Summary

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The project lies between birational geometry and mirror symmetry. This includes results in both these fields, together with applications of one of them to another.

The main objects of interest are Fano varieties and varieties that are close to them. Let me remind that the only smooth Fano variety in dimension one is a projective line. Two-dimensional Fano varieties are called del Pezzo surfaces. Smooth Fano threefolds are classified by Iskovskikh and Mori and Mukai. In higher dimensions only some series of examples, such as complete intersections in projective spaces and, more general, Grassmannians and smooth toric varieties are known; some sporadic examples are also known. In all examples it is easy to calculate the basic invariant, the anticanonical degree of the variety. In the case of smooth complete intersections in (weighted) projective spaces it is possible to compute other invariants, that are, Hodge numbers, using Griffiths approach.

One of the most brilliant recent ideas in mathematics, Mirror Symmetry, gives a promising approach to studying Fano varieties and their invariants. Mirror Symmetry came from physics, where physicists observed that any Calabi–Yau threefold has a pair whose Hodge diamond is a reflection of the Hodge diamond of the initial variety. Mirror Symmetry conjecture of variations of Hodge structures claims correspondence between Gromov–Witten invariants of a variety (expected numbers of rational curves lying on them) and periods of the dual family. Another conjecture, Kontsevich's Homological Mirror Symmetry, treats the duality in terms of derived categories. A dual object for a Fano variety is a so called Landau–Ginzburg model — a certain one-parameter family of varieties. As a combination of two conjectures mentioned above the applicant conjectured the existence of a toric Landau–Ginzburg model — a dual to a Fano variety Laurent polynomial satisfying certain numerical and geometric conditions. It is proven for del Pezzo surfaces, Fano threefolds, complete intersections.

The natural wish is to compute the invariants of Fano varieties via their toric Landau–Ginzburg models. Say, one can conjecturally compute some Hodge number of a Fano variety in terms of numbers of components of reducible fibers of the Landau–Ginzburg model. Another conjecture, that belongs to Katzarkov, Kontsevich, and Pantev, claims the duality of Hodge diamonds of the Fano variety and the dual Landau–Ginzburg model. For this they defined three collections of numbers that play role of Hodge numbers for Landau–Ginzburg models (conjecturally these collections coincide). Lunts and the applicant partially proved (and partially disproved) these conjectures for del Pezzo surfaces.

The project has 3 main directions. The first one is devoted to a classification of Fano weighted complete intersections. The second one studies how complicated their Hodge diamonds can be. The third one is aimed to prove Katzarkov–Kontsevich–Pantev conjectures in the threefold case.

Project 1. This project is common with C. Shramov. As it is mentioned above, the usual examples of Fano varieties in any dimension are complete intersections in projective spaces. The similar is true for weighted complete intersections. However there is no guarantee that general enough such complete intersection (if it exists!) is smooth. Roughly speaking, there should be enough "independent" weighted monomials of degrees defining the complete intersection for this. This gives bounds for weights and degrees (in addition to the Fano condition mentioned above) and enable one to classify smooth weighted complete intersections in each dimension. Chen, Chen, and Chen obtained such bound for codimension of a quasi-smooth weighted complete intersection. Chen bounded the degrees in terms of canonical volume and discrepancies. In the project we give upper bound for weights of a projective space admitting smooth Fano complete intersection therein. For this we solve some optimization problem using the standard down-to-earth method of Lagrange multipliers.

Project 2. This project is common with C. Shramov as well. The Lefschetz Theorem says that the only non-trivial Hodge numbers of a complete intersection are the middle ones. The question is: how complicated the middle Hodge numbers can be? Using Griffiths method we are going to find the minimal non-trivial middle Hodge numbers. In particular, this gives a classification of minimal, diagonal, of curve type, of K3 type, and of Calabi–Yau type weighted complete intersections, which are of special interest from derived categories point of view.

Project 3. This is a common project with A. Harder, L. Katzarkov, and V. Lunts. In this project we are going to prove Katzarkov–Kontsevich–Pantev conjectures for the case of Picard rank one Fano threefolds. The main ingredients of the proof are Hodge-theoretic and topological computations, as well as constructions of log Calabi–Yau compactifications and studying their reducible fibers. The compactification and the reducible fibers count are obtained by the applicant. Hopefully these methods can be applied in higher Picard rank case and in the case of complete intersections. We also are going to correct a broken part of Katzarkov–Kontsevich–Pantev conjectures and prove the corrected versions.