The IUM progress report to the Simons foundation, 2011

The Simons foundation supported three programs launched by the IUM:
Simons stipends for students and graduate students;
Simons IUM fellowships;
Simons awards for former winners of the Pierre Deligne and Dynasty contests.

13 applications were received for the Simons stipends contest. The selection committee consisting of *Yu.Ilyashenko (Chair), G.Dobrushina, G.Kabatyanski, S.Lando, I.Paramonova (Academic Secretary), A.Sossinsky, M.Tsfasman* awarded Simons stipends for 2011 year to the following students and graduate students:

1. Bibikov, Pavel Vital'evich

2. Bufetov, Alexei Igorevich

3. Gorin, Vadim Evgen'evich

4. Devyatov, Rostislav Andreevich

5. Efimov,Alexander Ivanovich

Each winner received $ 3600.

14 applications were received for the Simons IUM fellowships contest. The selection committee consisting of *Yu.Ilyashenko (Chair), G.Dobrushina, B.Feigin, I.Paramonova (Academic Secretary), A.Sossinsky, M.Tsfasman, V.Vassiliev* awarded

Simons IUM-fellowships for the first half year of 2011 to the following researches:

1. Burman, Yuri Mikhailovich

2. Kuznetsov, Alexander Gennad'evich

3. Olshanski, Grigori Iosifovich

4. Penskoi, Alexei Victorovich

5. Sobolevski, Andrei Nikolaevich
6. Timorin, Vladlen Anatol'evich

Each winner received $ 6000 in the first half of the year 2011;

Simons IUM-fellowships for the second half year of 2011 to the following researches:

1. Gusein-Zade, Sabir Medzhidovich
2. Kuznetsov, Alexander Gennad'evich
3. Olshanski, Grigori Iosifovich
4. Skopenkov, Arkady Borisovich
5. Smirnov, Evgeni Yur'evich
6. Verbitsky, Mikhail Sergeevich

Each winner received $ 6000 in the second half of the year 2011.

10 applications were received for the Simons awards from the former winners of the Pierre Deligne and Dynasty contests. The selection committee consisting of *Pierre Deligne (Co-Chair), V.Vassiliev (Co-Chair), M.Agranovich, V.Beloshapka, Yu.Burman, V.Buchstaber, A.Vershik, E.Vinberg, S.Gusein-Zade, V.Zelikin, Yu.Ilyashenko (Vice-Chair), S.Lando, V.Matveev, S.Natanzon, L.Pastur, A.Sergeev, I.Taimanov, D.Treshchev, B.Feigin (Vice-Chair), A.Khovanski, M.Tsfasman, A.Shen, V.Shekhtman, A.Shiryaev* awarded the Simons Awards to the following former winners of the Pierre Deligne and Dynasty contests:

1. Arzhantsev, Ivan Vladimirovich
2. Positselski, Leonid Efimovich
3. Talalaev, Dmitry Valer'evich

Positselski and Talalaev received $ 7500 each.

Arzhantsev was absent from Russia during two months. According to the rules of the contest, the Simons award was not payed to him during this period. He received $ 6250. The reminder of $ 1250 is kept for the year 2012.

The Modern Natural Science foundation got the overhead $ 12500 as planned.

A detailed financial report is attached separately.

The report below is split in three sections corresponding to the three programs above. The first subsection in each section is a report on the research activities. It consists of the titles of the papers published or submitted in the year of 2011, together with the corresponding abstracts. The last subsection of each section is devoted to conference and some most important seminar talks. The middle subsection of the middle section is devoted to the syllabi of the courses given by the winners of the Simons IUM fellowships. Most of these courses are innovative, as required by the rules of the contest for the Simons IUM fellowships.

1 Program: Simons stipends for students and graduate students

1.1 Research

P. Bibikov

[1] With V.V. Lychagin.
$GL_3(\mathbb{C})$-orbits of ternary rational forms
Doklady Mathematics, 2011, Vol. 84, No. 1, pp. 482–484.

Let R_n be the space of ternary rational forms of degree n. Consider the action of the group $GL_3(\mathbb{C})$ on the space R_n such that the subgroup $SL_3(\mathbb{C}) \subset GL_3(\mathbb{C})$ acts by standard changes of coordinates and the center $\mathbb{C}^* \subset GL_3(\mathbb{C})$ acts by homotheties $f \mapsto \lambda f$, where $f \in R_n$ and $\lambda \in \mathbb{C}^*$. In this paper, we describe the orbits of this action.

[2] Classification of ternary forms with zero Hessian
Izvestiya VUZov. Mathematics, 2011, No. 9, pp. 99–101.

In this paper we study the differential invariant algebra of the $GL_3(\mathbb{C})$-action on the space of ternary forms with zero Hessian and classify $GL_3(\mathbb{C})$-orbits of such forms.

[3] With V.V. Lychagin.
Projective classification of binary and ternary forms
J. Geometry and Physics, 2011, Vol. 61, No. 10, pp. 1914–1927.

In this paper we study orbits of $GL_2(\mathbb{C})$- and $GL_3(\mathbb{C})$-actions on the spaces of binary and ternary polynomials as well as rational forms and find criteria for their equivalence. Similar results are also valid for real forms.

[4] Affine differential invariants of n-webs
Reports of the International conference “Geometry. Control. Economy”. Astrahan, 2011, p. 8.

No abstract

[5] $SO_3(\mathbb{C})$-orbits of ternary forms
Reports of the International conference "Geometry. Control. Economy". Astrahan, 2011, p. 7.

No abstract

[6] Projective classification of algebraic projective curves
The Works of the Lobachevskii Mathematical Center, 2011, Vol. 41, pp. 92–95.

No abstract

[7] Metric classification of algebraic projective curves
Izvestiya PGPU. Mathematics, 2011, No. 26, pp. 36–42.

The aim of the paper is to classify algebraic projective curves with respect to the action of orthogonal group $SO_3(\mathbb{C})$. To solve this problem, we consider solutions of the differential Euler equation assigned to projective curves. This interpretation makes it possible to use theory of differential invariants. The differential invariant field of the group action on the Euler equation is found, and classification of projective curves in the terms of this field is given.

[8] On automorphic systems of differential equations and $GL_2(\mathbb{C})$-orbits of binary forms
Ufimskii Mathematical Journal, 2012, Vol. 4, No. 1, to appear.

In the work we introduce new method for studying classical algebraic problem of classifying $GL_2(\mathbb{C})$-orbits of binary forms with the help of differential equations. We construct and study automorphic system of differential equations $\mathcal{S}$ of the fourth order, whose solution space coincides with $GL_2(\mathbb{C})$-orbit of fixed binary form f. In cases of order 2 and 3 system $\mathcal{S}$ is integrable. In the most difficult case of order 4 we prove that system $\mathcal{S}$ may be reduced to the system of the differential Abel equation and the linear partial differential equation of order 1.

[9] With V.V. Lychagin.
Classification of linear action of algebraic groups on the spaces of homogenous forms.
Doklady Mathematics, 2012, to appear.

In the paper we study an action of an algebraic group G on the space of rational multivariable forms by linear changes of variables. We find the field of differential invariants of such an actions and an effective criterion to distinguish orbits of the forms with a non-zero Hessian.

A. Bufetov.
The central limit theorem for extremal characters of the infinite symmetric group
arXiv:1105.1519v2, *to appear in Functional Analysis and Its Applications.*

The asymptotics of the first rows and columns of random Young diagrams corresponding to extremal characters of the infinite symmetric group is studied. We consider rows and columns with linear growth in n, the number of boxes of random diagrams, and prove the central limit theorem for them in the case of distinct Thoma parameters. We also establish a more precise statement relating the growth of rows and columns of Young diagrams to a simple independent random sampling model.

R. Devyatov.
[1] Generically Transitive Actions on Multiple Flag Varieties
submitted to International Mathematics Research Notices in 2011, preprint published at arXiv:1007.1353v1 [math.AG].

Let G be a semisimple algebraic group whose decomposition into the product of simple components does not contain simple groups of type A, and $P \subseteq G$ be a parabolic subgroup. Extending the results of Popov, we enumerate all triples (G, P, n) such that (a) there exists an open G-orbit on the *multiple flag variety* $G/P \times G/P \times \ldots \times G/P$ (n factors), (b) the number of G-orbits on the multiple flag variety is finite.

[2] Unipotent Commutative Group Actions on Flag Varieties and Nilpotent Multiplications
in Russian, preprint published at Institue for Theoretical and Eexperimental Physics (ITEP)'s preprint directory: ITEP-TH-53/11.

Our goal is to classify all generically transitive actions of commutative unipotent groups on flag varieties up to conjugation. We establish relationship between this problem and classification of multiplications with certain properties on Lie algebra representations. We then classify multiplications with the desired properties and solve the initial classification problem.

A. Efimov
[1] Cohomological Hall algebra of a symmetric quiver.
arXiv:1103.2736v2, to appear in Compositio Mathematica.

In the paper [KS], Kontsevich and Soibelman in particular associate to each finite quiver Q with a set of vertices I the so-called Cohomological Hall algebra $\mathcal{H}$, which is $\mathbb{Z}^I_{\geq 0}$-graded. Its graded component $\mathcal{H}_\gamma$ is defined as cohomology of the Artin moduli stack of representations with dimension vector γ. The product comes from natural correspondences which parameterize extensions of representations.

In the case of a symmetric quiver, one can refine the grading to $\mathbb{Z}^I_{\geq 0} \times \mathbb{Z}$, and modify the product by a sign to get a super-commutative algebra $(\mathcal{H}, \star)$ (with parity induced by the $\mathbb{Z}$-grading). It is conjectured in [KS] that in this case the algebra $(\mathcal{H} \otimes \mathbb{Q}, \star)$ is free super-commutative generated by a $\mathbb{Z}^I_{\geq 0} \times \mathbb{Z}$-graded vector space of the form $V = V^{prim} \otimes \mathbb{Q}[x]$, where x is a variable of bidegree $(0, 2) \in \mathbb{Z}^I_{\geq 0} \times \mathbb{Z}$, and all the spaces $\bigoplus_{k \in \mathbb{Z}} V^{prim}_{\gamma,k}$, $\gamma \in \mathbb{Z}^I_{\geq 0}$. are finite-dimensional. In this paper we prove this conjecture.

We also prove some explicit bounds on pairs (γ, k) for which $V^{prim}_{\gamma,k} \neq 0$. Passing to generating functions, we obtain the positivity result for quantum Donaldson-Thomas invariants, which was used by S. Mozgovoy to prove Kac's conjecture for quivers with sufficiently many loops [M]. Finally, we mention a connection with the paper of Reineke [R].

[2] With Mohammed Abouzaid, Denis Auroux, Ludmil Katzarkov, Dmitri Orlov.
Homological mirror symmetry for punctured spheres.
arXiv:1103.4322v1.

We prove that the wrapped Fukaya category of a punctured sphere (S^2 with an arbitrary number of points removed) is equivalent to the triangulated category of singularities of a mirror Landau-Ginzburg model, proving one side of the homological mirror symmetry conjecture in this case. By investigating fractional gradings on these categories, we conclude that cyclic covers on the symplectic side are mirror to orbifold quotients of the Landau-Ginzburg model.

[3] Quantum cluster variables via vanishing cycles.
arXiv:1112.3601

In this paper, we provide a Hodge-theoretic interpretation of Laurent phenomenon for general skew-symmetric quantum cluster algebras, using Donaldson-Thomas theory for a quiver with potential. It turns out that the positivity conjecture reduces to the certain statement on purity of monodromic mixed Hodge structures on the cohomology with the coefficients in the sheaf of vanishing cycles on the moduli of stable framed representations.

As an application, we show that the positivity conjecture (and actually a stronger result on Lefschetz property) holds if either initial or mutated quantum seed is acyclic. For acyclic initial seed the positivity has been already shown by F. Qin [Q] in the quantum case, and also by Nakajima [Nak] in the commutative case.

[4] With Lunts, Valery A.; Orlov, Dmitri O.
Deformation theory of objects in homotopy and derived categories III: abelian categories.
Adv. Math. 226 (2011), no. 5, 38573911.

This is the third paper in a series. In part I we developed a deformation theory of objects in homotopy and derived categories of DG categories. Here we show how this theory can be used to study deformations of objects in homotopy and derived categories of abelian categories. Then we consider examples from (noncommutative) algebraic geometry. In particular, we study noncommutative Grassmanians that are true noncommutative moduli spaces of structure sheaves of projective subspaces in projective spaces.

[5] A proof of the Kontsevich-Soibelman conjecture.
Mat. Sb. 202 (2011), no. 4, 65–84; translation in Sb. Math. 202 (2011), no. 3-4, 527546

It is well known that "Fukaya category" is in fact an $A_\infty-$pre-category in sense of Kontsevich and Soibelman [KS]. The reason is that in general the morphism spaces are defined only for transversal pairs of Lagrangians, and higher products are defined only for transversal sequences of Lagrangians. In [KS] it is conjectured that for any graded commutative ring k, quasi-equivalence classes of $A_\infty-$pre-categories over k are in bijection with quasi-equivalence classes of $A_\infty-$categories over k with strict (or weak) identity morphisms.

In this paper we prove this conjecture for essentially small $A_\infty-$(pre-)categories, in the case when k is a field. In particular, it follows that we can replace Fukaya $A_\infty-$pre-category with a quasi-equivalent actual $A_\infty-$category.

We also present natural construction of pre-triangulated envelope in the framework of $A_\infty-$pre-categories. We prove its invariance under quasi-equivalences.

V. Gorin

[1] With Alexei Borodin
Markov processes of infinitely many nonintersecting random walks.
submitted. arXiv:1106.1299

Consider an N-dimensional Markov chain obtained from N one-dimensional random walks by Doob h-transform with the q-Vandermonde determinant. We prove that as N becomes large, these Markov chains converge to an infinite-dimensional Feller Markov process. The dynamical correlation functions of the limit process are determinantal with an explicit correlation kernel. The key idea is to identify random point processes on $\mathbb{Z}$ with q-Gibbs measures on Gelfand-Tsetlin schemes and construct Markov processes on the latter space. Independently, we analyze the large time behavior of PushASEP with finitely many particles and particle-dependent jump rates (it arises as a marginal of our dynamics on Gelfand-Tsetlin schemes). The asymptotics is given by a product of a marginal of the GUE-minor process and geometric distributions.

[2] With Alexander Gnedin, Sergei Kerov
Block characters of the symmetric groups.
submitted. arXiv:1108.5044

Block character of a finite symmetric group $S(n)$ is a positive definite function which depends only on the number of cycles in permutation. We describe the cone of block characters by identifying its extreme rays, and find relations of the characters to descent representations and the coinvariant algebra of $S(n)$. We also study counterparts of the block characters for the infinite symmetric group $S(\infty)$ along with their connection to the Thoma characters of the infinite linear group $GL(\infty, q)$ over a Galois field.

[3] With Omer Angel, Alexander E. Holroyd
A pattern theorem for random sorting networks.
submitted. arXiv:1110.0160

A sorting network is a shortest path from $12..n$ to $n..21$ in the Cayley graph of the symmetric group $S(n)$ generated by nearest-neighbor swaps. A pattern is a sequence of swaps that forms an initial segment of some sorting network. We prove that in a uniformly random n-element sorting network, any fixed pattern occurs in at least cn^2 disjoint space-time locations, with probability tending to 1 exponentially fast as n tends to infinity. Here c is a positive constant which depends on the choice of pattern. As a consequence, the probability that the uniformly random sorting network is geometrically realizable tends to 0.

[4] The q-Gelfand-Tsetlin graph, Gibbs measures and q-Toeplitz matrices.
Advances in Mathematics, 229 (2012), no. 1, 201–266, arXiv:1011.1769

The problem of the description of finite factor representations of the infinite-dimensional unitary group, investigated by Voiculescu in 1976, is equivalent to the description of all totally positive Toeplitz matrices. Vershik-Kerov showed that this problem is also equivalent to the description of the simplex of central (i.e. possessing a certain Gibbs property) measures on paths in the Gelfand-Tsetlin graph. We study a quantum version of the latter problem. We introduce a notion of a q-centrality and describe the simplex of all q-central measures on paths in the Gelfand-Tsetlin graph. Conjecturally, q-central measurets are related to representations of the quantized universal enveloping algebra $U_\epsilon(gl_\infty)$. We also define a class of q-Toeplitz matrices and show that every extreme q-central measure corresponds to a q-Toeplitz matrix with non-negative minors. Finally, our results can be viewed as a classification theorem for certain Gibbs measures on rhombus tilings of the halfplane. We use a class of q-interpolation polynomials related to Schur functions. One of the key ingredients of our proofs is the binomial formula for these polynomials proved by Okounkov.

1.2 Scientific conferences and seminar talks

P. Bibikov

1. Seminar "Geometry, topology and mathematical physics", academician Novikov, professor Buhstaber (Moscow, MSU, April 2011).

2. International conference "Lomonosov–2011" (Moscow, Russia, April 11–15, 2011).

3. Seminar on differential equations, professor Obnosov (Kazan, KFU, May 2011).

4. International conference "Geometry. Control. Economy" (Astrahan, Russia, August 15–26, 2011).

5. Seminar "Differential geometry and applications", academician Fomenko (Moscow, MSU, October 2011).

6. All-Russian conference "Lobachevskii readings" (Kazan, Russia, November 8–12, 2011).

7. Seminar "Cogomology aspects of differential equations" , professor Krasil'shik (Moscow, IMU, October 2011).

A. Bufetov

Visit to Utrecht University (with a talk), 23.11.2011 - 07.12.2011, Utrecht, Netherlands.

R. Devyatov

1. Conference "Lie algebras, algebraic groups and invariant theory", Moscow, January, 31 – February, 5, talk "Generically transitive actions on multiple flag varieties", see abstract here, in Russian:
   `http://new.math.msu.su/lie2011/abstracts/devyatov.pdf`
2. Summer School "Moduli of Curves and Gromov-Witten Theory", Grenoble–St Martin d'Hères, France, June, 20 – July, 8.
3. Talk "Unipotent commutative group actions on flag varieties and nilpotent multiplications" in Klaus Altmann's research group, Berlin, August, 17.
4. Conference "North German Algebraic Geometry Seminar", Hannover, November, 10–11.

A. Efimov

1) Bielefeld, Conference "Workshop on Matrix Factorizations"
Talk: "Reconstruction of hypersurface singularity from its triangulated category of singularities"
2) Vienna, Conference "Memorial conference for Maximilian Kreuzer"
Talk: "Quantum cluster algebras and motivic Hall algebras"
3) Cetraro, Conference "Mirror Symmetry and Tropical Geometry"
Talk: "Cohomological Hall algebra and Kac conjecture"
4) Split, Conference "Homological Mirror Symmetry and Category Theory"
Talk: "Quantum cluster algebras and motivic Hall algebras"
5) Tianjin, Conference "Geometry and Quantization"
Talk: "Quantum cluster variables via vanishing cycles"
6) Kashiwa, Conference "Curves and categories in geometry and physics"
Talk: "Quantum cluster variables via vanishing cycles"

V. Gorin

1. April: Visit to Utrecth, talk at "Stochastic colloquium" (Mathematical Institute, University of Utrecht)
2. September: Visit to Paris, talk at "Séminaire de Mathématique" (Institut des Hautes Études Scientifiques) and talk at "Processus Stochastiques, Matrices Aléatoires" (University Paris VI)
3. October: Visit to Utrecht, talk at "Utrecht mathematics colloquium" (Mathematical Institute, University of Utrecht)

4. November: Visit to Saint-Petersburg, talk at “Saint-Petersburg seminar on representation theory and dynamical systems” (St.Petersburg Department of Steklov Institute of Mathematics of RAS), talk at “Stochastics” (St.Petersburg Department of Steklov Institute of Mathematics of RAS), and talk at “Probability seminar of Chebyshev Laboratory” (Saint-Petersburg State University)

2 Program: Simons IUM fellowships

2.1 Research

Yu.Burman

With Andrey Ploskonosov, Anastasia Trofimova

Higher matrix-tree theorems.

submitted to Electronic Journal of Linear Algebra, preprint available as arXiv:1109.6625v1 [math.CO].

We calculate determinants of weighted sums of reflections and of (nested) commutators of reflections. The results obtained generalize the Kirchhoff's matrix-tree theorem and the matrix-3-hypertree theorem by G.Massbaum and A.Vaintrob.

S. Gusein-Zade

[]1 with W. Ebeling,

Saito duality between Burnside rings for invertible polynomials,

arXiv:1105.1964 Accepted for publication in the Bulletin of the London Mathematical Society.

There was given an equivariant version of the Saito duality which can be regarded as a Fourier transformation on Burnside rings. It was shown that (appropriately defined) reduced equivariant monodromy zeta functions of Berglund–Hübsch dual invertible polynomials are Saito dual to each other with respect to their groups of diagonal symmetries. Moreover it was shown that the relation between “geometric roots” of the monodromy zeta functions for some pairs of Berglund–Hübsch dual invertible polynomials described in a previous paper is a particular case of this duality.

[2] with W. Ebeling

Equivariant Poincaré series and monodromy zeta functions of quasihomogeneous polynomials.

arXiv:1106.1284 Submitted to Publications RIMS.

In earlier papers, there was described a relation between the Poincaré series and the classical monodromy zeta function corresponding to a quasihomogeneous polynomial. There was formulated an equivariant version of this relation in terms of the Burnside rings of finite abelian groups and their analogues.

3) with W. Ebeling,
Orbifold Euler characteristics for dual invertible polinomial,
arXiv:1107.5542 To appear in Moscow Mathematical Journal, 2012, v.12, no.1.)

To construct mirror symmetric Landau–Ginzburg models, P.Berglund, T.Hübsch and M.Henningson considered a pair (f, G) consisting of an invertible polynomial f and an abelian group G of its symmetries together with a dual pair $(\widetilde{f}, \widetilde{G})$. There was studied the reduced orbifold Euler characteristics of the Milnor fibres of f and $\widetilde{f}$ with the actions of the groups G and $\widetilde{G}$ respectively and it was shown that they coincide up to a sign.

A. Kuznetsov
with A.Polishchuk,
Exceptional collections on isotropic Grassmannians,
math.AG/1110.5607

We introduce a new construction of exceptional objects in the derived category of coherent sheaves on a compact homogeneous space of a semisimple algebraic group and show that it produces exceptional collections of the length equal to the rank of the Grothendieck group on homogeneous spaces of all classical groups.

G. Olshanski:
[1] with Alexei Borodin
The boundary of the Gelfand-Tsetlin graph: A new approach,
47 pages, submitted; arXiv:1109.1412.

The Gelfand–Tsetlin graph is an infinite graded graph that encodes branching of irreducible characters of the unitary groups. The boundary of the Gelfand–Tsetlin graph has at least three incarnations — as a discrete potential theory boundary, as the set of finite indecomposable characters of the infinite-dimensional unitary group, and as the set of doubly infinite totally positive sequences. An old deep result due to Albert Edrei and Dan Voiculescu provides an explicit description of the boundary; it can be realized as a region in an infinite-dimensional coordinate space.

The paper contains a novel approach to the Edrei–Voiculescu theorem. It is based on a new explicit formula for the number of semi-standard Young tableaux of a given skew shape (or of Gelfand–Tsetlin schemes of trapezoidal shape). The formula is obtained via the theory of symmetric functions, and new Schur-like symmetric functions play a key role in the derivation.

[2] with Alexei Borodin,
The Young bouquet and its boundary,
43 pages, submitted; arXiv:1110.4458.

The classification results for the extreme characters of two basic "big" groups, the infinite symmetric group $S(\infty)$ and the infinite-dimensional unitary group $U(\infty)$, are remarkably similar. It does not seem to be possible to explain this phenomenon using a suitable extension of the Schur-Weyl duality to infinite dimension. We suggest an explanation of a different nature that does not have analogs in the classical representation theory.

We start from the combinatorial/probabilistic approach to characters of "big" groups initiated by Vershik and Kerov. In this approach, the space of extreme characters is viewed as a boundary of a certain infinite graph. In the cases of $S(\infty)$ and $U(\infty)$, those are the Young graph and the Gelfand–Tsetlin graph, respectively. We introduce a new related object that we call the Young bouquet. It is a poset with continuous grading whose boundary we define and compute. We show that this boundary is a cone over the boundary of the Young graph, and at the same time it is also a degeneration of the boundary of the Gelfand–Tsetlin graph.

The Young bouquet has an application to constructing infinite-dimensional Markov processes with determinantal correlation functions.

A.Penskoi
[1] Extremal spectral properties of Lawson tau-surfaces and the Lame equation
Moscow Math. J., to appear

Extremal spectral properties of Lawson tau-surfaces are investigated. The Lawson tau-surfaces form a two-parametric family of tori or Klein bottles minimally immersed in the standard unitary three-dimensional sphere. A Lawson tau-surface carries an extremal metric for some eigenvalue of the Laplace-Beltrami operator. Using theory of the Lame equation we find explicitly these extremal eigenvalues.

[2] Extremal spectral properties of Otsuki tori
submitted to Mathematische Nachrichten, preprint arXiv:1108.5160

Otsuki tori form a countable family of immersed minimal two-dimensional tori in the unitary three-dimensional sphere. According to El Soufi-Ilias theorem, the metrics on the Otsuki tori are extremal for some unknown eigenvalues of the Laplace-Beltrami operator. Despite the fact that the Otsuki tori are defined in quite an implicit way, we find explicitly the numbers of the corresponding extremal eigenvalues. In particular we provide an extremal metric for the third eigenvalue of the torus.

A. Skopenkov
with D. Crowley

A classification of embeddings of non-simply-connected 4-manifolds into 7-space
2011, in preparation.

Let N be a closed connected orientable 4-manifold without torsion in homology. The main result is a complete readily calculable classification of embeddings $N \longrightarrow \mathbb{R}^7$, in the smooth and in the PL categories. Such a classification was earlier known only for simply-connected N, in the PL case by Boéchat-Haefilger-Hudson 1970, in the smooth case by the authors 2008. Our result for $N = S^1 \times S^3$ states that the set of smooth isotopy classes of smooth embeddings $S^1 \times S^3 \longrightarrow \mathbb{R}^7$ is in a (geometrically defined) 1–1 correspondence with the quotient set $\mathbb{Z}_{12} + \mathbb{Z} + \mathbb{Z}/(a, b, c) = (a, b, c + 2kb), k \in \mathbb{Z}$. We also obtain a PL classification, and we show how the two classification results are related to each other. We disprove the Multiple Haefliger-Wu invariant conjecture and the Melikhov conjecture.

E. Smirnov
[1] with Valentina Kiritchenko, Vladlen Timorin
Schubert calculus and Gelfand-Zetlin polytopes
arxiv.org, 01/2011. Submitted to "Annales de l'Institut Fourier"

We describe a new approach to the Schubert calculus on complete flag varieties using the volume polynomial associated with Gelfand-Zetlin polytopes. This approach allows us to compute the intersection products of Schubert cycles by intersecting faces of a polytope

[2] with Valentina Kiritchenko, Vladlen Timorin
Gelfand-Zetlin polytopes and Demazure characters
Proceedings of the International Conference "50 years of IITP", IITP RAS, Moscow, 2011.
see V. Timorin

[3] with Valentina Kiritchenko, Vladlen Timorin
Convex chains for Schubert varieties
Oberwolfach Reports, Switzerland: European Mathematical Society, 2011.
see V. Timorin

A. Sobolevski
With J. Delon and J. Salomon,
Minimum-weight perfect matching for non-intrinsic distances on the line
Zapiski nauchnykh seminarov POMI **390** (2011) 52–68
(in English, see also arxiv.org/abs/1102.1558).
The same article will appear in Springer's *Journal of Mathematical Sciences*, a collection of English translations of current Russian mathematical publicaions, in 2012.

Consider a real line equipped with a (not necessarily intrinsic) distance. We deal with the minimum-weight perfect matching problem for a complete graph whose points are located on the line and whose edges have weights equal to distances along the line. This problem is closely related to one-dimensional Monge-Kantorovich transport optimization (and is indeed motivated by our previous research on efficient algorithms for transport optimization in discrete setting). The main result of the present note is a "bottom-up" recursion relation for weights of partial minimum-weight matchings: a kind of Bellman equation, which streamlines and explicates a more involved construction proposed for the transportation problem in our work arXiv:1102.1795.

V. Timorin

[1] with E.Smirnov, V.A. Kiritchenko
Schubert calculus and Gelfand-Zetlin polytopes
arxiv.org, 01/2011. Submitted to "Annales de l'Institut Fourier"
See E.Smirnov

[2] with E.Smirnov, V.A. Kiritchenko,
Gelfand-Zetlin polytopes and Demazure characters
Proceedings of the International Conference "50 years of IITP", IITP RAS, Moscow, 2011.

An important feature of toric geometry is the interplay between a polarized projective toric variety and itsconvex polytope. For instance, the Hilbert polynomial can be computed by counting integer points in the dilations ofthe polytope. In a recent preprint, we explore the interplay between algebraic and convex geometry in a non-toric case,namely, for Schubert varieties in a complete flag variety. With a projective embedding of the flag variety, one can naturally associate a convex polytope, called the Gelfand–Zetlin polytope.Kogan assigned a collection of faces of the Gelfand–Zetlin polytope to each Schubert variety. Our main result is a formula for the Demazure character of a Schubert variety in terms of the exponential sums over the integer points in the union of these faces.As a corollary, we get a formula for the Hilbert functions of Schubert varieties via the number of integer points. This in turn implies a formula for the degrees of Schubert varieties via volumes similar to the Koushnirenko theorem in toric geometry.

[3] with V.A. Kiritchenko, E.Smirnov,
Convex chains for Schubert varieties
Oberwolfach Reports, Switzerland: European Mathematical Society, 2011.

We discuss a uniform construction of convex chains that can be regarded as analogs of Newtonpolytopes for the variety of complete flags in C^n and its Schubert varieties.The chains are given by the faces of a Gelfand-Zetlin polytope(the latter is a well-known Newton-Okounkov body for the flag variety). Kogan assigned a collection of faces of the Gelfand-Zetlinpolytope to each Schubert variety.We prove a formula for the Demazure character of a Schubert varietyin terms of the exponential sums over the integer points in the union ofthese faces.This implies that the union of faces captures the geometry of the Schubertvariety in the same way as Newton-Okounkov bodies do (that is,the Hilbert function of the Schubert variety can be computedvia the number of integer points). As a byproduct, we define analogs of Demazure operators acting on convex polytopes.This leads to a simple inductive construction of Gelfand-Zetlin polytopes and their generalizations.We also discuss applications of our results to Schubert calculus.

[4] With Alexander Blokh, Lex Oversteegen, Ross Ptacek
Topological polynomials with a simple core
arxiv.org, 2011. To appear in the proceedings of the international conference "Frontiers in Complex Dynamics"dedicated to J.Milnor's 80th Birthday

We define the (dynamical) core of a topological polynomial (and the associated lamination). This notion extends that of the core of a unimodal interval map. Two explicit descriptions of the core are given: one related to periodic objects and one related to critical objects. We describe all laminations associated with quadratic and cubic topological polynomials with a simple core (in the quadratic case, these correspond precisely to points on the Main Cardioid of the Mandelbrot set).

[5] Planarizations and maps taking lines to linear webs of conics
arxiv.org, 08/2011. Submitted to Mathematical Research Letters

Aiming at a generalization of a classical theorem of Moebius, we study maps that take line intervals to plane curves, and also maps that take line intervals to conics from certain linear systems

[6] with I. Mashsanova-Golikova,
Captures, matings and regluings,
arXiv, 11/2011. -25 .

In parameter slices of quadratic rational functions, we identify arcs represented by matings of quadratic polynomials. These arcs are on the boundaries of hyperbolic components.

[7] Cut and semi-conjugate
arXiv, 10/2011. -4 .

We define a very general class of rational functions $f : CP^1 \longrightarrow CP^1$ such that for every function f of this class, there exists a countable family of smooth curves γ_i and a critically finite hyperbolic function R such that the dynamical system obtained from f by cutting along the curves γ_i is topologically semi-conjugate to R.

M. Verbitsky

Preprints (2011)

[1] with Marcos Jardim,
Trihyperkahler reduction and instanton bundles on $\mathbb{C}P^3$
arXiv:1103.4431

A trisymplectic structure on a complex $2n$-manifold is a triple of holomorphic symplectic forms such that any linear combination of these forms has rank $2n$, n or 0. We show that a trisymplectic manifold is equipped with a holomorphic 3-web and the Chern connection of this 3-web is holomorphic, torsion-free, and preserves the three symplectic forms. We construct a trisymplectic structure on the moduli of regular rational curves in the twistor space of a hyperkaehler manifold, and define a trisymplectic reduction of a trisymplectic manifold, which is a complexified form of a hyperkaehler reduction. We prove that the trisymplectic reduction in the space of regular rational curves on the twistor space of a hyperkaehler manifold M is compatible with the hyperkaehler reduction on M.

As an application of these geometric ideas, we consider the ADHM construction of instantons and show that the moduli space of rank r, charge c framed instanton bundles on $\mathbb{C}P^3$ is a smooth, connected, trisymplectic manifold of complex dimension $4rc$. In particular, it follows that the moduli space of rank 2, charge c instanton bundles on $\mathbb{C}P^3$ is a smooth complex manifold dimension $8c-3$, thus settling a 30-year old conjecture.

[2] with Liviu Ornea, Victor Vuletescu,
Blow-ups of locally conformally Kaehler manifolds
arXiv:1108.4885

A locally conformally Kaehler (LCK) manifold is a manifold which is covered by a Kaehler manifold, with the deck transform group acting by homotheties. We show that the blow-up of a compact LCK manifold along a complex submanifold admits an LCK structure if and only if this submanifold is globally conformally Kaehler. We also prove that a twistor space (of a compact 4-manifold, a quaternion-Kaehler manifold or a Riemannian m anifold) cannot admit an LCK metric, unless it is Kaehler.

Published papers (2011)

[1] A CR twistor space of a $G2$-manifold,
Differential Geometry and its Applications, 2011, 29. pp. 101-107

Let M be a $G2$-manifold. We consider an almost CR-structure on the sphere bundle of unit tangent vectors on M, called the CR twistor space. This CR-structure is integrable if and only if M is a holonomy $G2$ manifold. We interpret $G2$-instanton bundles as CR-holomorphic bundles on its twistor space.

[2] with Ornea L.
Report on locally conformally Kaehler manifolds
Harmonic Maps and Differential Geometry, 2011, 542. pp. 284 pp

We present an overview of recent results in locally conformally Kaehler geometry, with focus on the topological properties which obstruct the existence of such structures on compact manifolds.

[3] Hodge theory on nearly Kaehler manifolds
Geometry and Topology, 2011, 15, pp 2111-2133

Let (M, I, ω, Ω) be a nearly Kaehler 6-manifold, that is, an $SU(3)$-manifold with the (3,0)-form Ω and the Hermitian form ω which satisfies $d\omega = 3\lambda \mathrm{Re}\Omega, d\mathrm{Im}\Omega = -2\lambda\omega^2$, for a non-zero real constant λ. We develop an analogue of Kaehler relations on M, proving several useful identities for various intrinsic Laplacians on M. When M is compact, these identities bring powerful results about cohomology of M. We show that harmonic forms on M admit the Hodge decomposition, and prove that $H^{p,q}(M) = 0$ unless $p = q$ or $(p = 1, q = 2)$ or $(p = 2, q = 1)$.

[4] Hyperholomorpic connections on coherent sheaves and stability
Central European Journal of Mathematics, 2011, 9 (3). pp. 535-557

Let M be a hyperkaehler manifold, and F a torsion-free and reflexive coherent sheaf on M. Assume that F (outside of its singularities) admits a connection with a curvature which is invariant under the standard $SU(2)$-action on 2-forms. If the curvature is square-integrable, then F is stable and its singularities are hyperkaehler subvarieties in M. Such sheaves (called hyperholomorphic sheaves) are well understood. In the present paper, we study sheaves admitting a connection with $SU(2)$-invariant curvature which is not necessarily square-integrable. This situation arises often, for instance, when one deals with higher direct images of holomorphic bundles. We show that such sheaves are stable.

[5] Manifolds with parallel differential forms and Kaehler identities for G_2-manifolds
Journal of Geometry and Physics, 2011, 61 (6). p. 1001-1016

Let M be a compact Riemannian manifold equipped with a parallel differential form ω. We prove a version of Kaehler identities in this setting. This is used to show that the de Rham algebra of M is weakly equivalent to its subquotient $(H^*_c(M), d)$, called **the pseudocohomology** of M. When M is compact and Kaehler and ω is its Kaehler form, $(H^*_c(M), d)$ is isomorphic to the cohomology algebra of M. This gives another proof of homotopy formality for Kaehler manifolds, originally shown by Deligne, Griffiths, Morgan and Sullivan. We compute $H^i_c(M)$ for a compact G_2-manifold, showing that it is isomorphic to cohomology unless $i = 3, 4$. For $i = 3, 4$, we compute $H^*_c(M)$ explicitly in terms of the first order differential operator $*d : \Lambda^3(M) \to \Lambda^3(M)$.

[6] with Jardim M
Moduli spaces of framed instanton bundles on $\mathbb{C}P^3$ and twistor sections of moduli spaces of instantons on $\mathbb{C}^2$,
Adv.Math., 2011, 227. pp. 1526-1538

We show that the moduli space M of holomorphic vector bundles on CP^3 that are trivial along a line is isomorphic (as a complex manifold) to a subvariety in the moduli of rational curves of the twistor space of the moduli space of framed instantons on $\mathbb{R}^4$, called the space of twistor sections. We then use this characterization to prove that M is equipped with a torsion-free affine connection with holonomy in $Sp(2n, \mathbb{C})$.

[7] with Ornea L.
Oeljeklaus-Toma manifolds admitting no complex subvarieties,
Mathematical Research Letters, 2011, 04 (18), pp. 747-754

The Oeljeklaus-Toma (OT-) manifolds are complex manifolds constructed by Oeljeklaus and Toma from certain number fields, and generalizing the Inoue surfaces S_m. On each OT-manifold we construct a holomorphic line bundle with semipositive curvature form and trivial Chern class. Using this form, we prove that the OT-manifolds admitting a locally conformally Kahler structure have no non-trivial complex subvarieties. The proof is based on the Strong Approximation theorem for number fields, which implies that any leaf of the null-foliation of w is Zariski dense.

2.2 Teaching

Yu. Burman

Spring semester 2011: topology II

At the spring semester of 2011 I taught (both lectures and excercises) the *Topology II* course for second-year students. The course programme:

1. Homology and cohomology.
   (a) Singular homology, homotopy invariance. Exercises: "informal" computations of homology, complexes, group homology.
   (b) Exact sequences of a pair and of a triple.
   (c) Meyer–Vietoris sequence. Exercises: computations of homology.
   (d) CW-complex, Gurevich theorem. Exercises: homotopy groups via killing by Cartan–Serre, $K(\pi, n)$.
   (e) Multiplicative structure of the cohomology.

2. Smooth manifolds.
   (a) Morse theory. Exercises: homology and homotopy type of manifolds.
   (b) Poincaré duality,
   (c) Intersection numbers, multiplication in cohomology and intersection of the Poincaré duals.
3. Applications
   (a) Hopf theorem.
   (b) Obstruction theory, characteristic classes. Exercises: computation of characteristic classes.
   (c) Other applications: embedding of manifolds into Euclidean spaces, manifolds as boundaries, and more.

Fall semester 2011: geometry (exercises)
In the fall semester of 2011 I worked as an assistant (organized exercises) at the *Geometry* course for first-year students. The course programme:

1. Klein's program.
2. Groups and group actions.
3. Euclidean transformation of the plane.
4. Coxeter polytopes.
5. Spherical geometry.
6. Inversion.
7. Lobchevsky plane (Poincaré models and Cayley–Klein model).
8. Projective plane. Projective duality.
9. Finite geometries.

S. Gusein-Zade

I. Courses read in 2011.

1. Calculus. Independent University of Moscow, I year students, September-December 2011, 2 hours per week.

Program.
1. The set of real numbers, its completeness.
2. Elements of general topology. Metric spaces. Completions and limits
3. Limits of sequences, sums of series. Remarkable limits.
4. The n-dimensional coordinate space and its topological properties. Continuous functions and maps.
5. Derivative of a function of one variable, its properties. Derivatives of elementary functions. Taylor series. Differential.
6. Riemann integral of a function in one variable. Central calculus theorem.
7. Differentiation of functions in several variables. "Commutativeness" of the differentiation. Implicit function theorem.
8. Limits of sequences of functions, sums of function series.
9. Uniform convergence, weak convergence.

2. Analytic geometry, Moscow State University, I year students, September-December 2011, 4 hours per week.

Program.
1. Coordinates on a plane and in the space. Coordinates of points and coordinates of vectors.
2. Coordinate change on a plane and in the space.
3. Scalar product of vectors. Scalar product in coordinates (in orthogonal and in arbitrary affine coordinates).
4. Orthogonal coordinate changes on a plane.
5. Division of a segment in a given ration.
6. Lines on a plane. Parametric definition and definition by an equation.
7. Definition of a semi-plane by an inequality. Systems of linear inequalities on a plane.
8. Distance from a point to a line on a plane. Normal equation of a line.
9. Pencils of lines on a plane. Proper and non-proper pencils.
10. A plane in the space. Parametric definition and definition by an equation.
11. Distance from a point to a plane. Normal equation of a plane.
12. Lines in the space. Parametric definition and definition by equations.
13. Pencils of planes in the space. The condition of belonging of a plane to the pencil defined by two planes.
14. Bunches of planes in the space. The condition of belonging of a plane to the bunch defined by three planes.
15. Vector product of vectors. Definition and main properties. Computation of the vector product in orthogonal coordinates.
16. The oriented area of a parallelogram on a plane and the oriented volume of a parallelepiped in the space. Expression of the oriented area and the oriented volume in terms of a determinant.
17. Expression of the volume of a parallelepiped through the scalar and the vector products (the mixed product).

18. Matrix expression of a coordinate change on a plane or in the space.
19. Orthogonal coordinate changes and orthogonal matrices.
20. Gram matrix of a system of vectors. Its connection with the area and with the volume.
21. Orthogonal coordinate changes in the space Euler angles.
22. Algebraic curves on a plane. Theorem on "splitting-off of a line".
23. Plane curves of degree two. The affine classification.
24. The orthogonal classification of the curves of degree two. Reduction of a curve equation to the canonical form.
25. Quadratic forms in two and three variables. The matrix of a quadratic form and its change under a coordinate change.
26. The invariants of curves of degree two.
27. The semi-invariant of curves of degree two.
28. Determination of the canonical equation of a curve of degree two through the values of the invariants and of the semi-invariant.
29. Conjugate diameters of a curve of degree two. Tangents to a curve of degree two.
30. The ellipse and its geometric properties.
31. The hyperbola and its geometric properties.
32. The parabola and its geometric properties.
33. Definition of a curve of degree two in the polar coordinates. Rational parametrization of a curve of degree two.
34. Surfaces of degree two. The affine classification.
35. The orthogonal classification of the surfaces of degree two. Reduction of a surface equation to the canonical form.
36. The invariants of surfaces of degree two. Partial classification of the surfaces of degree two with the help of the invariants.
37. The plane conjugate to a direction for a surface of degree two. Tangent planes to a surface of degree two. Straight line generators of a surface of degree two.
38. Curves of degree two as conical sections.
39. Affine transformations. Matrix definition of an affine transformation. Orthogonal affine transformations.
40. The change of the matrix of an affine transformation under a coordinate change. The determinant of an affine transformation and its geometric meaning.
41. Orthogonal transformations (isometries) of a plane.
42. Orthogonal transformations (isometries) of the space.
43. Representability of an arbitrary affine transformation as the composition of an isometry and tensions.
44. The correspondence between points and pencils of lines on a plane. The completion of an affine plane. The projective plane.
45. Projective transformations of a plane. Their connection with projections in the space.
46. The projective invariant of four points on a projective line (the double ratio).
47. Projective (homogeneous) coordinates on a projective plane. A line on a projective

plane.
48. Duality between points and lines on a projective plane. Dual statements.
49. Curves of degree two on a projective plane. The classification.
50. Curves of degree two passing through five and through four points on a projective plane.
51. The double ratio of four points on a curve of degree two.
52. Duality of curves of degree two on a projective plane. Pascal and Brianchon theorems.

3. Topological invariants of singularities, Moscow State University, III-V year students, September-December 2011, 2 hours per week.

Program.
1. Germs of analytic functions with isolated critical points.
2. Classification of analytic function germs with respect to different equivalencies.
3. The number of moduli of a germ. The classification of simple germs.
4. Milnor fibre and the classical monodromy transformation of a germ.
5. Homology groups of complex affine manifolds. The homotopy type of the Milnor fibre of an isolated singularity.
6. The monodromy group of an isolated singularity.
7. The variation operator of a singularity.
8. Picard–Lefschetz theorem.
9. Resolution of singularities. Quasi-unipotency of the monodromy operator.
10. Versal deformations of singularities.
11. The local degree of an analytic map in algebraic terms. Eisenbud–Levine–Khimshiashvili theorem.
12. Intersection matrices of function germs in two variables.
13. Toric resolutions.
13. Computation of topological invariants in terms of Newton diagrams.

A.Kuznetsov

Minicourse "Moduli spaces of curves and Gromov–Witten invariants", 3 lectures.

Program: Gromv–Witten invariants is a remarkable collection of numerical invariants of an algebraic (or more generally symplectic) variety, generalizing intersection indices of cohomological classes. They allow to enhance the cohomology ring of the variety with a new, so-called quantum, multiplication, which is a deformation of the usual multiplication and provide the first step to understanding of Mirror Symmetry, astonishing phenomenon discovered by physicists in late 80's of the last century. For an algebraic variety Gromov–Witten invariants are defined via the intersection theory on the moduli space of curves on this variety. I will try to explain what a moduli space of curves is, and how one can work with it; what kind of difficulties appear when one investigates Gromov–Witten invariants, and how one can deal with those.

A course on derived categories of coherent sheaves on algebraic varieties is scheduled for Spring 2012.

Program:

1. Exceptional collections of vector bundles (projective spaces, Grassmannians, quadrics, flag varieties);

2. Semiorthogonal decompositions (projectivizations of vector bundles, smooth blowups);

3. Homological projective duality;

4. Fano threefolds;

5. Stability conditions and autoequivalences;

6. Toric varieties.

G.Olshanski

Lectures

A series of 7 lectures for students at the seminar "Functional analysis and algebraic combinatorics" held at Independent University of Moscow and Steklov Mathematical Institute. The program:

1. Screw lines in Hilbert space, after works of Kolmogorov and von Neumann–Schoenberg (September 15 and 22).

2. Dirichlet measures and Rayleigh triangles (October 13).

3. Divided differences, B-splines, and projections of orbital measures in the spaces of Hermitian matrices (November 3).

4. Boundaries of branching graphs (November 24).

5. Mallows measure on the infinite symmetric group (December 1).

6. Hilbert spaces of holomorphic functions (December 8)

A.Penskoi

During 2011 I was teaching at three universities.

Independent University of Moscow.

I was giving lectures and exercise classes on Differential Geometry (Spring 2011) and Analysis on Manifolds (Fall 2011).

These courses are quite innovative. High mathematical potential of the IUM students permits us to explore differential geometry and analysis of manifolds deeper than it is made usually in standard russian university courses.

I was also giving lectures and exercise classes on Differential Geometry (Spring 2011) and Equations of Mathematical Physics and Topology-I (Fall 2011) for the Math in Moscow Program (a one-semester program for undergraduate students from the U.S.A. and Canada organized jointly by the IUM and the High School of Economics).

Moscow State University.

I was giving exercise classes on Classical Differential Geometry (Spring 2011) and Analytical Geometry and Differential Geometry (Fall 2011).

Bauman Moscow State Technical University.

I was lecturing on Numerical Methods (Spring 2011).

Since the most important courses were Differential Geometry (Spring 2011) and Analysis on Manifolds (Fall 2011) at the Independent University of Moscow, let me present here their sillabuses.

Differential Geometry (Spring 2011)

1. Curves and surfaces in the plane and the three-dimensional space. Curvature, torsion, Frenet frame. First and second fundamental forms. Principal curvatures, mean curvature and Gauß curvature. Mean curvature normal vector. Euler formula for the normal section curvature.

2. Surfaces in n-dimensional space. First and second fundamental forms. Connections in the tangent and normals bundles on a surface. Second fundamental form and Weingarten operator. Gauß-Weingarten derivational equations. Gauß-Bonnet theorem for surfaces.

3. Basic theory of Lie groups ans algebras.

4. Vector bundles and gluing cocycles. Structure group. Euclidean and hermitian bundles. Natural operations with bundles. Orientable bundles.

5. Connections in vector bundles. Connection local form, Christoffel symbols. Connections in euclidean and hermitian bundles. Connections compatible with metrics and their curvature.

6. Riemannian manifolds. Curvature, torsion. Levi-Civita connection. Symmetries of curvature tensor. Ricci tensor. Scalar curvature.

7. Riemannian manifolds II. Geodesics. Geodesic coordinates. Lagrangian approach to geodesics. Second variation.

8. Submanifolds of Riemannian manifolds. First and second fundamental forms.

9. Characteristic classes. Chern-Weil construction of characteristic classes. Chern, Pontryagin and Euler classes and their properties.

10. Vector bundles and their cohomologies. Thom class. Mathai-Quillen construction of the Thom class. The connection between the Thom class and the Euler class. Gauß-Bonnet theorem.

Analysis on Manifolds (Fall 2011)

1. Reminiscences from Calculus: implicit function theorem, inverse function theorem, rank theorem. Surfaces in affine spaces and different ways of their definition.

2. Smooth manifolds. Partition of unity. Maps of manifolds.

3. Tangent vectors and differential of a map. Tangent and cotangent spaces.

4. Immersions, submanifolds, submersions.

5. Sarde lemma. Transversality. Week Whitney theorem.

6. Vector fields. Commutator of vector fields. Integral curves of a vector filed. One-parametric group generated by a vector field.

7. Distributions and Frobenius theorem.

8. Tensor fields, differential forms. Riemann metric, volume form. Exterior differential.

9. Lie derivative. Cartan identity. Hodge operation. Relation between d and grad, rot and div.

10. Orientation of a manifold. Densities. Integration of densities and forms over manifolds and chains. Stokes theorem for integration over chains.

11. Manifolds with boundary. Stokes theorem for manifolds with boundary. Relation to Green, Stokes and Gauß-Ostrogradsky formulas in calculus.

12. Basics of Lie groups and algebras.

13. Actions of Lie groups. Homogeneous spaces.

14. De Rham cohomologies, de Rham cohomologies with compact support. Poincaré lemma. Mayer-Vietoris long exact sequence.

15. Properties of de Rham cohomologies (finite dimension, Künneth formula etc). De Rham theorem and Hodge theorem (without proofs).

A.Skopenkov

The following courses were delivered.

(A) Vector fields on manifolds and basic homology theory,
http://dfgm.math.msu.su/files/skopenkov/SPECKURS.pdf (in Russian)
1. Orientability of 2-manifolds: homology and the first Stiefel-Whitney class.
2. Classification of vector fields on subsets of the plane and on 2-manifolds.
3. Existence of tangents of vector fields on 2-manifolds.
4. Existence of normal vector fields for 2-manifolds.
5. Definition and examples of 3-manifolds.
6. The Hopf Theorem on existence of a nonzero tangent vector field on any 3-manifold.
7. Existence of tangents of vector fields on higher-dimensional manifolds.
8. Normal vector fields for manifolds of dimension 3 and higher.
9. Existence of orthonormal systems of vector fields.
10. Homology and Poincaré duality for 3-manifolds.
11. A simple proof of the Stiefel theorem on parallelizability of orientable 3-manifolds.

(B) Algorithmic recognition of realizability of hypergraphs,
http://ium.mccme.ru/f11/SKOPENKOV-SPECKURS.pdf (in Russian)

1. Definitions of a 2-dimensional complex (a hypergraph) and of a piecewise-linear embedding.

2. Embeddings in the plane. The Kuratowski and Halin-Jung Theorems.

3. Embeddings in the three-dimensional space. Ramsay link theory. Construction of Borromean rings from the torus.

4. Generalizations to embeddings in the four-dimensional space.

5. The van Kampen and the deleted product obstructions. Algorithmic recognition of embeddability.

6. The van Kampen and the deleted product invariants. Algorithmic recognition of knottings.

7. Obstruction theory of obstacles and algorithmic recognition of homotopic maps.

E.Smirnov

Programs of courses

Basic Representation Theory

This is a joint course of the IUM program for foreign students *Math in Moscow* and MSc program in Mathematics, Higher School of Economics.

Course outline:

1. Introduction. Main definitions. Irreducibility and indecomposability of representations. Maschke's theorem.
2. Schur's lemma. Representations of abelian groups. Tensor products of representations.
3. Symmetric and exterior powers of representations. Dual representations. Character of a representation.
4. First projection formula. Orthonormality of characters. Decomposition of the regular representation. Burnside's formula.
5. General projection formula. Characters as a basis in the space of class functions. Group algebra. Ideals in the group algebra, idempotent elements.
6. Representations of symmetric groups. Young symmetrizers.
7. Lie groups: definitions and basic properties.
8. The exponential map. Lie algebras.
9. Representations of Lie groups and Lie algebras. Compact groups, complete reducibity of their representations.
10. Reductive Lie groups; Weyl's unitary trick. Representations of SL_2. The Clebsch–Gordan problem for SL_2.

Algebra 1

Advanced undergraduate course, taught at the Independent University of Moscow.

Exercise sessins were organized jointly with Leonid Rybnikov and Grigory Merzon. Problems from the exercise sessions are available (in Russian) from
`http://ium.mccme.ru/f11/algebra1.html`.

Course outline:

Rings

1. Rings, fields, ring homomorphisms. Modular arithmetic. Fermat's little theorem. Chinese remainder theorem for remainder rings.

2. Ideals, quotient rings, the homomorphism theorem. Eucledian domains, principal ideal domains. GCD and LCD, associated elements. Chinese remainder theorem for principal ideal domains.

3. Unique factorization domains. Prime and irreducible elements. Integral principal ideal domains are unique factorization domains. Polynomials. Gauss's Lemma.

4. The polynomial ring over a UFD is a UFD. Fundamental theorem of symmetric polynomials.

Vector spaces 1. Vector spaces. Examples. The n-dimensional arithmetic vector space. Linear dependence and independence. Basis, dimension. Dual space.

2. Linear maps, isomorphisms. Basis as an isomorphism with the arithmetic vector space. Dual space. The canonical isomorphism between a vector space and its double dual. Annihilator of a vector subspace. Direct sums of vector spaces. Quotient space.

3. Matrix of a linear map. Product of matrices. Linear operators. Grassmann (skew) polynomials. Determinant. $\det AB = \det A \det B$.

4. The symmetric group. The sign of a permutation. Exterior powers of an operator. Minors as matrix entries of an exterior power. Computation of the determinant: an explicit formula, row/column decomposition.

5. Skew-symmetric multilinear forms. Determinant as a skew-symmetric multilinear form. Eigenvectors, diagonalizable operators, Cayley–Hamilton Theorem.

Modules 1. Modules over rings. Examples. Vector space with an operator is a $k[t]$-module. Homomorphisms. System of generators, basis. Free modules, rank. Submodule of a free module is free.

2. Description of finitely generated modules over principal ideal domains. Applications: classification of finitely generated abelian groups, Jordan normal form of an operator. Jordan normal form theorem.

3. Resultant and discriminant.

A.Sobolevski

No teaching this semester. Will give a course on Analysis for freshmen at the IUM in Spring 2012.

V.Timorin

Courses taught in 2011

In 2011, I have taught the following courses:

Analysis II (for 1st year students of the Independent University of Moscow)

Partial Differential Equations (for 3rd year math majors of the Higher School of Economics)

Seminar "Discrete dynamical systems" (organized jointly with P.Pushkar, for 2nd-3rd year math majors of the HSE)

Hamiltonian systems (for 3rd-4th year math majors of the HSE)

Seminar "Geometry and Dynamics" (organized jointly with A. Agrachev, A. Bufetov and A. Klimenko, based at the Steklov Mathematical Institute, open to all students).

Course syllabi:

Analysis II:

Norms on vector spaces

First differential

Inverse Function Theorem and Implicit Function Theorem

Multidimensional optimization problems

Basics of variational calculus

Differential forms in $\mathbb{R}^n$

Integrals of differential forms over chains

The Stokes theorem

Partial Differential Equations

First order PDEs, the initial value (Cauchy) problem

The Cauchy–Kowalevskaya theorem

Symbols of second order equations. Analytic normal forms for second order PDEs in the plane.

The wave equation: d'Alembert's and Fourier's methods.

Energy estimates and uniqueness theorems.

The Laplace and Poisson equations, Green's formulas.

Properties of harmonic and subharmonic functions, the maximum principle, Weierstrass' and Harnack's theorems.

Fundamental solutions of the Laplace equations, potentials.

Boundary value problems for Laplace equations.

The heat equation, the maximum principle.

Seminar "Discrete dynamical systems"

Symbolic dynamics, encoding.

Diffeomorphisms of the circle, rotation number, Arnold tongues.
Hyperbolic system, Anosov diffeomorphisms.
Isometries of the hyperbolic plane, cross-ratio.
Fuchsian groups and Riemann surfaces.
Billiards in polygons.
Billiards in ellipses and ellipsoids, complete integrability.
Invariant measures, Poincaré recurrence
Statistical properties of continued fractions and ergodic theory.

Hamiltonian systems
Geometric optics. Fermat's principle, generalized Snell's law.
The Huygens principle.
The Legendre transform. Hamiltonian equations.
The Hamilton–Jacobi theorem. Conservation of energy.
Manifolds and vector fields: a reminder.
Differential forms: a reminder.
Symplectic structures, Hamiltonian vector fields and flows.
Canonical invariants. The Poincaré recurrence theorem.
The Darboux linearization theorem.
Generating functions and the Hamilton-Jacobi method.
Complete integrability.

Seminar "Geometry and Dynamics"
The Rashevsky–Chow theorem.
How one surface rolls along another surface.
Euler's elastica and octonions.
Fractals, Julia sets, the Mandelbrot set.
Twisted rabbits.
Lattés' examples.
Classical orthogonal polynomials.
Interval exchange maps.
Quadratic maps, Hopf maps (e.g. complex, quaternionic, octonionic Hopf fibrations).
Dynamics of complex rational maps.
Cremona groups, Henon maps.

M.Verbitsky

A list of lecture courses

In 2011, I gave 4 lecture courses, "Kaehler manifolds and complex algebraic geometry" (10 lectures in Spring 2011 at Steklov Institute Educational Centre), "Mori Theory" (11 lectures at IUM, Fall 2011), "Basic algebraic geometry and commutative algebra" (9 lectures at Mathematical Faculty of HSE), "Geometric group theory", 4 lectures at the summer school of geometry and algebra (Yaroslavl).

"Kaehler manifolds and complex algebraic geometry"

It was a second part of a yearly course (19 lectures with special problem sets for every lecture and problem discussions after each lecture). The course's program:

1. Almost complex manifolds, complex manifolds, integrability
2. Vector bundles, connection, Levi-Civita connection
3. Kaehler manifolds. Kaehler condition and holonomy.
4. Sheaves and sheaf cohomology.
5. Poincare-Grothendieck-Dolbeault lemma. Positive currents and Cauchy kernel.
6. Kaehler relations and supersymmetry. Structure of the Hodge diamond.
7. Holomorphic vector bundles and Chern connection.
8. Kodaira-Nakano vanishing and Kodaira embedding theorem.
9. Holonomy, Ambrose-Singer theorem, Berger's list of holonomies.
10. Monge-Ampere equation and Calabi-Yau theorem
11. Clifford algebras, Bott periodicity for Clifford algebras, and spinorial representation
12. Spin-structures and Dirac operator
13. Elliptic operators, Weitzenböck formulas, Lichnerowicz-Weitzenböck formula.
14. Ricci curvature and canonical bundle. Geometry of Calabi-Yau manifolds. Bogomolov's decomposition theorem
15. Weierstrass preparation theorem. Structure of the ring of germs of analytic functions.
16. Noether's uniformization for germs of complex analytic varieties and Hilbert's Nullstellensatz.
17. Remmert and Remmert-Stein theorems. Chow's lemma and GAGA.

Mori Theory

Independent University, Fall 2011. Students: Artem Avilov, Roman Gaiduk, Svetlana Korobitsyna, Karine Kuyumzhiyan, Andrei Soldatenkov, Lev Sukhanov, Aleksandr Zakharov (also a few others I don't know personally).

1. Birational geometry and canonical bundle. Description of the minimal model program.
2. Kleiman's criterion of ampleness and Nakai-Moishezon theorem.
3. Minimal model program for surfaces.
4. Construction of a Hilbert scheme and Castelnuovo-Mumford regularity.
5. Mori's bend and break argument.
6. Mori's cone theorem.
7. Multiplier ideal sheaves and b-divisors.
8. Nadel vanishing theorem and Kawamata-Viehweg vanishing theorem.
9. Log-canonical centers and Kawamata's subadjunction.
10. Shokurov non-vanishing theorem
11. Kawamata base point free theorem

Basic algebraic geometry and commutative algebra

An entry-level course at HSE's math department, for students without any knowledge of geometry and complex analysis and very little algebra.

1. Basic set theory. Krull-van der Waerden's proof of Hilbert's Nullstellensatz over non-countable fields.
2. Category and functors. Equivalence of categories.
3. Strong Nullstellensatz. Localization. Equivalence of categories for affine varieties and finitely generated radical rings over $\mathbb{C}$.
4. Noetherian rings. Hilbert basis theorem. Irreducible varieties and irreducible decomposition.
5. Invariants of finite group actions and Noether's theorem.
6. Tensor product of modules and rings.
7. Finite morphisms and their properties.
8. Zariski topology, dominant morphisms and integral closures.
9. Finiteness of integral closures. The quotient space for finite group actions.

10. Normalization of affine varieties. Noether's normalization lemma.

Geometric group theory
4 lectures at a summer school in Yaroslavl, August 1-8, 2011.

1. Hahn-Banach theorem and amenability of abelian groups. Non-amenability of free groups, Tits alternative and Von Neuman problem.
2. Gromov-Hausdorff metric and its limits. Groups of polynomial growth and their amenability.
3. A sketch of Bruce Kleiner's proof of Gromov's theorem on group of polynomial growth.
4. Kazhdan's property T and Folner's theorem about properties of amenable groups.

2.3 Scientific conferences and seminar talks

S. Gusein-Zade

a) Conferences and Workshops.

1. IV Congress of the Turkic World Mathematical Society, Azerbaijan, Baku, July 1 - July 3. Member of the International Program Committee, plenary talk "Poincaré series of multi-index filtrations, integration with respect to the Euler characteristic and monodromy zeta functions".

2. Second International Conference and Workshop on Valuation Theory, Spain, Segovia-Escorial, July 18-29, 2011 (I was from July 18 till July 21). Invited talk "On equivariant versions of Poincaré series of filtrations and monodromy zeta functions".

3. CIMPA Research School "Singularities: Algebra, Geometry and Applications", Ukraina, Kiev, Ukraina, August 8-19, 2011 (I was from August 11 till 17). Series of 5 lectures "Poincaré series of filtrations, integrals with respect to the Euler characteristic, and monodromy zeta functions".

4. Interenational Conference "Analysis, Topology and Applications" in celebration of Professor A.S.Mishchenko's 70-th birthday, Harbin, China, August 22-26, 2011. Invited talk "Equivariant Saito duality and monodromy zeta functions of dual invertible polynomials".

5. International Workshop "Singularities and Monodromy". Germany, Hannover, October 6-7, 2011. Invited talk "Wolfgang Ebeling and indices of vector fields and 1-forms".

b) Seminar talks.

1. July 7, 2011, Germany, Hannover Leibniz University, Oberseminar: "Mirror symmetry of orbifold Landau-Ginzburg models".

2. July 12, 2011, Bielefeld University, Seminar: "Equivariant Saito duality and monodromy zeta functions of dual invertible polynomials".

A. Kuznetsov

- Conference ”Derived Categories 2011 Tokyo”, Tokyo, January 24–28, 2011.
- Conference ”Instantons in complex geometry”, Moscow, March 14–18, 2011.
- Conference ”Derived Categories”, Cambridge, April 11–15, 2011.
- Talks at Kings College and Imperial College, London, April 18, 2011.
- Visit to Bonn University with talk at Max-Planck Institute for Mathematics, April 19–22, 2011.
- Conference ”Homological Mirror Symmetry and Category Theory”, Split, Croatia, July 11–15, 2011.
- Conference “Automorphic Forms and Moduli Spaces”, CIRM, Luminy, October 10–14, 2011.
- Visit to Institut de Mathe'matiques de Jussieu with two talks, Paris, November 22–27.
- Visit to La Scuola Internazionale Superiore di Studi Avanzati di Trieste, Tireste, November 28–30.

G. Olshanski

- The Versatility of Integrability: Celebrating Igor Krichever's 60th Birthday, May 4–7, 2011, Columbia University, New York. Title of talk: *Integrable kernels and Markov dynamics.*
- International mathematical conference celebrating 50th anniversary of the Institute for Information Transmission Problems, July 25–29, 2011, Moscow. Title of talk: *Orthogonal polynomials in infinitely many variables and Markov processes.*

A. Penskoi

The above described results were presented in my talks on several seminars and conferences.

- Seminar on Geometry and Topology at the Weizmann Institute of Science, February 2011,
- Seminar on Spectral theory at the Université de Montréal, March 2011,

- International conference "Differential Equations and Related Topics" dedicated to Ivan G. Petrovskii, Moscow State University, June 2011.

I was also presenting my results from previous years at the conference

- La 88ème rencontre entre physiciens théoriciens et mathématiciens: Discrétisation en mathématiques et en physique, IRMA, Université de Strasbourg, September 2011.

A. Skopenkov

I delivered talks at research seminars at Faculty of Mechanics and Mathematics of the Lomonosov Moscow State University (headed by V. Buchstaber, N. Dolbilin, A. Mischenko, A. Raigorodskiy, I. Sabitov), at Faculty of Mathematics of Higher School of Economics (headed by M. Kazarian and S. Lando) and at Sharygin memorial seminar at MCCME (headed by V. Protasov).

E. Smirnov

Feb. 2011 Second school and conference on Lie algebras, algebraic groups, and invariant theory, Moscow, Russia;

Mar. 2011 Séminaire de géométrie, Université de Rennes 1, France;

Jul. 2011 Seminar über Gruppen und Geometrie, Universität Bielefeld, Germany;

Aug. 2011 Mini-course "Reflection groups" (three lectures), Summer school in algebra and algebraic geometry, Ekaterinburg, Russia;

A. Sobolevski

- *Applied Mathematics from Waves to Fluids* Nice, France, Feb 21–25, 2011, without a talk.
- *Search and Exploration* Cargèse, France, April 25–30, 2011, talk "Local and global search in transport optimization" (with J. Delon and J. Salomon)
- *Optimal Transport, algorithms and applications*, Institut Henri Poincaré, Paris, France, June 7–9, 2011, talk "From Bellman to Kardar–Parisi–Zhang: optimization and randomness in stochastic growth models" (with J. Delon and J. Salomon).
- *Extreme Value Statistics in Mathematics, Physics and Beyond*, talk "Cracks in $1 + 1$ ballistic deposition and discrete shocks in BurgersVtype equations" (with K. Khanin, S Nechaev, G. Oshanin, and O. Vasilyev).

- *Random Processes, Conformal Field Theory and Integrable Systems*, Moscow, September 19–23, 2011, talk "A random growth model in the perfect matching problem."

V. Timorin

February Mathematical Winter School of the HSE

February International Conference "Frontiers in Complex Dynamics", February 20 — February 25, Banff, Canada

March Texas Ergodic Theory Workshop, March 22 — March 23, Houston, USA

March Ahlfors-Bers Colloquium 2011, March 24 — March 27, Houston, USA

June International Conference "Polynomial Matings" June 8 — June 11, Toulouse, France

June Summer School "Dynamical Systems", June 25 — July 7, Dubna, Russia

July Summer School "Contemporary Mathematics", July 18 — July 29, Dubna, Russia

December Colloquium, University of Alabama at Birmingham, USA

M. Verbitsky

1. Complex and Riemannian Geometry: Extremal metrics: evolution equations and stability, 7.02.2011 - 11.02.2011, France, Luminy, with a talk "Extremal metrics in quternionic geometry".
2. Complex and Riemannian Geometry: Non-Kahlerian aspects of complex geometry, 21.02.2011 - 25.02.2011, France, Luminy, with a talk "Generalization of Inoue surfaces by Oeljeklaus-Toma and number theory".
3. Oberseminar Algebraische und Arithmetische Geometrie, University of Hannover, 24.03.2011, with a talk "Global Torelli theorem for hyperkaehler manifolds and the mapping class group"
4. Holomorphic symplectic varieties, Courant Institute, New York, 4.06.2011 - 5.06.2011, with a mini-course consisting of two talks: "Global Torelli theorem for hyperkehler manifolds" and "Subtwistor metric on the moduli of hyperkhler manifolds and its applications".
5. Oberseminar Algebraische und Arithmetische Geometrie, University of Hannover, 14.06.2011, with a talk "An intrinsic volume functional on almost complex 6-manifolds and nearly Kaehler geometry"

6. The Seventh Congress of Romanian Mathematicians, Brashov, Romania, 29.06.2011 - 5.07.2011, with a talk "Instanton bundles on $\mathbb{C}P^3$ and special holonomies"

7. French-Romanian Workshop on Complex Geometry, Bucharest, IMAR, 7.07.2011 - 9.07.2011, with a talk "Global Torelli theorem for hyperkaehler manifolds and the mapping class group"

8. Conference in honour of Fedor Bogomolov's 65th birthday, September 1-4, 2011, Moscow, Steklov institute, with a talk "Global Torelli theorem for hyperkahler manifolds"

9. Homological Mirror Symmetry and Category Theory, Split, Croatia, 11.07.2011 - 15.07.2011, with a talk "Global Torelli theorem for hyperkahler manifolds"

10. Moduli spaces and automorphic forms, 10.10.2011 - 14.10.2011, France, Luminy, with a talk "Any component of moduli of polarized hyperkahler manifolds is dense in its deformation space"

11. Complex geometry and uniformisation, 17.10.2011 - 21.10.2011, France, Luminy, with a talk "Morse-Novikov cohomology and Kodaira-type embedding theorem for locally conformally Kahler manifolds"

12. Algebraic geometry conference 19–23 December 2011, Chulalongkorn University, Bangkok, Thailand, with a talk "Global Torelli theorem for hyperkaehler manifolds".

3 Former winners of the Pierre Deligne and Dynasty contests

3.1 Research

I. Arzhantsev

[1] With A. Liendo.

Polyhedral divisors and $SL(2)$-actions on affine T-varieties.

Michigan Mathematical Journal, to appear. See also Prepublication de l'Institut Fourier, hal-00595725; arXiv:1105.4494v1, 26 pp.

In this paper we classify $SL(2)$-actions on normal affine T-varieties that are normalized by the torus T. This is done in terms of a combinatorial description of T-varieties given by Altmann and Hausen. The main ingredient is a generalization of Demazure's roots of the fan of a toric variety. As an application we give a description of special $SL(2)$-actions on normal affine varieties. We also obtain, in our terms, the classification of quasihomogeneous $SL(2)$-threefolds due to Popov.

[2] Torsors over Luna strata.

In "Torsors, étale homotopy and applications to rational points". Proceedings of the ICMS workshop in Edinburgh, 10-14 January 2011, London Mathematical Society Lecture Note Series, V. Batyrev and A. Skorobogatov, Editors, to appear. See also arXiv:1104.5581v1, 11 pp.

Let G be a reductive group and X be a Luna stratum on the quotient space $V//G$ of a rational G-module V. We consider torsors over X with both non-commutative and commutative structure groups. It allows us to compute the divisor class group and the Cox ring of a Luna stratum under mild assumptions. This techniques gives a simple cause why many Luna strata are singular along their boundary.

[3] With E.V. Sharoyko

Hassett-Tschinkel correspondence: Modality and projective hypersurfaces.

Journal of Algebra 348 (2011), no. 1, 217–232.

B. Hassett and Yu. Tschinkel (1999) introduced a remarkable correspondence between generically transitive actions of a commutative unipotent algebraic group G and finite-dimensional local algebras. In this paper we develop Hassett-Tschinkel correspondence and calculate modality of generically transitive G-actions on projective spaces, classify actions of modality one, and characterize generically transitive G-actions on projective hypersurfaces of given degree. In particular, actions on degenerate projective quadrics are studied.

[4] Flag varieties as equivariant compactifications of G_a^n.

Proceedings of the American Mathematical Society 139 (2011), no. 3, 783–786.

Let G be a semisimple affine algebraic group and P a parabolic subgroup of G. We classify all flag varieties G/P which admit an action of the commutative unipotent group G_a^n with an open orbit.

[5] With E.A. Makedonskii and A.P. Petravchuk.

Finite-dimensional subalgebras in polynomial Lie algebras of rank one.

Ukrainian Mathematical Journal 63 (2011), no. 5, 827–832.

Let $W_n(K)$ be the Lie algebra of derivations of the polynomial algebra $K[X] := K[x_1, ..., x_n]$ over an algebraically closed field K of characteristic zero. A subalgebra L of $W_n(K)$ is called polynomial if it is a submodule of the $K[X]$-module $W_n(K)$. We prove that the centralizer of every nonzero element in L is abelian provided L has rank one. This allows to classify finite-dimensional subalgebras in polynomial Lie algebras of rank one.

[6] With M. Zaidenberg.

Acyclic curves and group actions on affine toric surfaces.

arXiv:1110.3028v2, 29 pp. Submitted.

We show that every irreducible, simply connected curve on a toric affine surface X over the field of complex numbers is an orbit closure of a multiplicative group action on X. It follows that up to the action of the automorphism group $Aut(X)$ there are only finitely many non-equivalent embeddings of the affine line in X. A similar description is given for simply connected curves in the quotients of the affine plane by small finite linear groups. We provide also an analog of the Jung-van der Kulk theorem for affine toric surfaces, and apply this to study actions of algebraic groups on such surfaces.

L. Positselski

[1] Mixed Artin-Tate motives with finite coefficients.
Moscow Math. Journal 11, #2, p.317-402, 2011.

The goal of this paper is to give an explicit description of the triangulated categories of Tate and ArtinTate motives with finite coefficients $\mathbb{Z}/m$ over a field K containing a primitive m-root of unity as the derived categories of exact categories of filtered modules over the absolute Galois group of K with certain restrictions on the successive quotients. This description is conditional upon (and its validity is equivalent to) certain Koszulity hypotheses about the Milnor K-theory/Galois cohomology of K. This paper also purports to explain what it means for an arbitrary nonnegatively graded ring to be Koszul. Tate motives with integral coefficients are discussed in the "Conclusions" section.

[2] Two kinds of derived categories, Koszul duality, and comodule-contramodule correspondence.
Memoirs of the American Math. Society 212, #996, 2011. v+133pp.

The aim of this paper is to construct the derived nonhomogeneous Koszul duality. We consider the derived categories of DG-modules, DG-comodules, and DG-contramodules, the coderived and contraderived categories of CDG-modules, the coderived category of CDG-comodules, and the contraderived category of CDG-contramodules. The equivalence between the latter two categories (the comodule-contramodule correspondence) is established. Nonhomogeneous Koszul duality or "triality" (an equivalence between exotic derived categories corresponding to Koszul dual $(C)DG$-algebra and CDG-coalgebra) is obtained in the conilpotent and nonconilpotent versions. Various A-infinity structures are considered, and a number of model category structures are described. Homogeneous Koszul duality and $D-\Omega$ duality are discussed in the appendices.

[3] The algebra of closed forms in a disk is Koszul.
Electronic preprint arXiv:1007.5010 [math.KT], 9 pages. Accepted by Functional Analysis and its Applications in 2011.

We prove that the algebra of closed differential forms in an (algebraic, formal, or analytic) disk with logarithmic singularities along several coordinate hyperplanes is (both nontopologically and topologically) Koszul. The connection with variations of mixed Hodge-Tate structures, based on a preprint by Andrey Levin, is discussed in the introduction. 4.

Alexander Polishchuk and Leonid Positselski. Hochschild (co)homology of the second kind I. Electronic preprint arXiv:1010.0982 [math.CT], 67 pages. Accepted by Transactions of the American Math. Society in 2011.

We define and study the Hochschild (co)homology of the second kind (known also as the Borel-Moore Hochschild homology and the compactly supported Hochschild cohomology) for curved DG-categories. An isomorphism between the Hochschild (co)homology of the second kind of a CDG-category B and the same of the DG-category C of right CDG-modules over B, projective and finitely generated as graded B-modules, is constructed. Sufficient conditions for an isomorphism of the two kinds of Hochschild (co)homology of a DG-category are formulated in terms of the two kinds of derived categories of DG-modules over it. In particular, a kind of "resolution of the diagonal" condition for the diagonal CDG-bimodule B over a CDG-category B guarantees an isomorphism of the two kinds of Hochschild (co)homology of the corresponding DG-category C. Several classes of examples are discussed, and the case of matrix factorizations considered in detail.

[5] Artin-Tate motivic sheaves with finite coefficients over an algebraic variety.
Electronic preprint arXiv:1012.3735 [math.KT], 33 pages.

We propose a construction of a tensor exact category F_X^m of Artin-Tate motivic sheaves with finite coefficients $\mathbb{Z}/m$ over an algebraic variety X (over a field K of characteristic prime to m) in terms of etale sheaves of $\mathbb{Z}/m$-modules over X. Assuming the existence of triangulated categories of motivic sheaves $DM(X, \mathbb{Z}/m)$ over algebraic varities X over K and a weak version of the "six operations" in these categories, we identify F_X^m with the exact subcategory in $DM(X, \mathbb{Z}/m)$ consisting of all the iterated extensions of the Tate twists $M_c c^m(Y/X)(j)$ of the cohomological relative motives with compact support $M_c c^m(Y/X)$ of varieties Y quasi-finite over X. An isomorphism of the $\mathbb{Z}/m$-modules Ext between the Tate motives $\mathbb{Z}/m(j)$ in the exact category F_X^m with the motivic cohomology modules predicted by the Beilinson-Lichtenbaum etale descent conjecture (recently proven by Voevodsky, Rost, et al.) holds for smooth varieties X over K if and only if the similar isomorphism holds for Artin-Tate motives over fields containing K. When K contains a primitive m-root of unity, the latter condition is equivalent to a certain Koszulity hypothesis, as it was shown in our previous paper.

[6] Coherent analogues of matrix factorizations and relative singularity categories.
Electronic preprint arXiv:1102.0261 [math.CT], 67 pages.

We define the triangulated category of relative singularities of a closed subscheme in a scheme. When the closed subscheme is a Cartier divisor, we consider matrix factorizations of the related section of a line bundle, and their analogues with locally free sheaves replaced by coherent ones. The appropriate exotic derived category of coherent matrix factorizations is then identified with the triangulated category of relative singularities, while the similar exotic derived category of locally free matrix factorizations is its full subcategory. The latter category is identified with the kernel of the direct image functor corresponding to the closed embedding of the zero locus and acting between the conventional (absolute) triangulated categories of singularities. Similar results are obtained for matrix factorizations of infinite rank; and two different "large" versions of the triangulated category of singularities, due to Orlov and Krause, are identified in the case of a divisor in a smooth scheme. Contravariant (coherent) and covariant (quasi-coherent) versions of the Serre-Grothendieck duality theorems for matrix factorizations are established, and pull-backs and push-forwards of matrix factorizations are discussed at length. A number of general results about derived categories of the second kind for CDG-modules over quasi-coherent CDG-algebras are proven on the way.

D. Talalaev

Quantum generic Toda system, accepted for publication in Theor. Math. Phys. in 2011, arXiv:1012.3296.

The Toda chains take a particular place in the theory of integrable systems, in contrast with the linear group structure for the Gaudin model this system is related to the corresponding Borel group and mediately to the geometry of flag varieties. The main goal of this paper is to reconstruct a spectral curve in a wider context of the generic Toda system [1]. This appears to be an efficient way to find its quantization which is obtained here by the technique of quantum characteristic polynomial for the Gaudin model [2] and an appropriate AKS reduction. We discuss also some relations of this result with the recent consideration of the Drinfeld Zastava space [3], the monopole space and corresponding Borel Yangian symmetries [4].

3.2 Scientific conferences

I. Arzhantsev

[1] Session of Moscow Mathematical Society, 13 December 2011
talk "Local algebras and additive structures on projective varieties"

[2] International Conference "Toric Topology 2011 in Osaka", Japan, 28 November – 01 December 2011
talk "The automorphism group of a variety with torus action of complexity one"

[3] Sobolev Institute of Mathematics, Novosibirsk, Russia, August – September 2011; mini-course "Toric Geometry and Automorphisms of Affine Varieties" (3 lectures)

[4] Seminar "Geometry, Topology, and their Applications", Sobolev Institute of Mathematics, Novosibirsk, Russia, 24 August 2011; talk "Flexible Affine Algebraic Varieties"

[5] 3rd Summer Workshop on Algebraic Geometry, Tübingen, Germany, 21-22 July 2011

[6] Seminar on Algebra and Number Theory, University of Cologne, Germany, 12 July 2011; talk "Flexible Varieties and Automorphism Groups"

[7] Conference on Birational Transformation and Automorphisms of Affine Varieties, Basel, Switzerland, 20-22 June 2011

[8] Seminar on Algebra and Geometry, University of Basel, Switzerland, 13 May 2011; talk "Factorial algebraic group actions and categorical quotients"

[9] Journées de Géométrie Affine á Grenoble, France, 9-10 May 2011;
talk "Flexible Varieties and Cox Rings"

[10] Conference "Torsors: Theory and Applications", Edinburgh, Great Britain, 10-14 January 2011; talk "Two Applications of Cox Rings in Affine Geometry"

Work in Scientific Centers and International Groups

In the spring I visited Institute Fourier, Grenoble, France and collaborated with Mikhail Zaidenberg and Alvaro Liendo. In the summer I visited the University of Tuebingen, Germany and collaborated with Juergen Hausen. We were working, in particular, on our joint book project

L. Positselski

I participated in the workshop "New developments in noncommutative algebra and its applications" on Isle of Skye, Scotland, from June 26 to July 2, 2011. I did not give any talks there.

See `http://icms.org.uk/workshops/noncomalg`

I participated in the "International conference on Algebra and Geometry" in Yekaterinburg, Russia, August 22–27, 2011. I gave a 50-minute plenary talk titled "Absolute Galois groups and their properties".

See `http://algebra.imm.uran.ru`

I participated in the workshop "Derived categories in Algebraic Geometry", September 5–9, 2011, organized by the Laboratory of Algebraic Geometry and its Applications (headed by F. Bogomolov) in Moscow. I gave a 50-minute talk titled "Matrix factorizations and exotic derived categories".

See `http://bogomolov-lab.ru/DC-2011/workshop.html` and
`http://www.mathnet.ru/php/presentation.phtml?option_lang=rus&presentid=3709`

I gave a one-hour talk at the "Dynasty foundation Christmas mathematical meetings" (presided by P. Deligne) at the Independent University of Moscow on January 10, 2011. The talk was titled "Artin–Tate motivic sheaves with finite coefficients over a smooth variety".

See `http://www.mccme.ru/dfc/2011/Christmas_meetings_abstracts-2011.pdf` (Russian)

I gave a two-hour talk at the weekly seminar of the Laboratory Algebraic Geometry and its Applications at the Mathematics Department of the Higher School of Economics in Moscow on January 21, 2011. The talk was titled "Artin–Tate motivic sheaves with finite coefficients over a smooth variety".

See http://bogomolov-lab.ru/seminar.html (Russian)

I gave a two-hour talk at the Shafarevich seminar at the Steklov Math. Institute in Moscow on February 15, 2011. The talk was titled "Triangulated categories of relative singularities of Cartier divisors".

See http://www.mathnet.ru/php/seminars.phtml?option_lang=rus&presentid=3008 (Russian)

I gave a two-hour talk at the seminar "Homological and homotopical methods in Geometry" at the Mathematics Department of the Higher School of Economics in Moscow on November 2, 2011. The talk was titled "Derived categories of matrix factorizations".

See http://www.mathnet.ru/php/seminars.phtml?option_lang=rus&presentid=3967 (Russian)

I gave a 75-minute talk at the D. K. Faddeev St. Petersburg Algebraic Seminar at the Petersburg Division of Steklov Math. Institute on November 21, 2011. The talk was titled "Semi-infinite homology of associative algebraic structures".

D. Talalaev

1. Quantum Theory and Symmetries (QTS-7), Prague, Czech Republic, in August 7-13, 2011.
2. School and conference Geometry and Quantization Institute of Mathematics of Chinese Academy of Sciences in Beijing, September 4-10, 2011, Chern Institute of Mathematics in Tianjin, September 11-17, 2011.
3. Conference "Differential equations and related topics", Moscow, MSU,30.05-4.06.11.
4. Conference "Classical and quantum integrable systems", 24-27 January, Protvino, Institute of the High Energy Physics.