Short description of the research plan of Sergey Tikhomirov

The central theme of my research is, in a broad sense, the dynamics of approximations. I study which phenomena are common and different for original and approximate dynamical systems. The approximations that I consider include (but are not limited to) random perturbations of a dynamical system, systems with multiple time scales and hysteresis, interplay between spatially continuous and spatially discrete reaction-diffusion equations. My ongoing research focuses on three related projects:

Depinning bifurcation. I am studying differences between spatially discrete and spatially continuous reaction-diffusion equations. Let us consider spatial discretisation of a reaction-diffusion equation with a travelling wave solution, for instance discrete Nagumo equation If the step of discretisation h is too large often appears a stationary front. This phenomena is called pinning. The behaviour of such systems is well investigated far from the transition point. Surprisingly not much is known for parameter values near the phase transition. The main obstacle is that the system poses in a symmetry under action of a non-compact group, which makes the bifurcation analysis difficult. I plan to consider the bifurcation of a stationary front to a travelling wave in periodic and stochastic, discrete and continuous environments. Currently I obtained preliminarily results for the case of small quasiperiodic perturbations of continuous homogeneous environment.

One of the new ideas, that I am going to bring to the field is to use the shift invariance of the lattice and embed a trajectory of a spatially discrete system into an action of the group $\mathbb{Z} \times \mathbb{R}$ and adapt my results from shadowing section for those actions.

This project arised from attempts of mathematically rigorous description of fibrillation in a heart, which is related to front propagation in environments with critical percolation. Preliminarily investigations showed that currently front propagations in "critical environments" are so weakly investigated that I had to start with such a basic project.

Reaction-diffusion equations with hysteresis and multiple time scales. I consider reaction-diffusion equations involving a hysteretic discontinuity in the source term, describing chemical reactions and biological processes in which diffusive substances interact according to hysteresis law. I observed a connection with the free boundary problems and for the first time found sufficient geometric (transversality) conditions for initial data that guarantee the existence and uniqueness of solutions.

When this condition is violated I explore a new mechanism for pattern formation (different from travelling waves, Turing instability, etc.), which I called "rattling": the hysteretic nonlinearity, loosely speaking, takes a different value at every second spatial point. Rattling persists under most approximations such as spatial discretisation and slow-fast regularisation. The analog of this phenomena persists for multidimensional case and does not depend on the type of lattice, see Fig. 1. I rigorously proved this phenomena for some special cases in one spatial dimension. Currently I am working on the general case and on the description of universality of the rattling phenomenon, I believe it will be done in terms of Young measures.

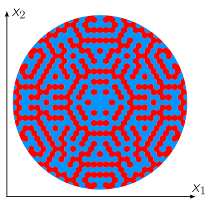

Fig. 1: Values of Hysteresis

Pseudotrajectories and shadowing. I investigate how similar the properties of pseudotrajectories are to those of exact trajectories (shadowing). I showed that the properties of infinite pseudotrajectories and exact trajectories are very alike only for uniformly hyperbolic systems. The aim of this research is to establish similarities between pseudotrajectories and exact trajectories for a much broader class than uniformly hyperbolic systems I consider dynamical systems with discrete and continuous time, as well as group actions.

Currently I study properties of pseudotrajectories arising from stochastic perturbations. Recently, I considered a special case – the linear skew product with the full shift in the base. I provided a sharp estimate for the precision of shadowing for a typical pseudotrajectory of finite length and showed the relation between the shadowing problem and the ruin problem for a random walk. Further consideration allows one to reduce general case to the mixing property of the Markov chain generated by pseudotrajectories.

Those results allow to understand which precision of calculation is needed for numerical simulation of chaotic dynamical systems and for how long time noisy system behaves similarly to a noiseless one.