Research plan. Short version. Yury Belov.

The project is devoted to the properties of *systems of reproducing kernels in Hilbert spaces of entire functions*. Reproducing kernels in Hilbert spaces of entire functions are among the most popular and powerful objects of modern complex analysis. Geometric properties of families of reproducing kernels express many analytic properties of the space. We will study in details the reconstuction property for formal Fourier series with respect to the exact (complete and minimal) system of repoducing kernels. Given an exact system $\{k_n\}$ in Hilbert space of entire functions H we consider its (unique) *biorthogonal* system $\{g_n\}_{n\in N}$. We associate to every element $f \in H$ its formal Fourier series

$$f \sim \sum_{n\in N} (f, g_n) k_n. \tag{0.1}$$

We will interesed in the hereditarily completeness, uniqueness, and N-approximation property (any N-vectors can be uniformly approximated by some finite sums elements of (0.1)) for systems of reproducing kernels.

It turns out that at least 3 long-standing problem were solved using this approach: hereditarily completeness problem for expnonentials (Advances in Mathematics 2013), D-invariant problem for $C^\infty(a, b)$ space (accepted for publication in Geometric and Functional Analysis), the Newman-Shapiro problem.

Paley-Wiener, de Branges spaces. The problem of existence of exact systems of exponentials in $L^2(-a, a)$ (or reproducing kernels in Paley-Wiener space) which is not hereditarily complete solved in 2013 (joint with A. Baranov and A. Borichev). Moreover it may happen that such system close to orthogonal basis ($\{e^{i\lambda_n t}\}_{\lambda_n\in\Lambda}, |\lambda_n - n| < 1$). We want to find answers on the following natural questions. *Is it true that any hereditarily complete system of exponentials satisfies 2-approximation property? What is the best possible constant δ such that any system $\{e^{i\lambda_n t}\}_{\lambda_n\in\Lambda}$, $|\lambda_n - n| \le \delta$ is hereditarily complete?*

Paley-Wiener space with disconnected spectrum. Let $\mathcal{PW}_E = L^2\hat{(}E)$, where E is some bounded disconnected subset of $\mathbb{R}$. All our questions are widely open for spaces $\mathcal{PW}_E$ even in the case when E is a union of two intervals. We want to prove (or disprove) the Young type theorem for this space (uniqueness of series (0.1)) for the case when E is a finite union of intervals.

Fock space, Fock type spaces. One of the most important classes important example are Gabor systems consisting of time-frequency shifts of "Gaussian window" $\gamma = e^{-\pi s^2}$, $\tau_{x,y}\gamma(s) = e^{2\pi i y s}\gamma(s - x), (x, y) \in \Lambda$, $\mathcal{G}_\Lambda = \{\tau_{x,y}\}_{(x,y)\in\Lambda}$. Such systems are unitary equivalent (via the Bargmann transform) to system of reproducing kernels in Fock space $\mathcal{F}$. The completeness of any biorthogonal (to exact Gabor system) system was proved by author in 2015. We are interested in the hereditarily completeness and N-approximation properties for classical Fock spaces and its weighted analogs (Fock type spaces).

The Newman-Shapiro problem. In 1966 D. Newman and H. Shapiro posed the following problem. Let $G \in \mathcal{F}$ be such that $e^{wz}G(z) \in \mathcal{F}$ for any $w \in \mathbb{C}$. Is it true that

$$\overline{\text{Span}}\{FG; FG \in \mathcal{F}\} = \overline{\text{Span}}\{e^{wz}G : w \in \mathbb{C}\}? \tag{0.2}$$

Recently the author (joint with A.Borichev) have constructed a counterexample to this conjecture and find some conditions on G which guarantee that (0.2) holds. It is interesting to study the same approximation question in weighted Fock spaces.