

Introduction into algebraic geometry - 3

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Program

The main idea of this course is to get basic results of the classical algebraic geometry while introducing only as much (or as little!) modern techniques as needed.

When do sheaf cohomology simplify things, and when do they obfuscate them? What is the meaning of the Serre duality without the cohomology groups? And so on.

The base field is always algebraically closed, unless specified otherwise. Characteristic of the base field is any, but sometimes we will assume that it is large enough (most often 5 is enough), or zero.

Algebraic curves - 2 1/2

- Curves of genus 5: detailed study, and trisecant lines. Plane models.
- From the Picard group of a curve to Picard variety: existence only. Detailed study of genus one case.
- Morphism from symmetric power of a curve to the Picard variety.
- Intersection index of curves in a surface. Bzout's theorem on a plane.
- Coarse moduli space of curves: definition. Dimension of the moduli space.

Algebraic surfaces - 1 1/2

- Adjunction formula – in more details.
- Intersection index of properly intersecting curves: case of non-transversal intersection. Intersection index and linear equivalence.
- Self-intersection of a curve on a surface.
- First Chern class of a line bundle (with values in the Neron-Severi group).
- Blow-up of a point on a plane.
- Digression: resolution of singularities of curves on a surface, and of surfaces – some examples.

Algebraic surfaces - 2

- Ruled surfaces, briefly.
- Elliptic surfaces: examples. Their degenerate fibers.
- Cubic surfaces in \mathbb{P}^3
 - Linear system of plane cubic curves through 6 points on \mathbb{P}^2
 - 27 lines: proof.
 - Their Picard group.
 - Relation to the root system E_6 and the Weyl group E_6
- Cremona transformations of \mathbb{P}^2 : Example.
- Space quartics, and the notion of K3-surfaces.
- Rational equivalence of 0-cycles on a surface. Severi-Chow group of a surface.
- (*) Characteristic classes of vector bundles on algebraic surfaces with values in Severi-Chow groups: Case of split vector bundles. Vector bundles with a filtration. Splitting principle. Porteus definition. B. Segre's definition of characteristic classes.
- Partial surface classification theorem: statements only.
- Surfaces of general type. Bogomolov's inequality (statement only). Some surfaces of general type. Hirzebruch's examples of surfaces with $c_1^2 = 3c_2$. Reid's conjecture.
- Digression: Zeta-function of a curve over a finite field: Weil's proof of rationality of zeta-function via correspondences.

Algebraic curves - 3

- Zeta-function of a curve over a finite field: examples. Statement of the Weil's conjecture.
- Moduli space of curves: Idea of the construction. Fine and coarse moduli spaces. Dimension of the moduli space.
- Vector bundles on curves. Moduli of vector bundles – idea of the construction; and an example.
- Space curves - geography. Halphen's results.
- Castelnuovo's results.

Grassmannian varieties - 1

- Projective embedding. Plucker quadric.
- Cell decomposition.
- Intersection theory on $\text{Gr}(2,4)$

Introduction into the quadratic complex of lines in

- Linear complex of lines
- Quadratic complex of lines: definition
- Associated Kummer surface.
- Associated K3 surface.
- Fano variety of X is an abelian surface.