Summary

Dmitry Itsykson

Propositional proof systems can be used in order to show that a CNF formula φ is unsatisfiable. Usually proof systems derive a contradiction from the clauses of φ. Every such proof system operates with proof lines (Boolean functions represented in some specific way). For example, Resolution operates with clauses, cutting planes operates with linear inequalities over Boolean variables with integer coefficients. New proof lines can be derived from the previous by syntactic derivation rules. The conclusion of a rule is semantically implied by the premises. A refutation of a formula ϕ is an inference of a constant false proof line from the clauses of ϕ.

The main research direction in proof complexity is the proving of lower bounds on size of derivations. The proposed project is devoted to proof systems where proof lines are read-once branching programs. A branching program (BP) is a data structure that represents of Boolean function; a function is represented by a directed acyclic graph that has one source and two sinks. One of the sinks is labeled with a constant 1, the other is labeled with a constant 0. All vertices except sinks are labeled with variables and each such vertex has exactly two outgoing edges. One of them corresponds to the value 0 and another one to the value of 1 of the variable. The value of the function can be calculated by passing through the graph from the source to a sink along the edges corresponding to the values of variables. A branching program is read-once if for every path each variable occurs most once.

In 2008 Krajicek proved an exponential lower bound for the proof system OBDD($\wedge$, weakening) that operates with OBDDs (ordering binary decision diagrams, an OBDD is a read-once branching programs such that on every path from the source to a sink all the variables appear in the same order) and all OBDDs are in the same order. In the paper [2] we introduced the reordering rule that allows to change the order in OBDDs. We have shown that the reordering rule makes this proof system strictly stronger [1]. There are no known superpolynomial lower bounds on sizes of OBDD($\wedge$, weakening, reordering)-derivations.

In the paper [2] we considered the proof system OBDD($\wedge$, reordering) that do not use the weakening rule and proved exponential lower bounds on sizes of derivations in this proof system. It turns out that the proof complexity of formulas are connected with the representations of some specific Boolean functions by 1-BP. In the paper [1] we construct an example of a family of formulas that have short OBDD($\wedge$, reordering) proofs but require superpolynomial (in fact quasipolynomial) resolution proofs. We will investigate whether Resolution (or bounded depth Frege) quasipolynomially simulates OBDD($\wedge$, reordering) or not. It is also interesting to investigate connections between Resolution proofs and 1-BPs for Boolean functions.

References

[1] Sam Buss, Dmitry Itsykson, Alexander Knop, and Dmitry Sokolov. Reordering rule makes OBDD proof systems stronger. In *33rd Computational Complexity Conference, CCC 2018, June 22-24, 2018, San Diego, CA, USA*, pages 16:1–16:24, 2018.

[2] Dmitry Itsykson, Alexander Knop, Andrei Romashchenko, and Dmitry Sokolov. On OBDD-based algorithms and proof systems that dynamically change order of variables. In Heribert Vollmer and Brigitte Vallée, editors, *34th Symposium on Theoretical Aspects of Computer Science, STACS 2017, March 8-11, 2017, Hannover, Germany*, volume 66 of *LIPIcs*, pages 43:1–43:14. Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, 2017.