Faculty of Mathematics, Higher School of Economics

Marco Mazzucchelli

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will give a minicours:

CLOSED GEODESICS IN RIEMANNIAN AND FINSLER MANIFOLDS

This series of lectures will be devoted to the problem of the existence and the multiplicity of closed geodesics in Riemannian and Finsler manifolds. The problem originated from the seminal work of Hadamard and Poincaré in the late 19th century, and inspired the development of critical point theory in the 20th century. The celebrated closed geodesics conjecture, which in its maximal generality is still open, claims that every closed Riemannian manifold of dimension at least 2 possesses infinitely many closed geodesics. In the course of the lectures, I will provide the background and the proofs of some among the most celebrated results that confirm this conjecture for several classes of manifolds. In the setting of Finsler manifolds, the analogous conjecture fails, as it was shown by Katok in his celebrated example on the 2-sphere. A more general setting, which includes the Finsler one as a special case, is the one of magnetic geodesic flows. The last part of this series of lectures will be devoted to present a few recent results on the multiplicity of closed magnetic geodesics, as well as related open problems from symplectic dynamics.

Schedule:

Mondays, April 11 and 18: 17:00—18:30, room 207,
Thursday, April 14: 17:00—18:30, room 207.
Friday, April 22: 15:30—16:50, room 311

Program of Mazzucchelli's cours

LECTURE 1: Introduction
- A crash course on critical point theory

- The closed geodesics problem: background, state of the art, and open problems
- The functional setting for the closed geodesics problem

LECTURE 2: Homological multiplicity results for closed geodesics
- Bott's and Gromoll-Meyer's iteration theory for the Morse index of closed geodesics
- Gromoll-Meyer's Theorem: closed geodesics on simply connected closed manifolds with non-monogenic cohomology

LECTURE 3: Beyond the homological multiplicity results
- Bangert-Klingenberg's Theorem: closed geodesics on product manifolds
- Bangert-Hingston's Theorem: closed geodesics on manifolds with infinite abelian fundamental group
- Closed geodesics on 2-spheres: Lusternik-Schnirelmann's Theorem and Bangert-Franks' Theorem ("A million ways to wrap a rubber band round a potato")

LECTURE 4: The problem of closed magnetic geodesics
- Failure of Lusternik-Schnirelmann's theorem in the Finsler setting: Katok's example
- The problem of periodic orbits of magnetic geodesic flows
- Ma–e's critical values and the free period action functional
- Some recent multiplicity results for closed magnetic geodesics, and open problems