Research plan. Short version. Yurii Belov.

The project is devoted to the properties of *systems of reproducing kernels in Hilbert spaces of entire functions*. Reproducing kernels in Hilbert spaces of entire functions are among the most popular and powerful objects of modern complex analysis. Geometric properties of families of reproducing kernels express many analytic properties of the space. We will study in details the reconstuction property for formal Fourier series with respect to an exact (complete and minimal) system of repoducing kernels. Given an exact system $\{k_n\}$ in Hilbert space of entire functions H we consider its (unique) *biorthogonal* system $\{g_n\}_{n\in N}$. We associate to every element $f \in H$ its formal Fourier series

$$f \sim \sum_{n\in N} (f, g_n) k_n. \tag{0.1}$$

We will be interested in the hereditary completeness and N-approximation property (any N-vectors can be uniformly approximated by some finite sums elements of (0.1)) for systems of reproducing kernels.

Paley-Wiener, de Branges spaces. The problem of existence of exact systems of exponentials in $L^2(-a, a)$ (or reproducing kernels in Paley-Wiener space) which is not hereditarily complete was solved in 2013 (joint with A. Baranov and A. Borichev). Moreover, it can happen that such systems are close to orthogonal basis ($\{e^{i\lambda_n t}\}_{\lambda_n\in\Lambda}, |\lambda_n - n| < 1$). We want to find answers on the following natural questions. *Is it true that any hereditarily complete system of exponentials satisfies 2-approximation property? How to describe all such systems? What is the best possible constant δ such that any system $\{e^{i\lambda_n t}\}_{\lambda_n\in\Lambda}$, $|\lambda_n - n| \le \delta$ is hereditarily complete?* For systems of eigenfunctions of second order differential operators (or system of reproducing kernels in de Branges spaces) the problem of existence of non-hereditarily complete systems was solved in 2015 (joint with A. Baranov and A. Borichev) but our questions for de Branges spaces are still open.

Fock space, Fock type spaces. One of the most important examples are Gabor systems consisting of time-frequency shifts of "Gaussian window" $\gamma = e^{-\pi s^2}$, $\tau_{x,y}\gamma(s) = e^{2\pi iys}\gamma(s-x), (x, y) \in \Lambda$, $\mathcal{G}_\Lambda = \{\tau_{x,y}\}_{(x,y)\in\Lambda}$. Such systems are unitary equivalent (via the Bargmann transform) to system of reproducing kernels in Fock space $\mathcal{F}$. The completeness of any biorthogonal (to exact Gabor system) system was proved by author in 2015. We are interested in the hereditary completeness and N-approximation properties for classical Fock spaces and its weighted analogs (Fock type spaces). One more interesting questions is to find some systems of reproducing kernels in $\mathcal{F}$ such that series (0.1) admits a linear summation method (this property implies hereditarily completeness). Some concrete examples were obtained by K.Seip and Yu. Lyubarskii in 90th. For Paley-Winer space some sufficient conditions were found by the author (joint with Yu. Lyubarskii).

Newman-Shapiro problem. In 1966 D. Newman and H. Shapiro posed the following problem. Let $G \in \mathcal{F}$ be such that $e^{wz}G(z) \in \mathcal{F}$ for any $w \in \mathbb{C}$. Is it true that

$$\overline{\operatorname{Span}}\{FG; FG \in \mathcal{F}\} = \overline{\operatorname{Span}}\{e^{wz}G : w \in \mathbb{C}\}? \tag{0.2}$$

Recently the author (joint with A.Borichev) have constructed a counterexample to this conjecture. On the other hand, if G is a polynomial, then (0.2) holds. It would be interesting to find some classes of functions G which satisfies (0.2).