PROJECT SUMMARY

The purpose of the project is to study finite subgroups of birational automorphism groups of algebraic varieties and rationality questions.

A group Γ is said to be Jordan if there is a constant $J = J(\Gamma)$ such that for any finite subgroup $G \subset \Gamma$ there is a normal abelian subgroup $A \subset G$ of index $[G : A] \leqslant J$. This property was established for linear algebraic groups by C.Jordan long ago. In 2007 it was suggested by J.-P.Serre that it should hold for certain birational automorphism groups; he also proved that it indeed holds for the group $\mathrm{Bir}(\mathbb{P}^2)$. Later V.Popov classified all surfaces with Jordan groups of birational automorphisms, and Yu.Prokhorov and C.Shramov showed that the groups $\mathrm{Bir}(\mathbb{P}^n)$ are Jordan. Sh.Meng and D.-Q.Zhang proved that this property holds for automorphisms of projective varieties, and T.Bandman and Yu.Zarhin obtained a similar result for quasi-projective surfaces.

Birational diffeomorphisms. A group of *birational diffeomorphisms* of a real variety X is a group of birational automrophisms of X inducing a diffeomorphism of the set of real points $X(\mathbb{R})$. I am going to prove that Jordan property holds in this case for varieties X of dimension up to 3. I have no prediction about higher dimensions, but hopefully understanding lower dimensions can provide some insight for this.

Automorphisms of quasi-projective threefolds. The second goal of the project is to prove that automorphism groups of quasi-projective threefolds are Jordan.

p-subgroups in the space Cremona group. The third goal of the project is to improve some bounds obtained earlier for p-subgroups of the Cremona group $\mathrm{Bir}(\mathbb{P}^3)$. Namely, I expect that for a p-subgroup G with $p \geqslant 5$ acting on a Gorenstein Fano threefold it is possible to show that G is abelian.

Verra threefolds. Verra threefolds are divisors of bi-degree $(2, 2)$ in $\mathbb{P}^2 \times \mathbb{P}^2$. The next goal of this project is to study rationality and $\mathbb{Q}$-factoriality of Verra threefolds with ordinary double singularities.

Quartic double solids. The last goal of this project is to classify quartic double solids with 10 singular points, and in particular to find the number of irreducible components of their parameter space.