

Topics in low dimensional topology

The goal of low dimensional topology is to understand manifolds of dimensions three and four, and this seminar will present a selection of techniques, new and old, which have been developed to achieve this. The course will begin from a topological perspective, but turning to recent developments in the subject (namely, Heegaard Floer homology) we will see that algebraic geometry (in particular, character varieties) has a significant role to play.

In the topological part, our emphasis will be on constructing examples and building geometric intuition. Note also that, regarding 3-manifold theory as consisting of two halves: knot theory and hyperbolic geometry, we will *not* touch much on hyperbolic geometry; moreover, our approach to knot theory will be topological, *rather than* combinatorial.

Selected topics

- Handlebody decompositions of manifolds, surgery and Morse theory
- Heegaard splittings and Heegaard diagrams of 3-manifolds
- Dehn surgery and surgery on a link in 3-space
- Kirby calculus on 4-manifolds
- Representation spaces, character varieties and invariants from Heegaard diagrams
- Special topics in 3-manifolds/knot theory (various invariants, cyclic and branched covers, open book decompositions, branched coverings, fibered knots, etc.)
- Special topics in 4-manifolds (various invariants, exotic 4-manifolds, trisections of 4-manifolds, knotted surfaces, topological classification, etc.)

Prerequisites: All students are welcome. Though familiarity with differential topology (e.g., transversality) and basic algebraic topology (e.g., homology, CW-complexes, fundamental group) will certainly be helpful, students may use this as an opportunity to develop some intuition and motivation for future courses in geometric topology.

Assessment: Students will be required to attend seminars regularly, and to give a presentation at the seminar (or to one of the advisors) on material appropriate to their level.

Selected reading:

- R. Gompf, A. Stipsicz, *4-manifolds and Kirby calculus*, American Math. Society, Providence 1999.
 - P. Ozsváth and Z. Szabó, *Heegaard diagrams and holomorphic disks*, *Different faces of geometry* (2004) 301-348.
 - D. Rolfsen, *Knots and Links*, Publish or Perish, Berkeley 1976
 - N. Saveliev, *Lectures on the Topology of 3-Manifolds*, Walter de Gruyter, Berlin (1999)
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