. The complex structure on $\mathbb{R}^2 = \mathbb{C}^1,\ GL(1, C),\ SL(1, C),\ U(1),\ Sp(1)$. Real projective plane, complex projective plane, complex projective line, corresponding groups, maps, transitions, invariants. Projective transformations, cross-ratio. Projective metrics, projective interpretation of Klein of the Lobachevsky plane.
6. $d = 3$: Real space $\mathbb{R}^3$. Motions of $T(3, R)$, linear $GL(3, R), SL(3, R), O(3)$, affine $Aff(3, R)$. Scalar, vector and triple product. Projective space, maps, transitions, invariants, orientation, subspaces of the dimensions 1 and 2.

The third unit: Groups and geometry.
7. Fuchsian groups. Poincare’s theorem on the fundamental polyhedron.
8. The group of motions of the Lobachevsky space. Klein’s groups.

9. Geometry of discrete groups. Gromov's hyperbolicity.
10. Examples of Lie groups. $SU(2)$ and $SO(3)$. Quaternions.

The forth unit: Geometry of $\mathbb{R}^4 = \mathbb{C}^2 = \mathbb{H}^1$.
11. $d = 4$: Real space $\mathbb{R}^4$, subspaces, ball and sphere, cube and its faces, convex polyhedra. Groups: $GL(4, R), SL(4, R), O(4), SO(4)$, motions, $Aff(3, R)$. Measuring of distances and volumes, orientation. $\mathbb{R}^4$ as $\mathbb{C}^2$, bidisk and sphere, complex lines and two-dimensional planes, complex part of real hyperplane, $GL(2, C), U(2), SU(2), Sp(2)$.
12. Compactifications. $\mathbb{RP}^4$ and $PSL(4, R)$, stereograpic projection, $\mathbb{CP}^2$ and $PSL(2, C)$; $\mathbb{C}^1 \times \mathbb{C}^1$, maps and transition functions. Grassmann manifold $Gr(2, R^4)$ and $Gr(1, C^2)$, dimensions, maps and transition functions. Algebraic sets and its closures, examples.
13. Minkowski spaces in dimensions 2, 3 and 4. Lorenz groups $O(1, 1)$, $O(1, 2)$, $O(1, 3)$. Special relativity theory. $U(1, 1)$ and $U(1, 2)$ groups.
14. Different realisations of a ball and a sphere in $\mathbb{C}^2$, the distribution of complex tangent lines. Projective automorphisms of a ball. Projective automorphisms of a bidisk in $\mathbb{C}^1 \times \mathbb{C}^1$.

3.3.2 Alexander Belavin

[1] Introduction to Superstring theory. Independent University of Moscow, 3-5 year students, February-May 2015, 4 hours per week.

Program.

1. Bosonic d=26 String;

2. Fermionic Neveu-Schwarz-Ramond string in 10-dimensional Space-Time;

3. BRST-quantization of Neveu-Schwarz-Ramond string;

4. N=2 superconformal symmetry and its action in Neveu-Schwarz-Ramond string on its world sheet;

5. Spectral flow isomorphysm of N=2 supeconformal algebra, GSO-projection of NSR string space of states and Space-Time supersymmetry;

6. Compactification on Calabi-Yau manifolds and N=2 superconformal symmetry on the world sheet , Space-Time supersymmetry in 4 dimensions.

3.3.3 Mikhail Bershtein

[1] Representation Theory and Knizhnik-Zamolodchikov Equations. Independent University of Moscow, February-May 2015, 2 hours per week.

Course was based on the book P. Etingof, I. Frenkel, A. Kirillov (Jr), Lectures on representation theory and Knizhnik-Zamolodchikov equations.

Program

1. Simple Lie algebras. Highest weight representation. Casimir operator.

2. Affine Lie algebras.
3. Intertwining operators.
4. Knizhnik-Zamolodchikov equations.
5. Integral formulas for solutions of Knizhnik-Zamolodchikov equations.
6. Free field realization. Wakimoto modules.
7. Quantum groups.
8. Local systems and configuration spaces.
9. Monodromy of Knizhnik-Zamolodchikov equations and braid group.
10. Drinfeld-Kono theorem.

[2] Introduction to group theory, Moscow , II year students, February-May 2015, 2 hours per week.

Program

Part 1.

1. Permuation group.
2. Abstract groups. Action on the set.
3. Order of the group element. Cosets. Lagranges theorem.
4. Group Isomorphism. Direct product of groups.
5. Conjugacy classes. Description of conjugacy classes for group S_n.

Part 2.

6. Group homomorphism. Group commutant.
7. Representation of groups. Direct sum of representations. Direct product of representations. Irreducible representations.
8. Characters of representations.
9. Tensor product of vector spaces. Restriction of representation on subgroup.

Part 3

10. Manifolds. Manifolds defined by system of equations. Tangent space.
11. Lie groups. Lie algebras. Tangent space to Lie group in 1 is an Lie algebra. Exponential map.
12. Isomorphism of Lie algebras $\mathfrak{so}(3)$, $\mathfrak{su}(2)$ and $\mathbb{R}^3$.
13. Representations of Lie groups. Representations of Lie algebras.
14. Irreducible representations of Lie algebra $\mathfrak{su}(2)$. Representations of Lie groups $SU(2)$ and $SO(3)$. Spin.
15. Characters of Lie groups representations. Clebsh-Gordon coefficients.

3.3.4 Yuri Chekanov

[1] Topology. Independent University of Moscow, 2 year students, February-May 2015, 2 hours per week.

1. Topological spaces, basic constructions with them. Elements of category theory: products, colimits.

2. CW spaces, their chain complexes. CW homology and cohomology. Long exact sequence of a pair. Künneth formula. Multiplication in cohomology.

3. Homotopy of mappings and chain homotopy. Homotopy equivalence. Homotopy groups. Long exact sequence of a pair. Weak homotopy equivalence. CW approximation. Hurewicz and Whitehead theorems. Eilenberg–MacLane spaces.

4. Simple homotopy equivalence of CW spaces. Invariance of simple homotopy type under homeomorphisms (without proof). Simple homotopy equivalence of chain complexes. Whitehead torsion. Reidemeister torsion.

5. Manifolds, homology, Morse theory: an overview. Two- and three-dimensional closed manifolds. Heegaard splitting. Three-dimensional lens spaces, their classification.

6. Locally trivial fibrations. Long exact sequence of a fibration. Lie groups. Principal bundles. Classifying spaces. Characteristic classes. Grassmanians as classifying spaces. Schubert cycles. Splitting principle.

3.3.5 Konstantin Fedorovsky

[1] Complex Analysis. Independent University of Moscow, II year students, February-May 2015, 2 hours per week.

Program:

1) Complex number, their properties and operations on them. Complex plane $\mathbb{C}$ and its compactification $\overline{\mathbb{C}}$. Topology in $\mathbb{C}$ and $\overline{\mathbb{C}}$, limits and continuity of functions of a complex variable. The function e^z and exponential form of complex numbers.

2) Paths and curves in $\mathbb{C}$. Jordan curve theorem, the concept of a simply connected domain. Rectifiable paths and curves. Increment of the argument along a path. Index of a path and its properties.

3) $\mathbb{R}$– and $\mathbb{C}$–differentiability of functions of a complex variable. Cauchy–Riemann conditions. Properties of a complex derivative. Directional derivative. Holomorphic functions. Conformity and its relationship with holomorphy.

4) Basic elementary functions of a complex variable and thier properties. Multi-valued functions and their continuous and holomorphic branches.

5) Integral over a path and over a curve by a complex variable and their properties. Goursat lemma. Cauchy integral theorem. Complex primitive, its properties and Newton–Leibnitz formula. Existence of holomorphic branches of the root function and of the logarithm in simply connected domains in $\mathbb{C} \setminus \{0\}$.

6) Cauchy integral formula, Cauchy formula for derivatives and infinite differentiability of holomorphic functions. Pompeiu formula. Mean value theorem, maximum modulus principle. Morera theorem. Local uniformly convergence of sequences of holomorphic functions. Weierstrass theorem.

7) Power series, Abel theorem, Cauchy–Hadamard formula. Termwise differentiability and integrability of power series. Singular points at the boundary of the disk of convergence.

Pringsheim theorem.

8) Taylor series. Expansion of holomorphic function into a power series. Cauchy inequalities for Taylor coefficients. Liouville theorem. Zeros of holomorphic functions. Uniqueness theorem. Approximation of holomorphic functions by polynomials. Runge theorem.

9) Laurent series. Expansion of holomorphic functions into a Laurent series. Cauchy inequalities for Laurent coefficients. Isolated singularities of holomorphic functions and their classification, Sokhotskii theorem. Infinity as a singular point, entire and meromorphic functions with poles at infinity. Schwarz lemma and conformal automorphisms of the basic domains.

10) Residues. Cauchy residues theorem. Residue at ∞. Evaluation of residues. Jordan lemma. Evaluation of integrals (including integrals in the sense of principal values) using the method of residues. Logarithmic residue. Argument principle. Rouche theorem. Domain preservation principle.

11) Inverse boundary correspondence principle for conformal mapping. Criteria for local univalence and local invertibility. Riemann–Schwarz symmetry principle. Construction of conformal mappings of a given domains. Hurwitz theorem and its corollaries. The classes S and Σ of univalent functions. Area theorem and Koebe theorem.

12) Compactness principle for families of holomorphic functions. Montel theorem. Accessibility of the upper about for continuous functional on a compact family of holomorphic functions. Riemann mapping theorem and its proof.

13) Boundary behavior of conformal mappings. Carathéodory theorems.

14) Analytic elements and their analytic continuation. Analytic continuation along a path and by a chain. Theorem about continuation by homotopic paths and monodromy theorem.

15) Complete analytic function in the sense of Weierstrass, its holomorphic branches and points of analyticity. Branch points of analytic functions and their classification. Complete analytic function 'root' and 'logarithm'. The concept of a Riemann surface.

[2] Theory of functions of a complex variable. Bauman Moscow State Technical University, II year students, February-June 2015, 3 hours per week.

Program:

1) Complex numbers, complex plane. Elements of topology of the complex plane. Increment of the argument along a path. Index of a path with respect to a point and its properties.

2) Differentiability of functions of a complex variable. Cauchy–Riemann conditions. Holomorphic functions. The notion of a conformal mapping. Elementary functions of a complex variable and their properties.

3) Integration of complex functions over a curve. Goursat lemma. Cauchy integral theorem for simply connected domain. Complex primitive. Admissible domains and their oriented boundaries. Cauchy integral theorem for admissible domains. Cauchy integral formula. Mean value theorem. Cauchy formula for derivatives and infinite differentiability

of holomorphic functions. Morera theorem.

4) Power series and their domains of convergence. Holomorphicity of the sum of a power series. Taylor series expansion of a holomorphic function. Cauchy inequalities. Liouville theorem.

5) Loran series and their domains of convergence. Cauchy inequalities for coefficients of Loran series. Isolated singularities of holomorphic functions and their description. Sokhotskii theorem. Infinity as a singular point.

6) Residues (definition, basic properties and formulae for evaluation of residues). Cauchy residue theorem. Residue at the infinity and theorem on the total sum of residues. Residue with respect to a domain. Jordan lemma. Evaluation of integrals using the method of residues.

7) Logarithmic residue and its properties. Argument principle. Rouché theorem. Maximum modulus principle and domain preservation principle.

8) Criteria of univalence and local invertibility. Hurwitz theorem and its corollaries. Properties of univalent functions.

9) The concept of a conformal mapping. Elementary functions and respective conformal mappings. Inverse boundary correspondence principle for conformal mappings. Riemann-Schwarz symmetry principle and its applications.

10) Schwarz's lemma and evaluation of groups of conformal automorphisms of basic domains. Riemann theorem and Carathéodory extension theorem.

11) The concept of analytic continuation. Weierstrass theory. Analytic functions and their singularities.

12) Laplace transform, and its properties. Using the operational calculus for solving ordinary differential equations.

[3] Linear algebra. Bauman Moscow State Technical University, I year students, February-June 2015, 2 hours per week.

Program:

1) Definition and basic properties of linear spaces. Basis and coordinates in a linear space. Dimension of a linear space. Isomorphism of linear spaces. Subspaces of linear spaces, sums, direct sums and intersection of subspaces. Linear spans of vector systems.

2) Definition and basic properties of linear operators. Kernel, image, rank and defect of a linear operator. Matrix representation of linear operators. Linear functionals, sesquilinear and bilinear forms in linear spaces. Dual space and its properties, adjoint operator.

3) Invariant subspaces of linear operators. Minimal and characteristic polynomials of a linear operator. Eigenvectors and eigenvalues of linear operators. Existence of invariant subspaces for operator in real and complex spaces. The Cayley-Hamilton theorem. Cyclic vectors of linear operators. Adjoined vectors, rooted subspaces of linear operators. Jordan form of a linear operator.

4) Scalar product and the concept of an euclidian space. Orthonormal basic of finite dimensional euclidian space and its properties. Gram matrix. Gram-Schmidt orthonormal-

ization procedure. Hermitian spaces and their properties. Cauchy-Bunyakovsky inequality in the real and complex cases. Theorems about representation of linear functionals and sesquilinear (bilinear) forms in Hermitian (Euclidian) spaces. Norm of a linear operator.

5) Self-adjoint operators. Properties of eigenvectors and eigenvalues of self-adjoint operators. Spectral decomposition of self-adjoint operators. Bilinear functions and self-adjoint operators in Euclidian spaces. Normal, unitary and orthogonal operators and their canonical representations.

6) Quadratic forms. Representation of quadratic form and of pair of quadratic forms as a sums of squares. Lagrange and Jacobi methds. Inertia law to quadratic forms and Sylvester criteria.

7) Equations of second order hypersurfaces and its change under affine transformations. Classification of equations of central and noncentral second order surfaces.

[4] Polynomial and rational approximation in a complex domain. St. Petersburg State University, IV year students, September-December 2015, 2 hours per week.

Program:

1) Pompeiu formula. Cauchy potential and its properties. Vitushkin localization operator (definition and basic properties).

2) The problem of uniform approximation of functions by polynomials in the complex variable on compact sets in the complex plane. Runge theorem.

3) Mergelyan theorem and its proof by the Vitushkin method.

4) Problems of uniform rational approximation for individual functions and for classes of functions. Examples of lacking of approximation.

5) Analytic capacity of sets and its properties. Vitushkin criterion of rational approximation.

6) Recent results about uniform and C^1-approximability of functions by polynomial and meromorphic solutions of homogeneous second order PDE with constant complex coefficients.

7)Dual approach to problems of rational approximation. Measures orthogonal to rational functions on compact subsets of the complex plane and their properties. Cauchy transform of measures. Bishop localization theorem. F. and M. Riesz theorem and the concept of an analytic balayage of measures.

[5] Algebra and elements of tensor calculus. Bauman Moscow State Technical University, II year students, September-December 2015, 2 hours per week.

Program:

1) Algebraic structures. Semigroups and groups. Basic matrix groups, symmetrical group, alternating group. Isomorphism of groups, Cayley theorem. Homomorphisms of groups. Cyclic groups, order of elements. Normal subgroup. Cosets and quotient groups. Description of groups of small orders. Theorems on group homomorphism. Commutator, the concept of a solvable group. Simple groups. Actions of groups on sets. Sylow theorems. Finitely generated Abelian groups and their structure.

2) Definition and basic properties of rings. Residue ring. Homomorphisms and ideals of rings. Quotient rings. Classes of rings, fields, characteristic of a field. Division in rings. The concept of a module.

3) Polynomials in one and several variables. Division of polynomials, irreducible polynomials. Irreducible polynomials over the fields $\mathbb{R}$ and $\mathbb{C}$. The field of rational fractions. Roots of polynomials, decomposition field of a polynomial. Symmetric polynomials, discriminant and resultant.

4) Tensors in linear space, their properties and basic tensor operations.

3.3.6 Evgeny Feigin

[1] Affine Lie algebras and applications, from 2nd year to PhD students, September-December 2015, 2 hours per week.

Program

Affine Lie algebras form one of the most well-known and popular classes of infinite-dimensional Lie algebras. During the last 20 years, they attracted attention of many mathematicians throughout the world due to the beautiful and rich structure theory and representation theory. The theory of affine Kac-Moody Lie algebras has important applications in the number theory, algebraic geometry and in mathematical physics.

Program:

1. Simple Lie algebras.
2. Affine Kac-Moody Lie algebras: basic definitions.
3. Affine Kac-Moody Lie algebras: integrable representations.
4. Affine $\mathfrak{sl}_2$: representations and theta functions.
5. Boson-fermion correspondence and Schur polynomials.
6. Affine algebras, theta functions and modular forms.
7. Vertex operator construction of representations.

Textbooks:

1. Kac, V. Innite dimensional Lie algebras, Cambridge University Press (1994).
2. Kac V., Raina A. Bombay lectures on Highest weight representations of infite-dimensional Lie algebras (WS, 1987)
3. Pressley, A., Segal, G. (1986), Loop groups.
4. Di Francesco, P., Mathieu, P., Senechal, D. (1997), Conformal Field Theory, Springer-Verlag.
5. Kumar, S., Kac-Moody Groups, their Flag Varieties and Representation Theory, 2012, Springer.

3.3.7 Sergey Galkin

[1] Proofs of Irrationality. Independent University of Moscow, students from 3 year, September-December 2015, 1.5 hours per week.

Program:

1. Rationality, stable rationality, retract-rationality, unirationality, rational connectedness.
2. Examples of rational varieties. Rationality of intersection of two quadrics.
3. Birational invariants. Resolution of singularities and weak decomposition theorem. Holomorphic contravariant tensors.
4. Castelnuovo's rationality criterium.
5. Rationality of surfaces over non-algebraically-closed fields. Del Pezzo fibrations.
6. Conic bundles, discriminant. Double covers and Prym varieties. Intermediate Jacobian of a conic bundle.
7. Artin–Mumford's example of stably irrational unirational threefold. Torsion in homology. Brauer group.
8. Clemens–Griffiths's proof of irrationality of a smooth cubic threefold. Variety of lines on a cubic hypersurface. Weil's intermediate Jacobian and Griffiths's component.
9. Iskovskikh–Manin's proof of irrationality of a smooth quartic threefold. Method of maximal singularities. Birational rigidity.
10. Kollár's method: holomorphic forms in finite characteristic.
11. Voisin's degeneration method. Stable irrationality of a very general double cover of $\mathbb{P}^3$ branched in a quartic.
12. Beauville's proofs using Voisin's degeneration method.
13. Work of Colliot-Thélène and Pirutka. Stable irrationality of a very general quartic threefold.
14. Stable irrationality of a very general quartic fourfold after Totaro.

[2] Algebraic Surfaces, National Research University Higher School of Economics (in IUM), students from 3 year, September 2015 - June 2016, 1.5 hours per week.

Program:

1. Basics of minimal model programme will be introduced as soon as they will be required. Canonical bundle, adjunction formula. Divisors and curves, Picard and Neron–Severi groups, Kähler and Mori cones. Criteria of ampleness. Hodge structure of surfaces, Hodge index theorem. Vanishing theorems, Serre duality. Kodaira dimension. Finite generation of canonical and Cox rings. Blowups and exceptional curves. Castelnuovo contraction theorem. Cone theorem, extremal rays, contraction theorem. Minimal models. Canonical models and canonical singularities.

2. Uniruled surfaces. Rational surfaces. Castelnuovo's rationality criterium. Hesse pencil and other interesting pencils. Del Pezzo surfaces, lines on them, exceptional root systems. Rational Jacobian elliptic surfaces. Du Val singularities. Rational elliptic surfaces without a section. Halphen pencils. Conic bundles, ruled surfaces, Tsen theorem, projectivization of vector bundles. Hirzebruch surfaces. Scrolls. Coble surfaces. Severi–Brauer varieties.

3. Kodaira dimension zero. Abelian surfaces. Bielliptic surfaces. Kummer surfaces. $K3$ surfaces. Torelli theorem. Enriques surfaces. Reye and Cayley models. Connection with Coble surfaces.

4. Kodaira dimension one, elliptic surfaces. Neron–Kodaira–Tate classification of minimal models of elliptic curves over a local field. Mordell–Weil group, Shioda–Tate formula. Theory of Jacobian elliptic surfaces. Ogg–Shafarevich theory, principal homogeneous spaces over elliptic curves.

5. Surfaces of general type. Surfaces with $p_g = q = 0$. Example: Godeaux surface. Campedelli surfaces. Barlow surface. Determinantal quintics and Catanese surfaces. Beauville surfaces. Bidisc quotients. Bogomolov–Miyaoka–Yau inequality. Ball quotients. Mostow rigidity. Fake projective planes. Rigid configurations of lines on a plane, and other rigid configurations. Fiberations by higher genus curves, Shafarevich and Mordell conjectures. A proof of Shafarevich conjecture. Parshin's trick and a proof of Mordell conjecture.

[3] Seminar "Diversity of manifolds", National Research University Higher School of Economics (in IUM), students from 3 year, September 2015 - June 2016, 1.5 hours per week.

[4] With V. Gritsenko

Seminar "Automorphic forms and their applications", National Research University Higher School of Economics (joint with Poncelet lab), students from 3 year, September 2015 - June 2016, 2 hours per week.

[5] With M. Verbitsky and V. Zhgoon

Seminar "Geometric structures on complex manifolds", National Research University Higher School of Economics (in IUM), students from 3 year, September 2015 - June 2016, 3 hours per week.

3.3.8 Alexei Gorodentsev

[1] Algebra. Independent University of Moscow, 2 year students, February–May 2015, 4 hours per week.
`http://gorod.bogomolov-lab.ru/ps/stud/algebra-3/1415/list.html`
Program:

1. Integral elements and integral extensions of commutative rings. Gauss lemma, normal rings. Examples: integral algebraic numbers; integrality of finite group characters, dimension of a complex irreducible representation divides the index of the center.

2. Finitely generated commutative algebras over a field. Transcendence degree. Algebraic elements, finitely generated commutative algebra with division is algebraic. Hilbert's Nullstellensatz.

3. Affine algebraic geometry. Antiequivalence between the category of affine algebraic varieties over algebraically closed field K and the category of finitely generated reduced commutative K-algebras. Maximal spectrum, Zariski topology. Rational functions. Geometric properties of algebra homomorphisms: closed embeddings, finite morphisms, dominant morphisms, normal varieties.

4. Algebraic manifolds. Examples: projective spaces and grassmannians, affine and projective varieties, closed submanifolds. Structure sheaf, regular and rational mappings, the graph of a regular morphism to a separable manifold is closed. Resultant systems, a regular morphism of a projective variety to a separable manifold is closed, a regular morphism of connected projective variety to an affine variety is constant is closed.

5. Dimension of algebraic manifold. Noether's normalization. Dimensions of subvarieties and dimensions of fibers of regular morphisms. How to compute the dimension of a projective variety via appropriate incidence manifold.

6. Algebraic field extensions: separability, primitive elements, continuation of field embeddings, normal extensions, Galois extensions. Splitting fields and algebraic closures.

7. Automorphisms of fields. Any field is the Galois extension of the subfield of invariants of an arbitrary finite group of automorphisms. The Galois correspondence. Each finite field is the Galois extension of the prime subfield with the cyclic Galois group spanned by Frobenius.

8. Galois theory applications: quadratic extensions and constructions by ruler and compasses, groups of polynomials and how to compute them, cyclotomic extensions and Frobenius elements, cyclic extensions and Kummer's theory, solvable extensions and Abel's theorem.

[2] Sheaves And Supplying Homological Algebra. National Research University Higher School of Economics, Faculty of Mathematics, 3-4 year of bachelor program and 1 year of master program, September–December 2015, 4 hours per week.
`http://gorod.bogomolov-lab.ru/ps/stud/sha/1516/list.html`
Program:

1. Categories, functors, pre-sheaves. The main working examples: open sets of a topology and simplicial sets. Cateory of functors, Yoneda's lemma. Adjoint functors. (Co) limits of diagrams.

2. Sheaves on topological spaces. Stalks and the étalé space of a sheaf. Sheafification. Pull back and push forward. Abelian sheaves.

3. Complexes and (co)homologies. Long exact sequence of cohomologies. The Koszul complexes. Cohomologies and filtered colimits. Spectral sequences of filtered complexes, bicomplexes, and exact couples.

4. Global sections, flabby sheaves, and the Godement resolution. Sheaf cohomology and hypercohomology. Acyclic resolutions. The Mayer–Vietoris exact sequence and Čech's resolution. Acyclic coverings and Cartan's criterion for acyclicity. The Čech cohomologies.

5. The simplicial cochain complex of a triangulated space coincides with the Čech complex of constant sheaf. Fine and soft sheaves, resolution of identity. The sheaves of differential forms, the Poincar lemma. DeRham theorem.

6. Higher direct images. The Leray spectral sequence. Čech cohomologies for coverings by closed subsets.

7. Coherent sheaves on algebraic manifolds. Acyclicity of affine coverings. Cohomologies of invertible and (co)tangent sheaves on projective spaces. Geometric examples and applications.

8. Grothendieck topologies and sheaves on sites. Canonical topology. Examples: fully faithful embedding of small abelian category into category of modules, site of G-sets and representation theory.

3.3.9 Maxim Kazarian

[1] Calculus. Independent University of Moscow, I year students 1st term), September-December 2015, 4 hours per week.

Program

Rational and Real numbers. *Rational number as an equivalence class of pairs of integers. Rational number as a line on the plane. Real number as a section on the set of rational numbers. Comparing real numbers. Operations on real numbers. Representation by decimal fractions. The theory of continued fractions, the best approximations of irrational numbers by rational ones. p-adic numbers.*

Limit of a sequence. Sum of a series. *Limit of a sum and of the product of sequences. Remarkable limits. Fundamental sequences. Cauchiy criterium. Limit of a monotone bounded sequence. Metric spaces. Limit of a sequence in a metric space. Complete metric spaces.*

Topology of a real line. *Open and closed sets on the line. Internal points. The closure. Compact sets and their characterizations. Dense sets. Connectness.*

Cardinality of a set. *Countable sets. Continuum. THe cardinality of the set of subsets.*

Continuous functions. *Equivalence of different definitions of a continuous functions. Inverse function. Theorem on the existence of a maximum of a function o a segment. Intermediate value theorem. Continuity of elementary functions.*

Power sums. *Convergency radius. Abel theorem. Cauchy and D'Alembert formulas.*

Differentiation. *Derivative of a product. Derivative of a composition. Derivative of the inverse function. . Mean value theorem. L'Hôpital rule. Taylor formula.*

[2] Differential Geometry, National Reserch University Higher School of Economics, III year students, September-December 2015, 4 hours per week.

Program.

Vector fields and differential forms. *Lie bracket, exterior differential and wedge product, distributions, Frobenius criterium of integrability.*

Plane and space curves. *Length, curvature, focal set of a plane curve, normal and geodesic curvature of a space curve on a surface.*

Surface geometry. *Riemann structure, IInd quadratic form, principal curvatures, Gaussian curvature.*

Gauss' Theorema Egregium. *Connection and curvature forms of a metric on a surface, Euclidean coordinates of a flat metric.*

Topological connection. *Fiber bundles, trivializations, parallel transport, curvatur as an infinitesimal holonomy.*

Covariant derivative. *Vector bundles, sections, connection matrix, structure Cartan equation, curvature tensor.*

Riemann manifolds. *Levi-Civita connection, Riemann tensor, geodesics.*

3.3.10 Anton Khoroshkin

[1] Quantum Groups. Independent University of Moscow, III year and higher level students, September-December 2015, 4 hours per week (2 hours lecture + 2 hours seminar).

The goal of the coarse is to understand the notions and ideas invented by V.Drinfeld developed in the series of papers on quantum groups.

program:

- Coalgebras, Hopf algebras and tensor categories;
- Quantization and classical limit, Poisson algebras;
- Poisson-Lie groups, Lie bialgebras;
- Coboundary, triangular and quasitriangular Lie bialgebras;
- Drinfeld Double and universal R-matrix;
- Operad theory and Tamarkin's quantization;
- Little discs operad and Deligne's conjecture;
- Braided tensor categories;
- KZ-equations and Drinfeld category;

3.3.11 Iosif Krasilshchik

[1] Linear differential operators over commutative algebras and geometry of jet spaces. Independent University of Moscow, II–V year and PhD students, September–December 2015, 4 hours per week.

Program

1. Categories and functors (introduction).
2. Linear differential operators with values in modules. Basic properties.
3. Derivations.
4. Representing objects: jets and differential forms.
5. Calculus over commutative algebras.
6. The Schouten–Nijenhuis bracket and related cohomologies. An algebraic model of Hamiltonian formalism.
7. The Frölicher–Nijenhuis bracket and related cohomologies. An algebraic model of nonlinear differential equations.
8. A geometric realization. Relations between the category of vector bundles over a smooth manifold and the category of projective modules over a commutative ring.
9. Jets of locally trivial bundles over smooth manifolds. The Cartan distribution.
10. Symmetries of the Cartan distribution and Lie–Bäcklund theorem.
11. Differential equations as geometrical objects and their symmetries.

12. Symmetries of ODEs and Lie–Bianchi theorem on integrability in quadratures.

[2] Calculus I, Russian State University for the Humanities, I year students, September–December 2015, 4 hours per week.

Program.

1. Sets and maps.
2. Real numbers, Euclidean plane, Cartesian coordinates polar coordinates.
3. Limits and their properties. The "remarkable limits" $\lim_{x\to 0} \frac{\sin x}{x}$ and $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x$.
4. Functions in one real variable. Continuity. Basic properties of continuous functions. Continuity of elementary functions.
5. Geometrical and dynamical meaning of the first derivative. The rigorous definition. First differential.
6. Basic properties of first derivative.
7. Derivatives of of the composite and inverse functions.
8. Derivatives of the elementary functions.
9. Higher derivatives. The Taylor formula.
10. L'Hôpital's rules.
11. The Newton method of solving equations $f(x) = 0$.
12. Derivatives and extrema.
13. Drawing graphs of functions using differential calculus.

[3] Ordinary differential equations, Russian State University for the Humanities, II year students, October–December 2015, 4 hours per week.

Program.

1. Problems in geometry and mechanics leading to ODEs.
2. General definition of ODEs and their solutions. Types of solutions.
3. The phase portrait and isolines.
4. Initial data and Cauchy problem. The uniqueness and existence theorem.

5. First-order equations.
   (a) Separation of variables.
   (b) Homogeneous equations.
   (c) Inhomogeneous equations.
   (d) Equations in total differentials. Integrating factor.
6. Second-order equations.
   (a) Equations $f(x, y', y'') = 0$.
   (b) Equations $f(y, y', y'') = 0$.
7. Linear equations of arbitrary order.
   (a) Homogeneous equations. Fundamental set of solutions. Wronskian.
   (b) Solving inhomogeneous equations by the Lagrange method.
   (c) Equations with constant coefficients. The characteristic polynomial.
      i. The case of simple real roots.
      ii. The case of multiple real roots.
      iii. The case of simple complex roots.
      iv. The case of multiple complex roots.
8. Solving ODEs by means of power series.

3.3.12 Andrei Kustarev

[1] Smooth, complex and symplectic torus actions. Independent University of Moscow, September-December 2015, 2 hours per week.

Program:

1. Convex geometry of cones and fans. Complex manifolds. Nonsingular simplicial fans and nonsingular toric varieties (defined as complex manifolds). Separability of nonsingular toric varieties.
2. Complete fans and compact toric varieties. Orbits of the torus action.
3. Simple polytopes and their moment-angle manifolds. The quotient construction of projective toric varieties as smooth manifolds.
4. Height functions on polytopes and Morse functions on toric varieties. Dehn-Sommerville relations.

5. Symplectic geometry, Darboux lemma and its equivariant version. Almost complex structures tamed by a given symplectic form.

6. Symplectic and Hamiltonian torus actions. Atiyah-Guillemin-Sternberg theorem (the image of a moment map is a convex polytope). Morse theory of almost periodic hamiltonians. Smooth actions of compact groups and their orbit types.

7. Projective toric varieties and Hamiltonian actions of half-dimensional torus (Delzant theorem).

8. Cox quotient construction of a toric variety. Equivariant monomials and projective embeddings of toric varieties.

9. ABBV localization theorem for Chern and Pontryagin numbers. Apllications: no torus actions on compact manifolds with exactly one fixed point, McDuff's theorem on symplectic circle actions in dimension four.

Lecture notes in Russian are available on the IUM website.

3.3.13 Taras Panov

[1] Topology-1. Independent Independent University of Moscow, I year students, February-May 2015, 2 hours per week.

Program:
1. Necessary facts from point-set topology.
2. Operations on topological spaces.
3. Homotopy and homotopy equivalence.
4. Cellular (CW) complexes.
5. Fundamental group.
6. Van Kampen Theorem.
7. Fundamental group of cellular complexes.
8. Coverings.
9. Fibrations.
10. Homotopy groups.

http://higeom.math.msu.su/people/taras/teaching/panov-topology1.pdf

[2] Topology-2. Independent Independent University of Moscow, II year students, September-December 2015, 2 hours per week.

Program:
1. Simplicial homology.
2. Singular homology.
3. Cellular homology.

4. Homotopy groups and homology groups.
5. Cohomology and multiplications.
http://higeom.math.msu.su/people/taras/teaching/panov-topology2.pdf

[3] Linear algebra and geometry, Kazakhstan Branch of Moscow State University, Astana, I year students, May 2015, 18 hours per week (two weeks total).

Program:
1. Vector spaces.
2. Linear operators.
3. Geometry of Euclidean and Hermitian spaces.
4. Operators in Euclidean and Hermitian spaces.
5. Bilinear and sesquilinear functions.
6. Tensors.
http://higeom.math.msu.su/people/taras/teaching/panov-linalg.pdf

[4] Introduction to topology, Moscow State University, I year students, September-December 2015, 2 hours per week.

Program: same as [1] Topology-1.

3.3.14 Alexei Penskoi

[1] Differential Geometry, Moscow State University, 3 year students, September-December 2015, 6 hours per week (lecture 2 hours + exersise class 4 hours).

Program

1. Reminiscences from Calculus: implicit function theorem, inverse function theorem, rank theorem. Surfaces in affine spaces and different ways of their definition.

2. Smooth manifolds. Partition of unity. Maps of manifolds.

3. Tangent vectors and differential of a map. Tangent and cotangent spaces.

4. Immersions, submanifolds, submersions.

5. Vector fields. Commutator of vector fields. Integral curves of a vector filed. One-parametric group generated by a vector field.

6. Tensor fields, differential forms. Riemann metric, volume form. Exterior differential.

7. Relation between d and grad, rot and div.

8. Orientation of a manifold. Integration of forms over manifolds.

9. Manifolds with boundary. Stokes theorem for manifolds with boundary. Relation to Green, Stokes and Gauß-Ostrogradsky formulas in calculus.

10. De Rham cohomologies. Poincaré lemma. Mayer-Vietoris long exact sequence.

11. Properties of de Rham cohomologies (finite dimension, Künneth formula etc).

12. Vector bundles.

13. Connections in vector bundles.

14. Levi-Civita connection.

15. Curvature operator, curvature tensor.

16. Parallel transport.

17. Geodesics, exponential map.

[2] Riemannian Geometry, Independent University of Moscow, 3+ year students, February-April 2015, 2 hours per week.

Program

1. Riemannian manifolds

2. Riemannian curvature

3. Riemannian coverings

4. Riemannian geometry of surfaces

5. Isoperimetric inequalities

6. Comparison theorems

[3] Analysis on Manifolds, Independent University of Moscow, 2 year students, September-December 2015, 4 hours per week (lecture 2 hours + exersise class 2 hours).

Program.

1. Reminiscences from Calculus: implicit function theorem, inverse function theorem, rank theorem. Surfaces in affine spaces and different ways of their definition.

2. Smooth manifolds. Partition of unity. Maps of manifolds.

3. Tangent vectors and differential of a map. Tangent and cotangent spaces.

4. Immersions, submanifolds, submersions.

5. Vector fields. Commutator of vector fields. Integral curves of a vector filed. One-parametric group generated by a vector field.

6. Tensor fields, differential forms. Riemann metric, volume form. Exterior differential.

7. Lie derivative. Cartan identity. Hodge operation. Relation between d and grad, rot and div.

8. Distributions and Frobenius theorem.

9. Orientation of a manifold. Integration of forms over manifolds.

10. Manifolds with boundary. Stokes theorem for manifolds with boundary. Relation to Green, Stokes and Gauß-Ostrogradsky formulas in calculus.

11. De Rham cohomologies, de Rham cohomologies with compact support. Poincaré lemma. Mayer-Vietoris long exact sequence.

12. Properties of de Rham cohomologies (finite dimension, Künneth formula etc).

13. Basics of Lie groups and algebras.

14. Actions of Lie groups. Homogeneous spaces.

15. Sarde lemma. Transversality. Whitney theorem.

[4] Topology-I. Math in Moscow program of the Independent University of Moscow for undergraduate students from the U.S. and Canada, February-May 2015, 4 hours per week (lecture 2 hours + exersise class 2 hours).

1. The language of topology. Continuity, homeomorphism, compactness for subsets of $\mathbb{R}^n$ (from the epsilon-delta language to the language of neighborhoods and coverings).

2. The objects of topology: topological and metric spaces, cell spaces, manifolds. Topological constructions (product, disjoint union, wedge, cone, suspension, quotient spaces, cell spaces, examples of fiber bundles).

3. Examples of surfaces (2-manifolds), orientability, Euler characteristic. Classification of surfaces (geometric proof for triangulated surfaces).

4. Homotopy and homotopy equivalence, fundamental groups and their elementary properties.

5. Fundamental group and covering spaces. Algebraic classification of covering spaces (via subgroups of the fundamental group of the base). Branched coverings, Riemann-Hurwitz theorem.

6. Knots and links in 3-space. Reidemeister moves. Polynomial invariants.

[5] Differential Geometry. Math in Moscow program of the Independent University of Moscow for undergraduate students from the U.S. and Canada, September-December 2015, 4 hours per week (lecture 2 hours + exersise class 2 hours).

1. Plane and space curves. Curvature, torsion, Frenet frame.
2. Surfaces in 3-space. Metrics and the second quadratic form. Curvature.
3. Connections in tangent and normal bundles to a k-surfaces in $\mathbf{R}^n$.
4. Parallel translations.
5. Geodesics.
6. Gauß and Codazzi formulas. "Theorema egregium" of Gauß.
7. Gauß-Bonnet theorem.
8. Extremal properties of geodesics. Minimal surfaces.
9. Vector bundles, connection in vector bundles.
10. Levi-Civita connection.
11. Connection curvature. Riemann curvature tensor.

[6] Geometry-I, National Research University — Higher School of Economics, January-March 2015, 4 hours per week (lecture 2 hours + exersise class 2 hours).

1. Bilinear and Sequilinear forms. Orthogonality w.r.t a form. Index, signature.
2. Lagrange algorithm. Classification of quadrics.
3. Scalar products. Gram matrix. Isotropic vectors and cones.
4. Sylvester criterion.
5. Isomorphism $\tau : V \longrightarrow V^*$ induced by a metric.
6. Orthogonalization. Jacobi theorem.
7. Angle between a vector and a subspace, angle between subspaces.
8. QR-decomposition.
9. Affine space. Distance.

10. Invariant subspaces of operators. Canonical form of unitary and orthogonal operator.
11. Adjoint, self-adjoint operators and their properties.
12. Projectors.
13. Skew-symmetric and skew-hermitian operators.
14. Positive definite operators, square root. Polar decomposition.
15. Eigenvalues of two forms. Angle between subspaces and eigenvalues of two forms
16. Canonical form of pair os forms, application to metric classification of quadrics.
17. Sections of quadric by planes.
18. Villarceau circles.
19. Ellipsoid, Staude theorem.
20. Classical quadrics and their properties, lines on quadrics.
21. Spheric geometry.
22. Projective plane. Klein and Poincaré models.

[7] Exercise classes for Classical Differential Geometry, Moscow State University, February-May 2015, 2 hours per week.

[8] Exercise classes for Calculus-I, National Research University — Higher School of Economics, September-December 2015, 2 hours per week.

3.3.15 Petr Pushkar'

[1] Differential Geometry-2, High School of Economics, Dept of Math, 3-4 year students January-June 2015, 4 hours per week.

The following topics were considered

1. Integrability of distributions 2. Elements of Symplectic Jeometry 3. Elements of Contact geometry 4. Connection, curvature 5. Riemanian Geometry, geodesics, Morse theory

[2] Differential geometry. Independent University of Moscow, 2 year students, February-May 2015, 4 hours per week.

Program

1. Bundles and connections
2. Curvature
3. Elements of symplectic and contact geometry

4. Riemanian Geometry and Morse theory for geodesics

[3] Mathematical methods of science High School of Economics, Dept of Math, Master program Fall 2015, 2 hours per week

Program From manifolds, Stokes theorem to elements of differential, symplectic and contact geometry

[4] Symplectic geometry. Independent University of Moscow, special course

The main purpose of the course is to give an introduction to Symplectic and Contact topology and geometry. I will try to cover following topics.

1. Symplectic and contact structures Darboux theorems. Examples of symplectic and contact manifolds.
2. Hamiltonian vector fields and Arnold conjecture.
3. Generating families, lagrangian and legendrian mappings.
4. Symplectic and contact reductions, examples.
5. Maslov class.
6. Chekanov theorem.
7. Examples and applications.

3.3.16 George Shabat

[1] Basic Algebra. Independent University of Moscow, I year students, September-December 2015, 2 hours per week.

Program

0. Introduction

0.0. Systems of linear equations with two unknowns
0.1. Equations of third and fourth degree sith one unknown
0.2. Generalization: systems of polynomial equations
0.3. What do we demand from the coefficients and from the components of solutions?
0.4. Semirings, rings, fields, skew-fields
0.5. On the Hilbert's tenth Problem

1. Language of categories in algebra

1.0. Categories
1.1. Category of sets
1.2. Small categories
1.3. Categories $\mathcal{SET}$; $\mathcal{MON}, \mathcal{GRP}, \mathcal{AB}$; $\mathcal{RING}, \mathcal{ANN}$
1.4. Isomorphisms
1.5. Monoids of endomorphisms and groups of automorphisms
1.6. Initial and final objects
1.7. Direct sums and direct products
1.8. Moderate categories

1.9. Classification problems in algebra
1.10. Functors and cofunctors
1.11. Embedding of categories, forgetful functors
1.12. Representable functors
1.13. Functors of points
1.14. Nonstandard examples: compositional rings, ternars

2. Groups -1

2.0. Enumeration of small groups, Cayley tables
2.1. Subgroups, cosets, Lagrange's theorem
2.2. Cyclic subgroup, generated by an element
2.3. Normal subgroups and factor-groups
2.4. Direct and semidirect products
2.5. Exact sequences of groups
2.6. Group extensions
2.7. Simple groups
2.8. Free groups
2.9. Presentation of groups
2.10. Free products of groups
2.11*. Fundamental groups of punctured topological spaces
2.12*. Fundamental groups of joins of topological spaces

3. Group actions -1

3.0. Definitions. Stationary groups and orbits.
3.1. Groups of permutations. Sign of a permutation.
3.2. Orbit-counting theorem and its applications
3.3. Automorphism groups in geometry
3.4. 3-transitive and 2-transitive groups
3.5. Groups, generated by two permutations
3.6. Categories of G-sets and G-modules

4. Rings-1

4.0. Double magmas and their "ideals"
4.1. Operations over ideals
4.2. Ideals and ring epimorphisms; factor-rings
4.3. Prime and maximal ideals; domains
4.4. Products of rings
4.5. Zero divisors and nilpotents
4.6. Finite rings and fields
4.7. Adjoining variables

4.8. Rings of polynomials, formal power series and Laurent series
4.9. Principal ideal rings and divisibility therein
4.10. Primary decomposition of ideals
4.11. Multiplicatively closed sets and rings of fractions
4.12. Field of fractions of a domain
4.13*. Rings of continuous functions
4.14. Systems of polynomial equations and functors of points

5. Fields -1
5.0. Characteristic of a field. Minimal fields.
5.1. Finite fields
5.2. Finite extensions; splitting field of a polynomial
5.3. Primitive elements
5.4. Algebraic closure of a field
5.5. Separable and normal extensions
5.6. Galois extensions
5.7. Fundamental theorem of Galois theory
5.8. Symmetric functions

6. Modules and vector spaces
6.0. Definitions
6.1. Dimension of a vector space; basis
6.2. Coordinate space
6.3. Classification of finite-dimensional vector spaces
6.4. Morphisms of modules
6.5. Matrix of a linear map
6.6. Dimension of an image and the rank of a matrix
6.7. Dual space
6.8. Modules over the principal ideal domains
6.7. Finitely generated abelian groups

7. Algebras
7.0. Definitions
7.1. Commutative and associative algebras
7.2. Lie algebras
7.3. Algebras of endomorphisms of vector spaces and algebras of square matrices
7.4. Trace of an endomorphism
7.5. Tensor algebra of a vector space
7.6. Symmetric algebra of a vector space
7.7. Exterior algebra of a vector space
7.8. The action of an endomorphism on the exterior algebra

7.9. Determinant
7.10. Matrix inversion

8. Systems of linear equations
8.0. Reducing of a system to the matrix equation
8.1. Dimension of the space of solutions and the rank of a matrix
8.2. Cramer's rule
8.3. Kronecker-Capelli theorem

9. Solubility of polynomial equations by radicals
9.0. Soluble equations and cyclic extensions of the fields
9.1. Solubility of the groups S_3 and S_4
9.2. The roots of cubics and quartics again
9.3. The group A_5 is simple

[2] Probabilistic models, Russian State University for the Humanities, II year students, September-December 2015, 4 hours per week.

Program.

1. Main concepts

1.0. Probability spaces with an equally likely outcomes. The set of outcomes, events. Random variables and their mean values.
1.1. General concept of a probability space. The set of outcomes. The algebra of events. Probability in the everyday language and the probability as a function on the algebra of events. Significant and impossible events. Incompatible events.
1.2. The algebra of random variables. Real-life examples.
1.3. Finite probability spaces. Conventions. Probabilities of outcomes. Structure theorem. Probability spaces with few outcomes.
1.4. The expected value of a random variable. Weighted averages. Linearity. Relations with minimal and maximal values.
1.5. The products of probability spaces. Definition. Examples. "Assotiativity". The powers of a probability space.
1.6. General additive functions. Cardinality, length, area, volume. Probability as a normed additive function.
1.7. The independence of events. Examples. Independent events in the products and powers of finite probability spaces.
1.8. Bernoulli trials. Successes and failures, their probability. The logs of trials. Number of successes; the probability of its occurrence in a prescribed interval.
1.9. The Central Limit Theorem. Case of Bernoulli trials. The histogram of the number of successes.

2. Models

2.1. Last digits of primes*. Empirics: observable frequencies. Relation with the Dirichlet's theorem on primes in arithmetic progressions.
2.2. First digits of powers of 2. Empirics: observable frequencies. Theoretical explanation. Further questions.
2.3. Statistics of the decimal digits of irrational numbers*. Empirics. Open problems.
2.4. Elements of statistical linguistics. Length of russian words. Frequencies of letters. Vowels and consonants in the european language.
2.5. Elements of geometric probability. The probability of a random point in a square to belong to the inscribed disc. Multidimensional generalizations. Observed frequencies and volumes of multidimensional balls.

3. Mathematical tools

3.1. Integrals. Motivations: areas and volumes, rough extrapolation of factorial, histograms.
3.2. Fundamental theorem of calculus. History of the terms "differential" and "integral"İntuition related to these objects in different times.
3.3. Simplest examples. $\int_0^{\pi/2} \sin^n x \mathrm{d}x$. Volumes of multidimensional balls. product. Extrapolation of factorial.
3.4. Stirling formula. Asymptotic of binomial coefficients.
3.5. Shape of graphs of rows of the Pascal triangle. Error function.

[3] Experimental Geometry, Moscow State Pedagogical University, II year students, September-December 2015, 2 hours per week.

Program.

1. Introduction. The concept of mathematical experiment. Interrelations between experiments and other forms of mathematical activity. The role of experiments in teaching. The role of computers in mathematical experiments. Special features of experiments in various domains of mathematics. Experiments in the planimetry. Dynamic geometry and the configurational perception. The computer tools currently available.
2. Overview of the simplest tools of GSP (Geometer's SketchPad). The objects of GSP: points; segments, rays, lines; circles, arcs; polygons, discs. Means of description and construction of objects. Incidence. Shown and hidden objects. Colors and thickness of lines. Captions. Animation.
3. Spaces of configurations. The simplest examples: triangles, circles, pairs of circles. Dimension as a number of parameters (informally). Degeneration of objects and boundaries of spaces of parameters.

4. Relation of configuration approach with the coordinate method; general concept of invariant. The simplest examples of invariants: lengths and areas. Invariant operations over the objects. Example: association to a triangle of a pair of inscribed and circumscribed circles. obtaining Euler's formula experimentally; relation with the Poncelet theorem. Variation: escribed circles.
5. Measurements. Technology of measurements and their precision. The units of measurements. Lengths, areas, angles. operations over the measurements. Simplest experiments: sum of the angles of polygon, area of a triangle, the product of the length of a secant segment and its external part. Conservation laws in geometry. Angles in a fixed segment.
6. Transformations. Translations, rotations, reflections, dilatations; their definition and realization. Variation of parameters. Iterations of transformations. Aesthetic applications, parametric design. Ornaments and other periodic structures; tilings.
7. Applications of transformations. Construction of a circle, inscribed in an angle and containing a given point. Inversion; construction of a circle, tangent to two given circles and containing a given point.
8. Advanced topics. Poncelet theorem and other porisms; generalizations. Ceva theorem and it particular cases. Euler line. Nine point circle. Napoleon theorem and its generalizations. Cubic curves related to a triangle; remarkable points on them.
9. Languages of geometry. General concept of formal languages. Language of GSP-objects. Synthetic and coordinate languages. Klein's Erlangen Program and the language of transformations. Generalized geometries; GSP-experiments in the spherical and hyperbolic planes.
10. Proofs in the GSP-environment. The general concept of a complete proof (demonstration). Sequences of sketches. Splitting demonstration into a sequence of intermediate statements.
11 GSP-presentations. Transformation of GSP-presentations into traditional text.
12. Some methodical problems. Sketches and traditional notebooks. Electronic notebooks and computer albums. Lessons and homeworks in the GSP environment; teachers comments inside the sketches. Assessment and corrections. Bookkeeping.
13. GSP-projects. Classification of topics. Research projects; cooperation with mathematicians-professionals. Organization of collective project work. Creation of databases of results and instruments. Using Internet; distant collective projects.

3.3.17 Stanislav Shaposhnikov

1) Mathematical calculus. Independent University of Moscow, 1 year students, February – May 2015, 4 hours per week.

Program:

1. Riemann integral and Lebesgue integral. Lebesgue measure.
2. Topology, metric and normed spaces.
3. Complete metric spaces. Baire category theorem.

4. Compact sets. Hausdorff criteria.
5. Menger-Nöbeling-Pontryagin theorem.
6. Kolmogorov-Arnold theorem.
7. The differentiability of real functions on normed linear spaces.
8. Implicit function theorem.

3.3.18 Mikhail Skopenkov

[1] Visual Geometry. Higher School of Economics, I-III year students, September-December-2015, 2 hours per week.

Program (short version).

1. Affine and Projective geometry.
2. Möbius geometry.
3. Spherical geometry.

[2] Visual potential theory, Independent Moscow University and Higher School of Economics, I-III year students, February-December-2015, 2 hours per week.

Program.

1. Definition of an electric network.
2. The existence and uniqueness of a potential in an electric network. Conductance.
3. Definition of a random walk. Physical interpretation of the hitting probability. The 1-dimensional random walk is recurrent.
4. Conductance and its probabilistic interpretation. Energy conservation. The variational principle. The short-cut principle. Conductance between the center and the boundary of a square lattice $n \times n$. The 2-dimensional random walk is recurrent.
5. Conductances of trees. The 3-dimensional random walk is not recurrent.
6. Conductance of regular graphs. The Neumann problem. Infinite electric networks.

[3] Distant courses for mathematical olympiads winners (http://school.dist-math.ru/moodle), 2007–2015. During 2014 supported by Independent University of Moscow. During 2015 supported by Higher School of Economics. Program in Russian available at the website
http://school.dist-math.ru/moodle

In addition to [1]–[3], throughout 2015 I am an assistance teacher in basic courses at Higher School of Economics, I-III year students, 9 hours per week, and a supervisor of 4 students.

3.3.19 Evgeni Smirnov

[1] Algebra, 1st year, 1st semester, Higher School of Economics, September–December 2015, 3 hours of lectures and 3 hours of exercise sessions per week

Course outline:

1. Fields. Systems of linear equation. Row-echelon form, Gaussian elimination.

2. Vector spaces. Subspaces. Linear independence, basis. Linear maps. Matrix of a linear map. Kernel, image, rank of a linear map. Equivalent definitions of rank.

3. Invertible linear maps. Determinant: abstract definition and explicit formula. Determinant of the composition of linear maps. Kramer's rule, formula for the inverse matrix.

4. Dual spaces. Dual bases. Annihilator of a subspace, its properties. Dual map, its matrix.

5. Arithmetic of integers: residues modulo m, invertibility, Euler function, Fermat's little theorem.

6. Rings. Integral domains, ideals, quotient rings, principal ideal domains, Euclidean rings, factorial rings. Factoriality of PIDs. Chinese remainder theorem.

7. Groups, subgroups. Groups acting on sets. Orbits, isotropy subgroups. Lagrange formula.

8. Abelian groups. Cyclic groups. Gaussian elimination revisited: structure of finitely generated abelian groups.

[2] Algebra, 2nd year, 3rd semester, Independent University of Moscow, September–December 2015, 2 hours of lectures and 2 hours of exercises per week

Course outline:

1. Modules over rings. Finitely generated modules over PIDs. Tensor products of modules. Injective and projective modules.

2. Basic notions of homological algebra. Ext and Tor.

3. Representations of finite groups. Group algebra, its semisimplicity. Schur lemma, Maschke and Burnside's theorems. Characters.

4. Representations of symmetric groups, after Vershik and Okounkov. Simple branching, Gelfand–Zetlin bases.

5. Representations of quivers. Finite, tame and wild types. Dynkin diagrams, Gabriel's theorem, Bernstein–Gelfand–Ponomarev reflection functor.

3.3.20 Mikhail Verbitsky

[1] Variations of Hodge structures, IUM, Spring 2015, 2 hours per week.

Variation of Hodge structures (VHS) is a flat bundle over a complex manifold equipped with a holomorphic filtration F^p ("Hodge filtration") and an anti-holomorphic involution, and satisfying the following two conditions: $V = \bigoplus_i F^p \cap \bar{F}^{n-p}$ ("grading") and Griffiths transversality condition $\nabla(F^p) \subset F^{p+1} \otimes \Lambda^1 M$. For any holomorphic family of Kähler manifolds over a base S, the cohomology of fibers give a VHS on S. Variations of Hodge structure is one of the main tools of agebraic geometry, very useful in a variety of different situations.

I will tell about construction and basic properties of variations of Hodge structures. Then I will define mixed Hodge structures (MHS) and construct MHS on a cohomology of a quasi-projective variety. I'll explain how one obtains mixed Hodge structures from degenerations of variations of Hodge structures. The course should be understandable to the students who know basics of topology, differential and complex geometry.

Program

1. Variations of Hodge structures: definition and constructions. Period map. Torelli theorems. Polarization.

2. Transversality condition: first applications. Griffiths rigidity theorem (two VHS on a compact manifold M, which are isomorphic at one point of M and with the same monodromy representations are isomorphic). Deligne semisimplicity theorem for monodromy representations.

3. Griffiths' theorem on period map being Lipschitz with respect to the Poincare metric on a disc. VHS over a punctured disk have quasi-unipotent monodromy.

4. Degeneration of Hodge structures. Nilpotent orbit theorem and SL(2)-orbit theorem of W. Schmid.

5. Mixed Hodge structures form an abelian category. Construction of the mixed Hodge structure on cohomology. Mixed Hodge structures and degenerations of VHS (Steenbrink).

References

[1] Claire Voisin: Hodge theory

[2] Topics in Transcendental Algebraic Geometry, ed. by Phillip A. Griffiths

[3] `http://arxiv.org/abs/math/0305090` Richard Hain, Periods of Limit Mixed Hodge Structures

[4] Fouad Elzein, Le Dung Trang, MIXED HODGE STRUCTURES, `https://hal.archives-ouvertes.fr/hal-00793699/document`

[5] E. Looijenga, Trento notes on Hodge theory `http://www.staff.science.uu.nl/~looij101/trentorendicontifinal.pdf`

[6] Donu Arapura, Building Mixed Hodge Structures `http://arxiv.org/abs/math/9908036`

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[9] C. A. M. Peters, "Curvature for period domains", "Complex Geometry and Lie Theory", ed. James A. Carlson, C. Herbert Clemens, David R. Morrison, Proceedings of Symposia of Pure Mathematics vol. 53, 1992.

[10] Phillip A. Griffiths, Periods of integrals on algebraic manifolds, III (some global differential-geometric properties of the period mapping) Publications de Mathematiques de IHES, January 1970, Volume 38, Issue 1, pp 125-180

[11] Eduardo Cattani, "Asymptotics of the Period Map", June 19, 2012, CBMS - TCU http://faculty.tcu.edu/gfriedman/CBMS2012/cattani.pdf

[12] Mikhail Kapranov, http://arxiv.org/abs/0802.0215 Real mixed Hodge structures.

[13] Akira Fujiki, Duality of Mixed Hodge Structures of Algebraic Varieties, Publ RIMS, Kyoto Univ., 16 (1980), 635-667

[14] Liviu Nikolaescu http://www3.nd.edu/~lnicolae/mhodge.pdf Mixed Hodge Structures, 2005.

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[16] O. Penaccio, MIXED HODGE STRUCTURES AND VECTOR BUNDLES ON THE PROJECTIVE PLANE I, http://www.crm.cat/en/Publications/Publications/2005/Pr654.pdf