The IUM report to the Simons foundation, 2012

The Simons foundation supported two programs launched by the IUM:
Simons stipends for students and graduate students;
Simons IUM fellowships.

12 applications were received for the Simons stipends contest. The selection committee consisting of *Yu.Ilyashenko (Chair), G.Dobrushina, G.Kabatyanski, S.Lando, I.Paramonova (Academic Secretary), A.Sossinsky, M.Tsfasman* awarded Simons stipends for 2012 year to the following students and graduate students:

1. Bazhov, Ivan Andreevich
2. Bibikov, Pavel Vital'evich
3. Bufetov, Alexei Igorevich
4. Netai, Igor Vitalevich
5. Ustinovsky, Yuri Mikhailovich
6. Fedotov, Stanislav Nikolaevich

27 applications were received for the Simons IUM fellowships contest. The selection committee consisting of *Yu.Ilyashenko (Chair), G.Dobrushina, B.Feigin, I.Paramonova (Academic Secretary), A.Sossinsky, M.Tsfasman, V.Vassiliev* awarded

Simons IUM-fellowships for the first half year of 2012 to the following researches:

1. Kuznetsov, Alexander Gennad'evich
2. Natanzon, Sergei Mironovich
3. Penskoi, Alexei Victorovich
4. Rybnikov, Leonid Grigorevich
5. Sadykov, Timur Mradovich

6. Skopenkov, Arkady Borisovich
7. Smirnov, Evgeni Yur'evich
8. Sobolevski, Andrei Nikolaevich
9. Zykin, Alexei Ivanovich

Simons IUM-fellowships for the second half year of 2012 to the following researches:

1. Arzhantsev, Ivan Vladimirovich
2. Bufetov, Alexander Igorevich
3. Feigin, Evgeny Borisovich
4. Gusein-Zade, Sabir Medzhidovich
5. Kuznetsov, Alexander Gennad'evich
6. Olshanski, Grigori Iosifovich
7. Prokhorov, Yuri Gennadevich
8. Skopenkov, Mikhail Borisovich
9. Timorin, Vladlen Anatol'evich
10. Vyugin, Ilya Vladimirovich

The report below is split in two sections corresponding to the two programs above. The first subsection in each section is a report on the research activities. It consists of the titles of the papers published or submitted in the year of 2012, together with the corresponding abstracts. The second subsection of each section is devoted to conference and some most important seminar talks. The last subsection of the second section is devoted to the syllabi of the courses given by the winners of the Simons IUM fellowships. Most of these courses are innovative, as required by the rules of the contest for the Simons IUM fellowships.

The support of the Simons foundation have drastically improved the financial situation at the IUM, and the whole atmosphere as well. On behalf of the IUM, I send my deep gratitude and the best New year wishes to Jim Simons, David Eisenbud, with whom the program was started, Yuri Tschinkel, with whom the program is run, and the whole team of the Simons foundation.

Yulij Ilyashenko

President of the Independent University of Moscow

1 Program: Simons stipends for students and graduate students

1.1 Research

Ivan Bazhov

[1] With I. V. Arzhantsev
On orbits of the automorphism group on an affine toric variety
arXiv:1203.2902 *to appear in Central European Journal of Mathematics.*

Let X be an affine toric variety. The total coordinates on X provide a canonical presentation of X as a quotient of a vector space by a linear action of a quasitorus. We prove that the orbits of the connected component of the automorphism group Aut(X) on X coincide with the Luna strata defined by the canonical quotient presentation.

Pavel Bibikov

[1] With V. Lychagin
Classification of linear action of algebraic groups on the spaces of homogenous forms
Doklady Mathematics, 2012, Vol. 85, No. 1, pp. 109–112.

This paper studies the orbits of an algebraic group G acting on the space of rational forms in many variables by linear changes of coordinates. The main result of the paper consists in finding the field of differential invariants of such an action and obtaining an effective criterion for distinguishing between the orbits of forms with nonzero Hessian.

[2] On affine classification of functions and foliations on the plane
Lobachevskii Journal of Mathematics, 2012, Vol. 33, No. 2, pp. 115–122.

In this paper we study problems of classifications of smooth and rational functions and foliations with respect to the actions of affine group SA(2) on them. Algebras of differential invariants of these actions are found and criteria of equivalence are obtained.

[3] Pseudogroup action of point transformations on space of smooth functions $C^\infty(J^1\mathbb{R})$
The Works of the Lobachevskii Mathematical Center, 2012, Vol. 45, pp. 25–27.

In this paper we study differential invariants and invariant derivatives for the pseudogroup action of point transformations on space of smooth functions $C^\infty(J^1\mathbb{R})$.

[4] On trivialization of symbols of linear differential operators
Izvestiya PGPU. Mathematics, 2012, Vol. 30, pp. 20–27.

In this paper we consider the problem of trivialization for symbols of linear differential operators. The criterion of triviality for regular symbols is proved and applications for web theory, symmetric differential forms and Abel equations are obtained. Methods used in work include theory of differential equations, jet spaces and differential invariants (recently used in classification of homogeneous forms) on the one hand, and differential geometry and geometry of tenzor operator fields on the other hand.

[5] Classification of rational functions in symplectic and metric spaces
To appear in Izvestiya VUZov. Mathematics.

Problems of symplectic and orthogonal classifications of rational functions are considered. The main idea is the use of methods from differential geometry and geometric theory of jet spaces. Group actions on the infinite jet space are considered. This makes it possible to find the fields of differential invariants. Finally, is is proved that the dependencies between basic differential invariants and their derivatives completely determine the orbit of the corresponding function.

[6] On automorphic systems of differential equations and $\mathrm{GL}_2(\mathbb{C})$-orbits of binary forms
To appear in Ufimskii Mathematical Journal

In the work we introduce new method for studying classical algebraic problem of classifying $\mathrm{GL}_2(\mathbb{C})$-orbits of binary forms with the help of differential equations. We construct and study automorphic system of differential equations $\mathcal{S}$ of the fourth order, whose solution space coincides with $\mathrm{GL}_2(\mathbb{C})$-orbit of fixed binary form f. In cases of order 2 and 3 system $\mathcal{S}$ is integrable. In the most difficult case of order 4 we prove that system $\mathcal{S}$ may be reduced to the system of the differential Abel equation and the linear partial differential equation of order 1.

Alexei Bufetov.

[1] A central limit theorem for extremal characters of the infinite symmetric group
Functional Analysis and Its Applications, 2012, 46:2, 83-93.

The asymptotics of the first rows and columns of random Young diagrams corresponding to extremal characters of the infinite symmetric group is studied. We consider rows and columns with linear growth in n, the number of boxes of random diagrams, and prove the central limit theorem for them in the case of distinct Thoma parameters. We also establish a more precise statement relating the growth of rows and columns of Young diagrams to a simple independent random sampling model.

[2] With A. Borodin
A central limit theorem for Plancherel representations of the infinite-dimensional unitary group

Zapiski Nauchnykh Seminarov POMI, Vol. 403, 2012, pp. 19-35.

We study asymptotics of traces of (noncommutative) monomials formed by images of certain elements of the universal enveloping algebra of the infinite-dimensional unitary group in its Plancherel representations. We prove that they converge to (commutative) moments of a Gaussian process that can be viewed as a collection of simply yet nontrivially correlated two-dimensional Gaussian Free Fields. The limiting process has previously arisen via the global scaling limit of spectra for submatrices of Wigner Hermitian random matrices.

[3] Kerov's interlacing sequences and random matrices
arXiv:1211.1507, preprint.

To a $N \times N$ real symmetric matrix Kerov assigns a piecewise linear function whose local minima are the eigenvalues of this matrix and whose local maxima are the eigenvalues of its $(N-1) \times (N-1)$ submatrix. We study the scaling limit of Kerov's piecewise linear functions for Wigner and Wishart matrices. For Wigner matrices the scaling limit is given by the Verhik-Kerov-Logan-Shepp curve which is known from asymptotic representation theory. For Wishart matrices the scaling limit is also explicitly found, and we explain its relation to the Marchenko-Pastur limit spectral law.

Igor Netai

[1] Syzygy algebras for the Segre embedding
arXiv:1108.3733, *submitted to in Functional Analysis and Its Applications.*

In this paper A_∞-structures for the Segre embeddings are calculated. Also relations with canonical resolutions of sheaves on $\mathbb{P}(U) \times \mathbb{P}(V) \to \mathbb{P}(U \otimes V)$ are found. All resolutions are equivariant w. r. t. the action of $\mathrm{GL}(U) \times \mathrm{GL}(V)$.

Yuri Ustinovsky

[1] On almost free torus actions and Horrocks conjecture
Far Eastern Mathematical Journal, 12:1 (2012), 98–107
arXiv:1203.3685

We construct a model for cohomology of a space X equipped with a torus T action, whose homotopy orbit space X_T is formal. This model represents Koszul complex of its equivariant cohomology. Studying homological properties of modules over polynomial ring we derive new bounds on homological rank (dimension of cohomology ring) of X equipped with almost free torus action. We give a proof of toral rank conjecture for spaces with formal quotient in the case of torus dimension ≤ 5.

Stanislav Fedotov

[1] Semi-invariants of 2-representations of quivers
Math. Notes, 2012, Vol. 92, no. 1-2, pp. 99-107

In the paper, an analog of the Procesi-Razmyslov theorem for the algebra of semi-invariants of representations of an arbitrary quiver with dimension vector (2, 2, ..., 2) is obtained.

[2] Framed moduli spaces and tuples of operators
Fundamental‘naya i Prikladnaya Matematika, 2012, Vol. 17, no. 5, pp. 187209

In this work we address the problem of classifying tuples of linear operators and linear functions on a finite dimensional vector space up to base change. It is considered in a broader framework of theory of of framed representations. As the main result constructed is a complete classification of such tuples in a Zariski open subset. We introduce families of normal forms and an embedding of the moduli space into a projective space.

1.2 Scientific conferences and seminar talks

Ivan Bazhov

[1] Conference “Algebra and Geometry”, Moscow, June, 4 – 9
Talk “On orbits of the automorphism group on a complete or affine toric variety”
[2] Visit to Geneve, June
Talk “Authomorphisms of toric varieties” (Universite de Geneva)
[3] School “Algebra and Geometry”, Yaroslavl, August, 25 – 31
[4] School “Subgroups of Cremona groups”, Poland, September, 23 – 30

Pavel Bibikov

[1] Conference “Lomonosov”, Moscow, April 10–15
Talk “Classification of rational Hamiltonians in symplectic spaces”
[2] Conference “Probability theory and its applications”, Moscow, June 26–30
Talk “On Arnold randomness of integer sequences”
[3] Conference “Geometric methods in physics and control theory”, Odessa, July 15–25
Talk “Classification of binary forms with control parameter”
[4] Conference “Algebra and geometry of PDE’s”, Tromso, August 26–31
Talk “Differential invariants and classification of linear actions on homogeneous forms”
[5] Conference “Lobachevskii readings”, Kazan, November 1–6
Talk “Point transformations of smooth functions $C^\infty(J^1\mathbb{R})$”
[6] Seminar “Modern problems in number theory”

Talk “Differential invariants of affine actions of algebraic groups”

Alexei Bufetov.

[1] Visit to Massachusetts Institute of Technology, Boston, April, 23 — May, 6.

[2] Visit to Alfred Renyi Institute of Mathematics, Budapest, November, 1 - November, 10.

Igor Netai

[1] Conference “Birational and affine geometry”, Moscow, April, 23–27.

[2] Conference “Algebra and Geometry”, Moscow, June, 4–9.

[3] Traditional counrtyside workshop on dacha of Valera Lunts, July, 22–29.

[4] The first conference on algebraic geometry, particle physics and string theory “Relation of String Theory to Gauge Theories and Moduli Problems of Branes”, Moscow, September, 10–14.

[5] Workshop “Homological projective duality and non-commutative geometry”, October, 8–14, 2012, Coventry (Warwick university).

Talk “On syzygy A_∞-algebras of highest weight orbits”

[6] Summer conference “Lie algebras, algebraic group and invariant theory”, Tolyatti, June, 24–31

Talk “A_∞-algebras of the Segre embeddings”

[7] One-day conference in honour of 61 anniversary of K. Shramov and S. Galkin, Moscow, September, 28.

[8] Iskovskikh Seminar, Moscow, October, 4.

Talk “Mori dream spaces of Calabi–Yau type and the log canonicity of the Cox rings (following the paper of Yujiro Kawamata and Shinnosuke Okawa) ”

Yuri Ustinovsky

[1] Conference “Toric topology meeting”, Osaka, November 16 – 19

Talk “Complex geometry of moment-angle-manifolds”.

[2] Conference “Tranformation groups”, Tokyo, November 23 – 35

[3] Conference “Alexandroff readings”, Moscow, May 21 – 25

Talk “On almost free torus actions and Horrocks conjecture”

[4] Seminar “Geometry, topology and mathematical physics” at Moscow State University

Talk “On almost free torus actions and Horrocks conjecture”

Stanislav Fedotov

[1] Workshop and International Conference on Representations of Algebras “ICRA 2012”, Bielefeld, August, 8 – 17

Talk “Framed moduli spaces, Grassmannians and tuples of operators”

[2] Conference “Lie algebras, algebraic groups and invariant theory”, Tolyatti, June, 25 – 30

Talk “Moduli spaces of framed representations of quivers and classification of tuples of operators”

2 Program: Simons IUM fellowships

2.1 Research

Ivan Arzhantsev

[1] With A. Liendo. Polyhedral divisors and SL_2-actions on affine T-varieties. Michigan Mathematical Journal, 2012, Vol. 61, No. 4, pp. 731–762.

In this paper we classify SL_2-actions on normal affine T-varieties that are normalized by the torus T. This is done in terms of a combinatorial description of T-varieties given by Altmann and Hausen. The main ingredient is a generalization of Demazure's roots of the fan of a toric variety. As an application we give a description of special SL_2-actions on normal affine varieties. We also obtain, in our terms, the classification of quasihomogeneous SL_2-threefolds due to Popov.

[2] With K. Kuyumzhiyan and M. Zaidenberg. Flag varieties, toric varieties, and suspensions: three instances of infinite transitivity. Mat. Sbornik, 2012, Vol. 203, No. 7, pp. 3–30; English transl.: Sbornik: Math., 2012, Vol. 203, No. 7, pp. 923–949.

We say that a group G acts infinitely transitively on a set X if for every $m \in \mathbb{N}$ the induced diagonal action of G is transitive on the cartesian mth power $X^m \setminus \Delta$ with the diagonals removed. We describe three classes of affine algebraic varieties such that their automorphism groups act infinitely transitively on their smooth loci. The first class consists of normal affine cones over flag varieties, the second of non-degenerate affine toric varieties, and the third of iterated suspensions over affine varieties with infinitely transitive automorphism groups of a reinforced type.

[3] With H. Flenner, S. Kaliman, F. Kutzschebauch, and M. Zaidenberg. Flexible varieties and automorphism groups. arXiv:1011.5375, *to appear in Duke Mathematical Journal.*

Given an irreducible affine algebraic variety X of dimension ≥ 2, we let $\mathrm{SAut}(X)$ denote the special automorphism group of X i.e., the subgroup of the full automorphism group $\mathrm{Aut}(X)$ generated by all one-parameter unipotent subgroups. We show that if $\mathrm{SAut}(X)$

is transitive on the smooth locus X_{reg} then it is infinitely transitive on X_{reg}. In turn, the transitivity is equivalent to the flexibility of X. The latter means that for every smooth point $x \in X_{\text{reg}}$ the tangent space T_xX is spanned by the velocity vectors at x of one-parameter unipotent subgroups of $\text{Aut}(X)$. We provide also various modifications and applications.

[4] With H. Flenner, S. Kaliman, F. Kutzschebauch, and M. Zaidenberg. Infinite transitivity on affine varieties. arXiv:1210.6937, *to appear in: Birational Geometry, Rational Curves, and Arithmetic – Simons Symposium 2012, F. Bogomolov, B. Hassett, and Yu. Tschinkel, Editors, Springer Verlag.*

In this paper we survey recent results on automorphisms of affine algebraic varieties, infinitely transitive group actions and flexibility. We present related constructions and examples, and discuss geometric applications and open problems.

[5] With I. Bazhov. On orbits of the automorphism group on an affine toric variety. arXiv:1203.2902, *to appear in Central European Journal of Mathematics.*

Let X be an affine toric variety. The total coordinates on X provide a canonical presentation $\overline{X} \to X$ of X as a quotient of a vector space $\overline{X}$ by a linear action of a quasitorus. We prove that the orbits of the connected component of the automorphism group $\text{Aut}(X)$ on X coincide with the Luna strata defined by the canonical quotient presentation.

[6] Torsors over Luna strata. arXiv:1104.5581, *to appear in "Torsors, 'etale homotopy and applications to rational points". Proceedings of the ICMS workshop in Edinburgh, 10-14 January 2011, London Mathematical Society Lecture Note Series, A.Skorobogatov, Editor, Cambridge University Press.*

Let G be a reductive group and X_H be a Luna stratum on the quotient space $V//G$ of a rational G-module V. We consider torsors over X_H with both non-commutative and commutative structure groups. It allows us to describe the divisor class group and the Cox ring of a Luna stratum under mild assumptions. This approach gives a simple cause why many Luna strata are singular along their boundary.

[7] With M. Zaidenberg. Acyclic curves and group actions on affine toric surfaces. arXiv:1110.3028, *to appear in: Proceedings of the Conference on Affine Algebraic Geometry in Osaka (2011), World Scientific Publishing Co.*

We show that every irreducible, simply connected curve on a toric affine surface X over $\mathbb{C}$ is an orbit closure of a $\mathbb{G}_m$-action on X. It follows that up to the action of the automorphism group $\text{Aut}(X)$ there are only finitely many non-equivalent embeddings of the affine line $\mathbb{A}^1$ in X. A similar description is given for simply connected curves in the quotients of the affine plane by small finite linear groups. We provide also an analog of the Jung-van der Kulk theorem for affine toric surfaces, and apply this to study actions of algebraic groups on such surfaces.

[8] With J. Hausen, E. Herppich, and A. Liendo. The automorphism group of a variety with torus action of complexity one. arXiv:1202.4568, *submitted to Moscow Mathematical Journal.*

We consider a normal complete rational variety with a torus action of complexity one. In the main results, we determine the roots of the automorphism group and give an explicit description of the root system of its semisimple part. The results are applied to the study of almost homogeneous varieties. For example, we describe all almost homogeneous (possibly singular) del Pezzo $\mathbb{K}^*$-surfaces of Picard number one and all almost homogeneous (possibly singular) Fano threefolds of Picard number one having a reductive automorphism group with two-dimensional maximal torus.

Alexander Bufetov

[1] A. I. Bufetov, Limit theorems for translation flows, Annals of Mathematics, published online at http://annals.math.princeton.edu/articles/7111 paper issue pending

The aim of this paper is to obtain an asymptotic expansion for ergodic integrals of translation flows on flat surfaces of higher genus (Theorem 1) and to give a limit theorem for these flows (Theorem 2).

[2] A. Bufetov, B. Solomyak, Limit theorems for self-similar tilings , 2012, 36 pp., *to appear in Comm. Math.Phys.*, arXiv: 1201.6092

We study deviation of ergodic averages for dynamical systems given by self-similar tilings on the plane and in higher dimensions. The main object of our paper is a special family of finitely-additive measures for our systems. An asymptotic formula is given for ergodic integrals in terms of these finitely-additive measures, and, as a corollary, limit theorems are obtained for dynamical systems given by self-similar tilings.

[3] A. Bufetov, On multiplicative functionals of determinantal processes"

Rus.Math.Surveys , 67 :1(403) (2012), 177-178 .

It is shown that a determinantal measure times a multiplicative functional is again determinantal.

[4] A. I. Bufetov, A. V. Klimenko, Maximal inequality and ergodic theorems for Markov groups, Proc. Steklov Inst. Math. , 277 (2012), 27-42

Proof of the maximal inequality for actions of free groups.

[5] A. I. Bufetov, A. V. Klimenko, On Markov operators and ergodic theorems for group actions, European Journal of Combinatorics , 33 :7 (2012), 1427-1443

A survey of the method of Markov operators in the study of actions of free groups.

[6] A. I. Bufetov, M. Khristoforov, A. Klimenko, Cesro convergence of spherical averages for measure-preserving actions of Markov semigroups and groups, International Mathematics Research Notices , 2012 :21 (2012), 47974829.

Cesaro convergence of spherical averages is proven for measure-preserving actions of Markov semigroups and groups. Convergence in the mean is established for functions in L^p, $1 \leq p < \infty$, and pointwise convergence for functions in L^∞. In particular, for measure-preserving actions of word hyperbolic groups (in the sense of Gromov) we obtain Cesaro convergence of spherical averages with respect to any symmetric set of generators.

[7] J. Athreya, A. Bufetov, A. Eskin, M. Mirzakhani, Lattice point asymptotics and volume growth on Teichmller space, Duke Mathematical Journal , 161 :6 (2012), 1055-1111, arXiv: math/0610715

We apply some of the ideas of the Ph.D. Thesis of G. A. Margulis to Teichmuller space. Let x be a point in Teichmuller space, and let $B_R(x)$ be the ball of radius R centered at x (with distances measured in the Teichmuller metric). We obtain asymptotic formulas as R tends to infinity for the volume of $B_R(x)$, and also for for the cardinality of the intersection of $B_R(x)$ with an orbit of the mapping class group.

[8] A. I. Bufetov, On the VershikKerov Conjecture Concerning the ShannonMcMillan-Breiman Theorem for the Plancherel Family of Measures on the Space of Young Diagrams, Geometric and Functional Analysis , 22 :4 (2012), 938975, arXiv: 1001.4275

Vershik and Kerov conjectured in 1985 that dimensions of irreducible representations of finite symmetric groups, after appropriate normalization, converge to a constant with respect to the Plancherel family of measures on the space of Young diagrams. The statement of the Vershik-Kerov conjecture can be seen as an analogue of the Shannon-McMillan-Breiman Theorem for the non-stationary Markov process of the growth of a Young diagram. The limiting constant is then interpreted as the entropy of the Plancherel measure. The main result of the paper is the proof of the Vershik-Kerov conjecture. The argument is based on the methods of Borodin, Okounkov and Olshanski.

Evgeny Feigin

[1] $\mathbb{G}_a^M$ degeneration of flag varieties
Selecta Mathematica, 2012, Vol. 18 (3), pp. 513–537.

Let $\mathcal{F}_\lambda$ be a generalized flag variety of a simple Lie group G embedded into the projectivization of an irreducible G-module V_λ. We define a flat degeneration $\mathcal{F}_\lambda^a$, which is a $\mathbb{G}_a^M$ variety. Moreover, there exists a larger group G^a acting on $\mathcal{F}_\lambda^a$, which is a degeneration of the group G. The group G^a contains $\mathbb{G}_a^M$ as a normal subgroup. If G is of type A, then the degenerate flag varieties can be embedded into the product of Grassmanians and thus to the product of projective spaces. The defining ideal of $\mathcal{F}_\lambda^a$ is generated by the set of degenerate Plücker relations. We prove that the coordinate ring of $\mathcal{F}_\lambda^a$ is isomorphic to a direct sum of dual PBW-graded $\mathfrak{g}$-modules. We also prove that there exist bases in multi-homogeneous components of the coordinate rings, parametrized by the semistandard PBW-tableux, which are analogues of semistandard tableux.

[2] Systems of correlation functions, coinvariants and the Verlinde algebra
Funkts. Anal. Prilozh., 2012, Vol. 46 (1), pp. 4964

We study the Gaberdiel-Goddard spaces of systems of correlation functions attached to an affine Kac-Moody Lie algebra $\widehat{\mathfrak{g}}$. We prove that these spaces are isomorphic to the spaces of coinvariants with respect to certain subalgebras of $\widehat{\mathfrak{g}}$. This allows to describe the Gaberdiel-Goddard spaces as direct sums of tensor products of irreducible $\mathfrak{g}$-modules with multiplicities given by fusion coefficients. We thus reprove and generalize Frenkel-Zhu's theorem.

[3] With G. Cerulli Irelli and M. Reineke
Quiver Grassmannians and degenerate flag varieties
Algebra & Number Theory, 2012, Vol. 6 (1) (2012), pp. 165–194

Quiver Grassmannians are varieties parametrizing subrepresentations of a quiver representation. It is observed that certain quiver Grassmannians for type A quivers are isomorphic to the degenerate flag varieties investigated earlier by the second named author. This leads to the consideration of a class of Grassmannians of subrepresentations of the direct sum of a projective and an injective representation of a Dynkin quiver. It is proven that these are (typically singular) irreducible normal local complete intersection varieties, which admit a group action with finitely many orbits, and a cellular decomposition. For type A quivers explicit formulas for the Euler characteristic (the median Genocchi numbers) and the Poincare polynomials are derived.

[4] The median Genocchi numbers, Q-analogues and continued fractions
European Journal of Combinatorics, 2012, Vol. 33, pp. 1913–1918.

The goal of this paper is twofold. First, we review the recently developed geometric approach to the combinatorics of the median Genocchi numbers. The Genocchi numbers appear in this context as Euler characteristics of the degenerate flag varieties. Second, we prove that the generating function of the Poincaré polynomials of the degenerate flag varieties can be written as a simple continued fraction. As an application we prove that the Poincaré polynomials coincide with the q-version of the normalized median Genocchi numbers introduced by Han and Zeng.

[5] Degenerate SL_n: representations and flag varieties
arXiv:1202.5848, *to appear in Functional Analysis and Its Applications*.

The degenerate Lie group is a semidirect product of the Borel subgroup with the normal abelian unipotent subgroup. We introduce a class of the highest weight representations of the degenerate group of type A, generalizing the PBW-graded representations of the classical group. Following the classical construction of the flag varieties, we consider the closures of the orbits of the abelian unipotent subgroup in the projectivizations of the representations. We show that the degenerate flag varieties $\mathcal{F}_n^a$ and their desingularizations R_n can be obtained via this construction. We prove that the coordinate ring of R_n is isomorphic to the direct sum of duals of the highest weight representations of the degenerate group. In the end, we state several conjectures on the structure of the highest weight representations.

[6] With G. Cerulli Irelli and M. Reineke
Degenerate flag varieties: moment graphs and Schröder numbers
arXiv:1206.4178, *to appear in Journal of Algebraic Combinatorics.*

We study geometric and combinatorial properties of the degenerate flag varieties of type A. These varieties are acted upon by the automorphism group of a certain representation of a type A quiver, containing a maximal torus T. Using the group action, we describe the moment graphs, encoding the zero- and one-dimensional T-orbits. We also study the smooth and singular loci of the degenerate flag varieties. We show that the Euler characteristic of the smooth locus is equal to the large Schröder number and the Poincaré polynomial is given by a natural statistics counting the number of diagonal steps in a Schröder path. As an application we obtain a new combinatorial description of the large and small Schröder numbers and their q-analogues.

[7] With G. Cerulli Irelli and M. Reineke
Desingularization of quiver Grassmannians for Dynkin quivers
arXiv:1209.3960, *submitted to Advances in Matrhematics.*

A desingularization of arbitrary quiver Grassmannians for representations of Dynkin quivers is constructed in terms of quiver Grassmannians for an algebra derived equivalent to the Auslander algebra of the quiver.

[8] With G. Fourier, P. Littelmann
PBW–filtration over $\mathbb{Z}$ and compatible bases for $V_{\mathbb{Z}}(\lambda)$ in type $\mathtt{A}_n$ and $\mathtt{C}_n$
arXiv:1204.1854, *submitted to the proceedings of the conference Symmetries, Integrable Systems and Representations, Lyon, France, December 2011.*

We study the PBW-filtration on the highest weight representations $V(\lambda)$ of the Lie algebras of type $\mathtt{A}_n$ and $\mathtt{C}_n$. This filtration is induced by the standard degree filtration on $\mathrm{U}(\mathfrak{n}^-)$. In previous papers, the authors studied the filtration and the associated graded algebras and modules over the complex numbers. The aim of this paper is to present a proof of the results which holds over the integers and hence makes the whole construction available over any field.

Sabir Gusein-Zade

[1] With A.F. Costa and S.M. Natanzon.
Klein Foams.
Indiana University Mathematics Journal, 2011, Vol.60, No.3, pp. 985–996 (published in July 2012).

Klein foams are analogues of Riemann and Klein surfaces with one-dimensional singularities. We prove that the field of dianalytic functions on a Klein foam Ω coincides with the field of dianalytic functions on a Klein surface K_Ω. We construct the moduli space of Klein foams and we prove that the set of classes of topologically equivalent Klein foams form an analytic space homeomorphic to $\mathbb{R}^n/\mathsf{Mod}$, where Mod is a discrete group.

[2] With W. Ebeling.
Orbifold Euler characteristics for dual invertible polynomials.
Moscow Mathematical Journal, 2012, Vol.12, No.1, pp. 49–54.

To construct mirror symmetric Landau–Ginzburg models, P. Berglund, T. Hübsch and M. Henningson considered a pair (f, G) consisting of an invertible polynomial f and an abelian group G of its symmetries together with a dual pair $(\widetilde{f}, \widetilde{G})$. Here we study the reduced orbifold Euler characteristics of the Milnor fibres of f and $\widetilde{f}$ with the actions of the groups G and $\widetilde{G}$ respectively and show that they coincide up to a sign.

[3] With W. Ebeling.
Saito duality between Burnside rings for invertible polynomials.
Bulletin of the London Mathematical Society, 2012, Vol.44, No.4, pp. 814–822.

In this paper we give an equivariant version of the Saito duality which can be regarded as a Fourier transformation on Burnside rings. We show that (appropriately defined) reduced equivariant monodromy zeta functions of Berglund–Hübsch dual invertible polynomials are Saito dual to each other with respect to their groups of diagonal symmetries. Moreover we show that the relation between “geometric roots” of the monodromy zeta functions for some pairs of Berglund–Hübsch dual invertible polynomials described in a previous paper is a particular case of this duality.

[4] With W. Ebeling.
On a Newton filtration for functions on a curve singularity.
Journal of Singularities, 2012, Vol.4, pp. 180–187.

In a previous paper, there was defined a multi-index filtration on the ring of functions on a hypersurface singularity corresponding to its Newton diagram generalizing (for a curve singularity) the divisorial one. Its Poincaré series was computed for plane curve singularities non-degenerate with respect to their Newton diagrams. Here we use another technique to compute the Poincaré series for plane curve singularities without the assumption that they are non-degenerate with respect to their Newton diagrams. We show that the Poincaré series only depends on the Newton diagram and not on the defining equation.

[5] With W. Ebeling.
Equivariant Poincaré series and monodromy zeta functions of quasihomogeneous polynomials.
Publications of the Research Institute for Mathematical Science, 2012, Vol. 48, No.3, pp. 653–660.

In earlier work, the authors described a relation between the Poincaré series and the classical monodromy zeta function corresponding to a quasihomogeneous polynomial. Here we formulate an equivariant version of this relation in terms of the Burnside rings of finite abelian groups and their analogues.

[6] With I. Luengo and A. Melle Hernández.
On the pre-λ ring structure on the Grothendieck ring of stacks and on the power structures over it.
arXiv 1008:5063 (a preliminary version), *to appear in Bulletin of the London Mathematical Society.*

In this paper we discuss a generalization ("extension") of the pre-λ structure on the Grothendieck ring of quasi-projective varieties and of the corresponding power structure over it to the Grothendieck ring of stacks, discuss some of their properties and give some explicit formulae for the Kapranov zeta function for some stacks. In particular we show that the kth symmetric power of the class of the classifying stack $BGL(1)$ of the group $GL(1)$ coincides, up to a power of the class L of the affine line, with the class of the classifying stack $BGL(k)$.

On an equivariant analogue of the monodromy zeta function.
arXiv 1207.2282, *to appear in Functional Analysis and Its Applications.*

In this paper we offer an equivariant analogue of the monodromy zeta function of a germ invariant with respect to an action of finite group G as an element of the Grothendieck ring of finite $(Z \times G)$-sets. We formulate equivariant analogues of the Sebastiani–Thom theorem and of the A'Campo formula.

Alexander Kuznetsov

[1] Instanton bundles on Fano threefolds,
CEJM 2012, 10(4), 1198-1231.

We introduce the notion of an instanton bundle on a Fano threefold of index 2. For such bundles we give an analogue of a monadic description and discuss the curve of jumping lines. The cases of threefolds of degree 5 and 4 are considered in a greater detail.

[2] Height of exceptional collections and Hochschild cohomology of quasiphantom categories,
preprint math.AG/1211.4693

We define the normal Hochschild cohomology of an admissible subcategory of the derived category of coherent sheaves on a smooth projective variety X — a graded vector space which controls the restriction morphism from the Hochschild cohomology of X to the Hochschild cohomology of the orthogonal complement of this admissible subcategory. When the subcategory is generated by an exceptional collection, we define its new invariant (the height) and show that the orthogonal to an exceptional collection of height h in the derived category of a smooth projective variety X has the same Hochschild cohomology as X in degrees up to $h-2$. We use this to describe the second Hochschild cohomology of quasiphantom categories in the derived categories of some surfaces of general type. We also give necessary and sufficient conditions of fullness of an exceptional collection in terms of its height and of its normal Hochschild cohomology.

[3] With V. Lunts,
Categorical resolutions of irrational singularities,
preprint, to appear.

We show that the derived category of any singularity over a field of characteristic 0 can be embedded fully and faithfully into a smooth triangulated category which has a semiorthogonal decomposition with components equivalent to derived categories of smooth varieties. This provides a categorical resolution of the singularity.

Sergei Natanzon

[1] With A.Mironov, A.Morozov
Algebra of differential operators associated with Young diagrams
Journal of Geometry and Physics, 2012. 62. pp. 148-155

In this paper we establish a correspondence between Young diagrams and differential operators of infinitely many variables. These operators form a commutative associative algebra isomorphic to the algebra of the conjugated classes of finite permutations of the set of natural numbers. The Schur functions form a complete system of common eigenfunctions of these differential operators, and their eigenvalues are expressed through the characters of symmetric groups. The structure constants of the algebra are expressed through the Hurwitz numbers.

[2] With A.Alexandrov, A.D.Mironov, A.Morozov
Integrability of Hurwitz Partition Functions. I. Summary
Journal of Physics A: Mathematical and Theoretical, 2012. 45. C. 10

In this paper we considered a special form for the generating function of Hurwitz numbers, which depends on two infinite systems of variables. We prove that one of them, gives a solution of the equations of WDVV. The other system variables give a solution of the cut-and-join equation and its analogues.

[3] With A. Felikson
Moduli via double pants decompositions
Differential Geometry and its Applications, 2012, 30, pp. 490-508

In this paper we consider special sets of (6g-6) geodesics on a Riemann surface of genus g that describe all the possible Heegaard splitting of a 3-dimensional sphere. We prove that every such set determines the Riemann surface up to a finite number of possibilities.

[4] With A. Pratoussevitch
Topological invariants and moduli of Gorenstein singularities
arXiv:1010.1111, *to appear in the Proceedings of the London Math. Society*

In this paper we describe all connected components of the space of hyperbolic Gorenstein quasi-homogeneous surface singularities. We prove that any connected component is homeomorphic to a quotient of a cell by a discrete group.

[5] With A. Mironov, A. Morozov
A Hurwitz theory avatar of open-closed strings
arXiv:1208.5057 *submitted to European Physical Journal*

In this paper we review and explain an infinite-dimensional counterpart of the Hurwitz theory realization of algebraic open-closed string model a, where the closed and open sectors are represented by conjugation classes of permutations and the pairs of permutations, i.e. by the algebra of Young diagrams and bipartite graphes respectively.

Grigori Olshanski

[1] Laguerre and Meixner symmetric functions.
International Mathematics Research Notices, Vol. 2012 (2012), No. 16, pp. 3615–3679.

Analogs of Laguerre and Meixner orthogonal polynomials in the algebra of symmetric functions are studied. The work is motivated by a connection with a model of infinite-dimensional Markov dynamics.

[2] With A. Borodin
The boundary of the Gelfand–Tsetlin graph: a new approach.
Advances in Mathematics vol. 230 (2012), no. 4-6, 1738–1779.

The Gelfand–Tsetlin graph is an infinite graded graph that encodes branching of irreducible characters of the unitary groups. The boundary of the Gelfand-Tsetlin graph has at least three incarnations as a discrete potential theory boundary, as the set of finite indecomposable characters of the infinite-dimensional unitary group, and as the set of doubly infinite totally positive sequences. An old deep result due to Albert Edrei and Dan Voiculescu provides an explicit description of the boundary; it can be realized as a region in an infinite-dimensional coordinate space. The paper contains a novel approach to the Edrei–Voiculescu theorem. It is based on a new explicit formula for the number of semi-standard Young tableaux of a given skew shape (or of Gelfand–Tsetlin schemes of trapezoidal shape). The formula is obtained via the theory of symmetric functions, and new Schur-like symmetric functions play a key role in the derivation.

[3] With A. Borodin
Markov processes on the path space of the Gelfand-Tsetlin graph and on its boundary.
Journal of Functional Analysis vol. 263 (2012), no. 1, 248–303.
We construct a four-parameter family of Markov processes on infinite Gelfand–Tsetlin schemes that preserve the class of central (Gibbs) measures. Any process in the family induces a Feller Markov process on the infinite-dimensional boundary of the Gelfand–Tsetlin graph or, equivalently, the space of extreme characters of the infinite-dimensional unitary group $U(\infty)$. The process has a unique invariant distribution which arises as the decomposing measure in a natural problem of harmonic analysis on $U(\infty)$ posed in Olshanski (2003). As was shown in Borodin and Olshanski (2005), this measure can also be described as a determinantal point process with a correlation kernel expressed through the Gauss hypergeometric function.

[4] With A. Gnedin
The two-sided infinite extension of the Mallows model for random permutations
Advances in Applied Mathematics vol. 48 (2012), no. 5, 615–639.

We introduce a probability distribution Q on the infinite group of permutations of the set of integers. The distribution Q is a natural extension of the Mallows distribution on the finite symmetric group. A one-sided infinite counterpart of Q, supported by the group of permutations of the set of natural numbers, was studied previously in our paper [A. Gnedin, G. Olshanski, q-Exchangeability via quasiinvariance, Ann. Probab. 38 (2010) 21032135, arXiv:0907.3275]. We analyze various features of Q such as its symmetries, the support, and the marginal distributions.

[5] With E. Lytvynov

Equilibrium Kawasaki dynamics and determinantal point processes.
Zapiski Nauchnyh Seminarov POMI RAN, vol. 403 (2012), 81–94

Let μ be a point process on a countable discrete space X. Under assumption that μ is quasi-invariant with respect to any finitary permutation of X, we describe a general scheme for constructing an equilibrium Kawasaki dynamics for which μ is a symmetrizing (and hence invariant) measure. We also exhibit a two-parameter family of point processes μ possessing the needed quasi-invariance property. Each process of this family is determinantal, and its correlation kernel is the kernel of a projection operator in $\ell^2(X)$.

[6] With A. Osinenko
Multivariate Jacobi polynomials and Selberg's integral.
Functional Analysis and its Applications, vol. 46 (2012), no. 4

The paper is motivated by the problem of harmonic analysis on "big" groups and can be viewed as a continuation of first author's paper in Funct. Anal. Appl. **37** (2003), no. 4, 281–301. Our main result is the proof of existence of a family of probability distributions with infinite-dimensional support; these distributions are analogs of the multidimensional Euler's beta-distributions that appear in the Selberg integral.

Alexei Penskoi

[1] Extremal spectral properties of Lawson tau-surfaces and the Lameé equation
Moscow Math. J., 2012, Vol. 12, No. 1, pp. 173-192 .

In this paper extremal spectral properties of Lawson tau-surfaces are investigated. The Lawson tau-surfaces form a two-parametric family of tori or Klein bottles minimally immersed in the standard unitary three-dimensional sphere. A Lawson tau-surface carries an extremal metric for some eigenvalue of the Laplace-Beltrami operator. Using theory of the Lamé equation we find explicitly these extremal eigenvalues.

[2] Extremal spectral properties of Otsuki tori
arXiv:arXiv:1108.5160, to appear in Mathematische Nachrichten.

In this paper we study Otsuki tori. Otsuki tori form a countable family of immersed minimal two-dimensional tori in the unitary three-dimensional sphere. According to El Soufi-Ilias theorem, the metrics on the Otsuki tori are extremal for some unknown eigenvalues of the Laplace-Beltrami operator. Despite the fact that the Otsuki tori are defined in quite an implicit way, we find explicitly the numbers of the corresponding extremal eigenvalues. In particular we provide an extremal metric for the third eigenvalue of the torus.

Yuri Prokhorov

[1] Simple finite subgroups of the Cremona group of rank 3

J. Algebraic Geom., 2012, Vol. 21, No. 3, pp. 563-600

We classify all finite simple subgroups of the Cremona group $\mathrm{Cr}_3(\mathbb{C})$.

[2] On birational involutions of $\mathbf{P}^3$

arXiv:1206.4985 *to appear in Izv. RAN, Ser. Mat.*

Let X be a rationally connected three-dimensional algebraic variety and let τ be an element of order two in the group of its birational selfmaps. Suppose that there exists a non-uniruled divisorial component of the τ-fixed point locus. Using the equivariant minimal model program we give a rough classification of such elements.

[3] G-Fano threefolds, II.

arXiv:1101.3854 *to appear in Advances in Geometry*

We classify Fano threefolds with only Gorenstein terminal singularities and Picard number greater than 1 satisfying an additional assumption that the G-invariant part of the Weil divisor class group is of rank 1 with respect to an action of some group G.

[4] With S. Mori

Threefold extremal contractions of types (IC) and (IIB)

arXiv:1106.5180 *to appear in Proc Edinburgh Math. Soc.*

Let (X, C) be a germ of a threefold X with terminal singularities along an irreducible reduced complete curve C with a contraction $f : (X, C) \to (Z, o)$ such that $C = f^{-1}(o)_{red}$ and $-K_X$ is ample. Assume that (X, C) contains a point of type (IC) or (IIB). We complete the classification of such germs in terms of a general member $H \in |\mathcal{O}_X|$ containing C.

[5] With M. Reid

On $\mathbf{Q}$-Fano threefolds of Fano index 2

arXiv:1203.0852 *submitted to Advanced Study of Pure Mathematics*

We show that, for a $\mathbf{Q}$-Fano threefold X of Fano index 2, the inequality $\dim |-\frac{1}{2}K_X| \leq 4$ holds with a single well understood family of varieties having $\dim |-\frac{1}{2}K_X| = 4$.

[6] With C. Shramov

Jordan property for Cremona groups

arXiv:1211.3563, *will be submitted soon*

Assuming Borisov–Alexeev–Borisov conjecture, we prove that there is a constant $J = J(n)$ such that for any rationally connected variety X of dimension n and any finite subgroup $G \subset \mathrm{Bir}(X)$ there exists a normal abelian subgroup $A \subset G$ of index at most J. In particular, we obtain that the Cremona group $\mathrm{Cr}_3 = \mathrm{Bir}(\mathbf{P}^3)$ enjoys the Jordan property.

Leonid Rybnikov

[1] With M.Finkelberg. Quantization of Drinfeld Zastava in type A.
arXiv:1009.0676 (revised version 17 Nov 2012) *to appear in Journal of the European Mathematical Society, 2013.*

In this paper, we quantize the natural Poisson bracket on Drinfeld Zastava spaces in type A. Drinfeld Zastava is a certain closure of the moduli space of maps from the projective line to the Kashiwara flag scheme of the affine Lie algebra $\hat{sl}_n$. We introduce an affine, reduced, irreducible, normal quiver variety Z which maps to the Zastava space bijectively at the level of complex points. The natural Poisson structure on the Zastava space can be described on Z in terms of Hamiltonian reduction of a certain Poisson subvariety of the dual space of a (nonsemisimple) Lie algebra. The quantum Hamiltonian reduction of the corresponding quotient of its universal enveloping algebra produces a quantization Y of the coordinate ring of Z. The same quantization was obtained in the finite (as opposed to the affine) case generically in arXiv:math/0409031. We prove that, for generic values of quantization parameters, Y is a quotient of the affine Borel Yangian. The paper generalizing these results to type C is in preparation.

Timur Sadykov

[1] With V. Krasikov
On the analytic complexity of discriminants
Proceedings of the Steklov Institute of Mathematics, 2012, Vol. 279, pp. 78-92. (Published in Russian in Trudy Matematicheskogo Instituta imeni V.A. Steklova, 2012, Vol. 297, pp. 86-101.)

The paper deals with the notion of the analytic complexity introduced by V.K. Beloshapka in [V.K. Beloshapka, *Analytic complexity of functions of two variables*, Russian Journal of Mathematical Physics. **14:3** (2007), 243-249.]. We give an algorithm which allows one to check whether a bivariate analytic function belongs to the second class of analytic complexity. We also provide estimates for the analytic complexity of classical discriminants and introduce the notion of the analytic complexity of a knot.

[2] With V. Krasikov

Linear differential operators for generic algebraic curves

This is a revised and extended version of the 2010 preprint:

arXiv:1001.2607 *to appear in Journal of Siberian Federal University. Mathematics and Physics.*

We give a computationally efficient method for constructing the linear differential operator with polynomial coefficients whose space of holomorphic solutions is spanned by all the branches of a function defined by a generic algebraic curve. The proposed method does not require solving the algebraic equation and can be applied in the case when its Galois group is not solvable. We investigate the structure of the Newton polytopes of the coefficients of annihilating operators and describe their apparent singularities.

[3] With S. Tanabe

Maximally reducible monodromy of bivariate hypergeometric systems

Work in progress

We investigate branching of solutions to hypergeometric systems of partial differential equations. Special attention is paid to the invariant subspace of Puiseux polynomial solutions and to the systems defined by simplicial configurations. We prove a necessary and sufficient condition for the monodromy representation to be maximally reducible, that is, for the space of holomorphic solutions to split into the direct sum of one-dimensional invariant subspaces.

[4] Book in preparation

Hypergeometric systems of partial differential equations and their applications (Russian, 326 pages)

A preliminary version of the manuscript is available

Arkady Skopenkov

[0] D. Crowley and A. Skopenkov, A classification of embeddings of non-simply-connected 4-manifolds into 7-space, 2012, in preparation.

Abstracts of conference talks:

[1] D. Crowley and A. Skopenkov, Classification of smooth embeddings of non-simply-connected 4-manifolds into R^7, Abstracts of the International Alexandroff Conference, Moscow, 2012

Let N be a closed connected orientable 4-manifold with torsion free integral homology. The main result is *a complete readily calculable classification of embeddings* $N \to R^7$, in the smooth and in the piecewise-linear (PL) categories. Such a classification was earlier known only for simply-connected N, in the PL case by Boéchat-Haefilger-Hudson 1970, in the smooth case by the authors 2008. In particular, for $N = S^1 \times S^3$ we define geometrically a 1–1 correspondence between the set of PL isotopy classes of PL embeddings $S^1 \times S^3 \to R^7$ and the quotient set of $Z \oplus Z_6$ up to equivalence $(l, b) \sim (l, b')$ for $b \equiv b' \mod 2GCD(3, l)$. This particular case allows us to disprove the conjecture on the completeness of the Multiple Haefliger-Wu invariant, as well as the Melikhov informal conjecture on the existence of a geometrically defined group structure on the set of PL isotopy classes of PL embeddings in codimension 3. For $N = S^1 \times S^3$ and the smooth case we identify the isotopy classes of embeddings with an explicitly defined quotient of $Z_{12} \oplus Z \oplus Z$.

[2] D. Crowley and A. Skopenkov, Classification of smooth embeddings of non-simply-connected 4-manifolds into R^7, Abstracts of European Congress of Mathematicians, Krakow, 2012

Same content as in [1].

Internet publications:

[3] D. Goncalves and A. Skopenkov, Embeddings of homology equivalent manifolds with boundary http://arxiv.org/abs/1207.1326

We prove a theorem on equivariant maps implying the following two corollaries:

(1) Let N and M be compact orientable n-manifolds with boundaries such that $M \subset N$, the inclusion $M \to N$ induces an isomorphism in integral cohomology, both M and N have $(n-d-1)$-dimensional spines and $m > \max\{n+2, (3n+1-d)/2\}$. Then the restriction-induced map $E^m(N) \to E^m(M)$ is bijective. Here $E^m(X)$ is the set of embeddings $X \to R^m$ up to isotopy (in the PL or smooth category).

(2) For a 3-manifold N with boundary whose integral homology groups are trivial and such that $N \not\cong D^3$ (or for its special 2-spine N) there exists an equivariant map from the deleted product of N to S^2, although N does not embed into R^3. The second corollary completes the answer to the following question: for which pairs (m, n) for each n-polyhedron N the existence of an equivariant map from the deleted product of N to S^{m-1} implies the embeddability of N into R^m? An answer was known for each pair (m, n) except $(3, 3)$ and $(3, 2)$.

[4] A. Skopenkov, A classification of smooth embeddings of 4-manifolds in 7-space, I, http://arxiv.org/abs/0804.4357, v8

We work in the smooth category. Let N be a closed connected n-manifold and assume that $m > n+2$. Denote by $E^m(N)$ the set of embeddings $N \to R^m$ up to isotopy. The group $E^m(S^n)$ acts on $E^m(N)$ by embedded connected sum of a manifold and a sphere. If $E^m(S^n)$ is non-zero (which often happens for $2m < 3n+4$) then no results on this action and no complete description of $E^m(N)$ were known. Our main results are examples of the triviality and the effectiveness of this action, and a complete isotopy classification of embeddings into R^7 for certain 4-manifolds N. The proofs are based on the Kreck modification of surgery theory and on construction of a new embedding invariant.

Corollary. (a) There is a unique embedding $CP^2 \to R^7$ up to isoposition.

(b) For each embedding $f : CP^2 \to R^7$ and each non-trivial knot $g : S^4 \to R^7$ the embedding $f\#g$ is isotopic to f.

Expository publications for university students:

[5] A. Skopenkov, Ambient Homogeneity, in Russian, MCCME, Moscow, 2012, http://arxiv.org/abs/

This paper is purely expository. A subset N of the plane is affine ambient homogeneous if for each x, y in N there exists an affine transformation taking x to y and N to itself. The result of D. Repovs, E. V. Scepin and the author on such subsets is presented, together with discussion, corollaries and generalizations. At the end some non-elementary corollaries are given (including a simple proof of the smooth version of the Hilbert-Smith conjecture on topological groups). Most part of the text is accessible to undergraduates familiar with the notion of continuity. The text could be an interesting easy reading for mature mathematicians.

[6] I. Arzhantsev, V. Bogachev, A. Garber, A. Zaslavsky, V. Protasov and A. Skopenkov, Students' mathematical olympiades at Moscow State University 2010-2011, in Russian, Mat. Prosveschenie, 16 (2012), 214-227.

We present problems and solutions of the final round's students' mathematical olympiades at Moscow State University, for years 2010-2011.

[7] A. Skopenkov, Algebraic Topology From Geometric Viewpoint, in Russian, MCCME, Moscow, to appear. arXiv:0808.1395 update version: www.mccme.ru/circles/oim/obstruct.pdf

This book is purely expositional and is in Russian. It is shown how in the course of solution of interesting geometric problems (close to applications) naturally appear main notions of algebraic topology (homology groups, obstructions and invariants, characteristic classes). Thus main ideas of algebraic topology are presented with minimal technicalities. Familiarity of a reader with basic notions of topology (such as 2-dimensional manifolds and vector fields) is desirable, although definitions are given at the beginning. The book is accessible to undergraduates and could also be an interesting easy reading for mature mathematicians.

[8] A. Skopenkov, Some more proofs from the Book: solvability and insolvability of equations in radicals, in Russian, submitted http://arxiv.org/abs/0804.4357 v2

This paper is purely expository and is in Russian. We present short elementary proofs of

• the Gauss Theorem on constructibility of regular polygons;
• the existence of a cubic equation unsolvable in real radicals;
• the existence of a quintic equation unsolvable in complex radicals (Galois Theorem).

We do not use the terms 'Galois group' or even 'group'. However, our presentation is a good way to learn (or recall) starting idea of the Galois theory. The paper is accessible for students familiar with elementary algebra (including complex numbers), and could be an interesting easy reading for mature mathematicians. The material is presented as a sequence of problems, which is peculiar not only to Zen monasteries but also to serious mathematical education; most problems are presented with hints or solutions.

Expository internet-publications for university students:

[9] A. Skopenkov, A two-page disproof of the Borsuk partition conjecture, http://arxiv.org/abs/0712.4009, v2

It is presented the simplest known disproof of the Borsuk conjecture stating that if a bounded subset of n-dimensional Euclidean space contains more than n points, then the subset can be partitioned into $n+1$ nonempty parts of smaller diameter. The argument is due to N. Alon and is a remarkable application of combinatorics and algebra to geometry. This note is purely expository and is accessible for students.

[10] S. Avvakumov, A. Berdnikov, A. Rukhovich and A. Skopenkov, How do curved spheres intersect in 3-space, or two-dimensional meandra, http://olympiads.mccme.ru/lktg/2012/

This is an exposition of recent results of S. Avvakumov and A. Rukhovich on the Lando conjecture on embeddings of two spheres in 3-space (http://arxiv.org/abs/1012.0925, http://arxiv.org/abs/1210.7361).

[11] A. Skopenkov, Algorithms for recognition of the realizability of hypergraphs, in Russian, www.mccme.ru/circles/oim/algor.pdf

This is an exposition of recent result of J. Matoušek, M. Tancer and U. Wagner on hardness of embedding simplicial complexes in R^d (http://arxiv.org/abs/0807.0336), and of related topics.

Mikhail Skopenkov

[1] With F. Nilov
A surface containing a line and a circle through each point is a quadric
Geom. Dedicata (2012), DOI: 10.1007/s10711-012-9750-0;

We prove that a surface in 3-dimensional Euclidean space containing a line and a circle through each point is a quadric. We also give some particular results on the classification of surfaces containing several circles through each point.

[2] With M. Cencelj, D. Repovš
Classification of knotted tori in the 2-metastable dimension
Sb. Math.+ 203:11 (2012), 129-158;

This paper is devoted to the classical Knotting Problem: for a given manifold N and number m describe the set of isotopy classes of embeddings $N \to S^m$. We study the specific case of *knotted tori*, i. e. the embeddings $S^p \times S^q \to S^m$. The classification of knotted tori up to isotopy in the *metastable* dimension range $m \geq p + \frac{3}{2}q + 2$, $p \leq q$, was given by A. Haefliger, E. Zeeman and A. Skopenkov. We consider the dimensions below the metastable range, and give an explicit criterion for the finiteness of this set of isotopy classes in the *2-metastable* dimension:

Theorem. Assume that $p + \frac{4}{3}q + 2 < m < p + \frac{3}{2}q + 2$ and $m > 2p + q + 2$. Then the set of isotopy classes of smooth embeddings $S^p \times S^q \to S^m$ is infinite if and only if either $q + 1$ or $p + q + 1$ is divisible by 4.

Our approach to the classification is based on an analogue of the U. Koschorke exact sequence from the theory of link maps. Our sequence involves a new *β-invariant* of knotted tori. The exactness is proved using embedded surgery and the N. Habegger–U. Kaiser techniques of studying the complement.

[3] With H. Pottmann, L. Shi
Darboux cyclides and webs from circles
Computer Aided Geom. Design 29:1 (2012), 77-97[1].

Motivated by potential applications in architecture, we study Darboux cyclides. These algebraic surfaces of order a most 4 are a superset of Dupin cyclides and quadrics, and they carry up to six real families of circles. Revisiting the classical approach to these surfaces based on the spherical model of 3D Moebius geometry, we provide computational tools for the identification of circle families on a given cyclide and for the direct design of those. In particular, we show that certain triples of circle families may be arranged as so-called hexagonal webs, and we provide a complete classification of all possible hexagonal webs of circles on Darboux cyclides.

[4] With H. Pottmann, P. Grohs,
Ruled Laguerre minimal surfaces,
Math. Z. 272 (2012), 645-674[1].

A Laguerre minimal surface is an immersed surface in the Euclidean space being an extremal of the functional $\int (H^2/K - 1) dA$. In the present paper, we prove that the only

[1]This paper contains no reference to Simons–IUM fellowship because it was published online before the contest

ruled Laguerre minimal surfaces are up to isometry the surfaces $R(u,v) = (Au, Bu, Cu + D\cos 2u) + v(\sin u, \cos u, 0)$, where A, B, C, D are fixed real numbers. To achieve invariance under Laguerre transformations, we also derive all Laguerre minimal surfaces that are enveloped by a family of cones. The methodology is based on the isotropic model of Laguerre geometry. In this model a Laguerre minimal surface enveloped by a family of cones corresponds to a graph of a biharmonic function carrying a family of isotropic circles. We classify such functions by showing that the top view of the family of circles is a pencil.

[5] With V. Smykalov, A. Ustinov,
Random walks and electric networks
Mat. Prosv. 3rd ser. 16 (2012), 25-47[1];

This is a popular science paper devoted to an elementary proof of the following beautiful result:
The Polya Theorem. (a) A man which is randomly walking in a 2-dimensional lattice will return to the initial point with probability 1.
(b) A man which is randomly walking in a 3-dimensional lattice will return to the initial point with probability strictly less than 1.

The approach to the proof is based on a physical interpretation. The exposition goes along the lines of P. Doyle and L. Snell.

[6] Boundary value problem for discrete analytic functions
Adv. Math. (2012), conditionally accepted;

This paper is on further development of discrete complex analysis introduced by R. Isaacs, R. Duffin, and C. Mercat. We consider a graph lying in the complex plane and having quadrilateral faces. A function on the vertices is called discrete analytic, if for each face the difference quotients along the two diagonals are equal.

We prove that the Dirichlet boundary value problem for the real part of a discrete analytic function has a unique solution. In the case when each face has orthogonal diagonals we prove that this solution converges to a harmonic function in the scaling limit (under certain regularity assumptions).

This solves a problem of S. Smirnov from 2010. This was proved earlier by R. Courant–K. Friedrichs–H. Lewy for square lattices, by D. Chelkak–S. Smirnov and implicitly by P.G. Ciarlet–P.-A. Raviart for rhombic lattices.

In particular, our result implies uniform convergence of finite element method on Delauney triangulations. This solves a problem of A. Bobenko from 2011. The methodology is based on energy estimates inspired by alternating-current networks theory.

[7] With A. Bobenko
Discrete Riemann surfaces: linear discretization and its convergence
submitted. http://arxiv.org/abs/1210.0561

We develop linear discretization of complex analysis, originally introduced by R. Isaacs, J. Ferrand, R. Duffin, and C. Mercat. We prove convergence of discrete period matrices and discrete Abelian integrals to their continuous counterparts. We also prove a discrete counterpart of the Riemann–Roch theorem. The proofs use energy estimates inspired by electrical networks.

[8] When the set of embeddings is finite?
submitted. http://arxiv.org/abs/1106.1878

Given a manifold N and a number m, we study the following question: *is the set of isotopy classes of embeddings* $N \to S^m$ *finite*? In case when the manifold N is a sphere the answer was given by A. Haefliger in 1966. In case when the manifold N is a disjoint union of spheres the answer was given by D. Crowley, S. Ferry and the author in 2011.

We consider the next natural case when N is a product of two spheres. In the following theorem, $FCS(i,j) \subset \mathbb{Z}^2$ is a concrete set depending only on the parity of i and j which is defined in the paper.

Theorem. Assume that $m > 2p+q+2$ and $m < p+3q/2+2$. Then the set of isotopy classes of smooth embeddings $S^p \times S^q \to S^m$ is infinite if and only if either $q+1$ or $p+q+1$ is divisible by 4, or there exists a point (x,y) in the set $FCS(m-p-q, m-q)$ such that $(m-p-q-2)x + (m-q-2)y = m-3$.

Our approach is based on a group structure on the set of embeddings and a new exact sequence, which in some sense reduces the classification of embeddings $S^p \times S^q \to S^m$ to the classification of embeddings $S^{p+q} \sqcup S^q \to S^m$ and $D^p \times S^q \to S^m$. The latter classification problems are reduced to homotopy ones, which are solved rationally.

Evgeni Smirnov

[1] With V. Kiritchenko, V. Timorin
Schubert calculus and Gelfand–Zetlin polytopes
Uspekhi Matematicheskikh Nauk 67:4(406), 89–128 (2012) (in Russian). English translation: Russian Mathematical Surveys, 64:4, 685–719 (2012)

We describe a new approach to the Schubert calculus on complete flag varieties using the volume polynomial associated with Gelfand-Zetlin polytopes. This approach allows us to compute the intersection products of Schubert cycles by intersecting faces of a polytope.

Andrei Sobolevski

[1] with Julie Delon, Julien Salomon
Local matching indicators for transport problems with concave costs
SIAM J. Discrete Math., 2012, Vol. 26, No. 2, pp. 801–827 (also available as arXiv:1102.1795).

In this paper, we introduce a class of combinatorial quantities called "local indicators" that enable to efficiently compute optimal transport plans associated to arbitrary distributions of N demands and M supplies in $\mathbb{R}$ in the case where the cost function is concave.

The computational cost of these indicators is small and independent of N. A hierarchical computation of local indicators gives an efficient way to construct the optimal transport plan.

[2] with Matteo Novaga and Eugene Stepanov
Droplet condensation and isoperimetric towers,
CVGMT preprint 1784 (`http://cvgmt.sns.it/paper/1784/`) *to appear in Pacific J. Math.*

We consider a variational problem in a planar convex domain, motivated by statistical mechanics of crystal growth in a saturated solution. The minimizers are constructed explicitly and are completely characterized.

[3] with S. K. Nechaev and O. V. Valba
Planar diagrams from optimization for concave costs
arXiv:1203.3248 *to appear in Phys. Rev. E*

We propose a new toy model of a heteropolymer chain capable of forming planar secondary structures typical for RNA molecules. In this model the sequential intervals between neighboring monomers along a chain are considered as quenched random variables. Using the optimization procedure for a special class of concave-type potentials, borrowed from optimal transport analysis, we derive the local difference equation for the ground state free energy of the chain with the planar (RNA-like) architecture of paired links . We consider various distribution functions of intervals between neighboring monomers (truncated Gaussian and scale-free) and demonstrate the existence of a topological crossover from sequential to essentially embedded (nested) configurations of paired links.

[4] with Konstantin Khanin
On dynamics of Lagrangian trajectories for Hamilton–Jacobi equations
arXiv:1211.7084 *submitted to Discrete Cont. Dynamical Systems*

Characteristic curves of a Hamilton–Jacobi equation can be seen as action minimizing trajectories of fluid particles. This description, however, is valid only for smooth solutions. For non-smooth "viscosity" solutions, which give rise to discontinuous velocity fields, this picture holds only up to the moment when trajectories hit a shock and cease to minimize the Lagrangian action. In this paper we show that for any convex Hamiltonian, a viscous regularization allows to construct a non-smooth flow that extends particle trajectories and determines dynamics inside the shock manifolds. This flow consists of integral curves of a particular velocity field, which is uniquely defined everywhere in the flow domain and is discontinuous on shock manifolds.

Vladlen Timorin

[1] With V. Kirichenko and E. Smirnov
Schubert calculus and Gelfand–Zetlin polytopes
Russian Mathematical Surveys 2012, Vol. 67, No. 4, pp. 685–719

A new approach is described to the Schubert calculus on complete flag varieties, using the volume polynomial associated with Gelfand–Zetlin polytopes. This approach makes it possible to compute the intersection products of Schubert cycles by intersecting faces of a polytope.

[2] With E. Ghys and S. Tabachnikov
Osculating curves: around the Tait-Kneser Theorem
Mathematical Intelligencer, doi:10.1007/s00283-012-9336-6

The Tait-Kneser theorem states that the osculating circles of a plane curve with monotonic curvature are pairwise disjoint and nested. We discuss this theorem and a number of its variations.

[3] With I. Mashanova
Captures, matings and regluings,
to appear in the Annales de Toulouse

In parameter slices of quadratic rational functions, we identify arcs represented by matings of quadratic polynomials. These arcs are on the boundaries of hyperbolic components.

[4] Planarizations and maps taking lines to linear webs of conics,
to appear in the Mathematical Research Letters

Aiming at a generalization of a classical theorem of Moebius, we study maps that take line intervals to plane curves, and also maps that take line intervals to conics from certain linear systems.

[5] With P. Gusev and V. Kirichenko
Number of vertices in Gelfand-Zetlin polytopes
arXiv:1205.6336, *Submitted to Journal of Combinatorial Theory, Series A*

We discuss the problem of counting vertices in Gelfand-Zetlin polytopes. Namely, we deduce a partial differential equation with constant coefficients on the exponential generating function for these numbers. For some particular classes of Gelfand-Zetlin polytopes, the number of vertices can be given by explicit formulas.

Ilya Vyugin

[1] (With I.D. Shkredov) On additive shifts of multiplicative subgroups
Sbornik: Mathematics, 2012, 203(6):844

It is proved that for an arbitrary subgroup $R \subseteq Z/pZ$ and any distinct nonzero elements $\mu_1, \ldots, \mu_k$ we have

$$|R \cap (R + \mu_1) \cap \ldots \cap (R + \mu_k)| \ll |R|^{1/2+\alpha_k}$$

under the condition that $1 \ll_k |R| \ll_k p^{1-\beta_k}$, where $\{\alpha_k\}$, $\{\beta_k\}$ are some sequences of positive numbers such that $\alpha_k, \beta_k \to 0$ as $k \to \infty$. Furthermore, it is shown that the inequality $|R \pm R| > |R|^{5/3} \log^{-1/2} |R|$ holds for any subgroup R such that $|R| > p^{1/2}$.

[2] (With R.R. Gontsov) On the question of solubility of Fuchsian systems by quadratures
Russian Mathematical Surveys, 2012, 67:3, 585587

The criterion of solubility in quadratures of a Fuchsian system from some class was obtained. This is a generalization of the Ilyashenko-Khovanskii result.

[3] Expansions for Solutions of the Schlesinger Equation at a Singular Point, P. 151-158, Chapter 18 in the book: "Painlev? Equations and Related Topics", De Gruyter, 2012 (arxiv:1212.2176).

A local behavior of solutions of the Schlesinger equation is studied. We obtain expansions for this solutions, which converge in some neighborhood of a singular point. As a corollary the similar result for the sixth Painlevé equation was obtained.

Alexei Zykin

[1] Editor with Y. Aubry and C. Ritzenthaler
Arithmetic, Geometry, Cryptography and Coding Theory
Contemporary Mathematics, Vol. 574 (2012), 183 pp;

This volume contains the proceedings of the 13th conference AGC2T which takes place in Marseilles (France) every two years together with the proceedings of the conference Geocrypt. The international conference AGC2T has been a major event in the area of applied arithmetic geometry for more than 25 years and can be proud of the presence of J.-P. Serre (Fields medal, Abel prize winner), H. Stichtenoth, M. Hindry, Y. Zarhin, E. Howe and other leading researchers in the field among its more than 80 participants. In this book are gathered 15 original research articles on various topics ranging from algebraic number theory to diophantine geometry, curves and abelian varieties over finite fields and applications to codes, boolean functions or cryptography.

2.2 Scientific conferences and seminar talks

Ivan Arzhantsev

[1] Workshop "Rational points and rational curves", University of Zurich, Switzerland, 21–21 May 2012

Talk "Flexible varieties and automorphism groups"

[2] Workshop on Lie Groups and Algebraic Groups, University of Bielefeld, Germany, 23–25 July 2012

Talk "The automorphism group of a variety with torus action of complexity one"

[3] Conference "Groups of Automorphisms in Birational and Affine Geometry", CIRM, Trento, Italy, 28 October – 3 November 2012

Talk "The automorphism group of a variety with torus action of complexity one"

[4] The Third School-Conference "Lie Algebras, Algebraic Groups and Invariant Theory", Tolliati, Russia, 25-30 June 2012

Lecture Course "Geometric Invariant Theory and Cox Rings"

[5] Conference "Algebraic groups and related structures", Sankt-Peterburg Mathematical Institute, Russia, 17-22 September 2012

Talk "Algebraically generated groups"

[6] Conference "Christmas Mathematical Meetings of Dynasty Foundation", Moscow, Russia, 8 – 10 January 2012

Talk "Flexible affine algebraic varieties"

[7] Oberseminar on Algebraic Geometry, Ludwig-Maximilains University, Munich, Germany, 25 April 2012

Talk "Additive structures on projective varieties and local algebras"

[8] Sankt-Peterburg Seminar on Representation Theory and Dynamical Systems, Sankt-Peterburg, Russia, 28 March 2012

Talk "Automorphisms of algebraic varieties and toric geometry"

[9] Postnikov Memorial Seminar "Algebraic topology and its applications", Moscow State University, Russia, 3 April 2012

Talk "The automorphism group of a variety with torus action of complexity one"

[10] Iskovskih Seminar, Steklov Mathematical Institute, Moscow, Russia, 18 October 2012

Talk "Geometric Invariant Theory and Cox Rings"

Alexander Bufetov

[1] ICTP-ESF School and Conference in Dynamical Systems. 4-8 June 2012, Italy, Talk "On the Vershik-Kerov conjecture"

[2] Dynamics in infinite-dimensions: ergodic theory and PDEs, 21-25 May 2012 ., Edinburgh, UK

Talk "On ergodic decomposition for actions of infinite-dimensional groups"

[3] Laminations et Dynamique symbolique, 2-6 April 2012 ., CIRM, Marseille

Talk "Limit theorems for tilings"

[4] Aspects of representation theory in low-dimensional topology and 3-dimensional invariants, 5-9 November 2012,

Talk "Limit theorems for parabolic flows".

Evgeny Feigin

[1] Conference "Classical and Quantum Integrable Systems", Russia, Dubna, January, 23 – January, 27

Talk "Abelianized representations of simple Lie algebras"

[2] Conference "Algebra and Geometry", Russia, Moscow, May, 4 – May, 9

Talk "PBW degeneration of flag varieties in type A"

[3] Conference "Symmetric Spaces and their Generalisations II", Italy, Trento, June, 25 – June, 29

Talk "Degenerate flag varieties"

[4] Conference "Lie Theory and quantum analogues", France, Marseille, April, 23 – April, 27

Talk "Degenerate flag varieties"

[5] Conference "Enveloping algebras and geometric representation theory", Germany, Oberwolfach, March, 4 – Marrch, 10

Talk "PBW degeneration: representations and flag varieties"

Sabir Gusein-Zade

[1] Visit to Spain, February-March.

Talk: "Equivariant Saito duality and monodromy zeta functions of dual invertible polynomials" at the Seminar on Algebraic Geometry and Singularities, University of Valladolid.

Talk "Equivariant Saito duality and monodromy zeta functions of dual invertible polynomials" at the Seminar of the Dept. of Algebra, University Complutense de Madrid.

[2] Visit to Baku, Azerbaijan, April.

Talk "What is a power series whose coefficients are algebraic varieties in a power which is an algebraic variety as well?" at the seminar of the Baku branch (filial) of the Moscow State University.

[3] Visit to Germany, Bonn, Max Planck Institute for Mathematics (MPIM), June-July.

Talk “Poincaré series of multi-index filtrations, integration with respect to the Euler characteristic and monodromy zeta functions” at the Oberseminar of MPIM.

Talk “Power structure over the Grothendieck ring of complex quasi-projective varieties and its applications” at the Seminar of Yu.I.Manin at MPIM.

Talk “Poincaré series of multi-index filtrations, integration with respect to the Euler characteristic and monodromy zeta functions” at the Seminar of the Group of Algebraic Geometry at the Kaiserslautern University.

[4] International scientific school for young scientists “Modelling and control of social-economic processes”, Vladimir, October 1-5.

Series of lectures “Modelling of distribution of the population and populated places”.

Alexander Kuznetsov

[1] Conference “Workshop on Homological Mirror Symmetry and Related Topics”, Miami, January 23–27, 2012.

Talk “Categorical resolutions of singularities”

[2] Conference “Algorithmic and Experimental Methods in Algebra, Geometry and Number Theory”, Hanover, February 27–March 1, 2012.

Talk “Categorical resolutions of singularities”

[3] “Noncommutative Algebraic Geometry and its Applications to Physics”, Leiden, March 19–23, 2012.

Talk “Categorical resolutions of singularities”

[4] Conferemce “Homological Projective Duality and Noncommutative Geometry”, Warwick, October 8–13, 2012.

Minicourse “Homological Projective Duality”

[5] Conference “Homological Projective Duality and Quantum Gauge Theory”, Tokyo, IPMU, November 12–16, 2012.

Talks “Homological Projective Duality I,II”

[6] Conference “Birational And Affine Geometry”, Moscow, April 23–27, 2012.

Talk “Categorical resolutions of singularities”

[7] Conference “Algebraic and Differential Geometry of Andrei Tyurin”, Moscow, October 24–26, 2012.

Talk “Heights of exceptional collections and Hochschild cohomology of quasiphantoms”

[8] Conference “Relation of String Theory to Gauge Theories and Moduli Problems of Branes”, Moscow, September 10–14, 2012.

Talk “Homological Projective Duality”

[9] Visit to Indiana University, Bloomington, January 2012

[10] Visit to SISSA, Trieste, August 2012

Sergei Natanzon

[1] Workshop "Synthesis of integrabilities in the context of duality between the string theory and gauge theories", Moscow, September, 17-21.

Talk "A Hurwitz theory avatar of open-closed strings".

[2] Conference "An international workshop in Singularity Theory, its Applications and Future Prospects", The University of Liverpool, Liverpool, July, 18-22.

Talk "Disk Hurwitz numbers and Half-Cut-and-Join operators"

[3] International Topology Conference "Alexander Reading", Moscow, May, 21-25.

Talk "Disk Hurwitz numbers and Half-Cut-and-Join operators".

[4] Conference "3rd International Workshop on Combinatorics of moduli spaces, cluster algebras, knots, and topological recursion" , Moscow, May 28 – June 02.

Talk "Disk Hurwitz numbers and Half-Cut-and-Join operators"

Grigori Olshanski

[1] Workshop "Discrete Random Structures, Representation Theory and Interacting Particle Systems", Germany, Bielefeld, Center for Interdisciplinary Research, July 16, 2012 – July 19, 2012.

Two talks "From Representations to Point Processes", "Infinite-Dimensional Analogs of Non-Colliding Processes".

[2] Conference "Infinite Dimensional Analysis and Representation Theory", Germany, University of Bielefeld, December 10, 2012 – December 14, 2012.

Two talks "Determinantal measures and Markov dynamics"

Alexei Penskoi

[1] Workshop: "Integrability - modern variations", Hausdorff Research Institute for Mathematics, Universität Bonn, January, 9 – 13, 2012.

Talk "Extremal metrics: recent developments".

[2] "Workshop on Geometry of Eigenvalues and Eigenfunctions", Centre de recherches mathématiques, Université de Montréal, June, 4 – 8, 2012.

Talk "Extremal metrics for Laplace eigenvalues on tori".

[3] Workshop "Applications of Analysis: Game Theory, Spectral Theory and Beyond" in honor of Yakar Kannai's 70th birthday, December, 25 – 27, 2012.

Talk "Geometric optimization of eigenvalues of the Laplace operator".

[4] Symposium "Adventures in mathematical physics", Centre Jacques Cartier, Lyon, November, 19 – 21, 2012.

Poster talk "Spectral Geometry and Mathematical Physics — joint quest for extremal metrics".

[5] Workshop “Geometric Structures in Integrable Systems”, Moscow, October, 30 – November, 2, 2012.

Talk “Extremal metrics for Laplace eigenvalues on tori”.

[6] Visit to Institut de Mathématiques de Bourgogne, Dijon, November, 22 – 25, 2012.

Talk “Extremal metrics for Laplace eigenvalues on tori” at “Séminaire Mathématique Physique”.

Yuri Prokhorov

[1] London Mathematical Society - EPSRC Durham Symposium “Interactions of birational geometry with other fields”, July 2 - 7, 2012, University of Durham, UK

Invited Talk “Subgroups of Cremona groups and Fano varieties”

[2] “Birational Geometry and Derived Categories”, August 1–6, 2012, Vienna

Talk “Subgroups of Cremona groups and Fano varieties”

[3] Essential Dimension and Cremona Groups, June 11 - 15, 2012, Chern Institute of Mathematics Nankai University, Tianjin, China

Talk “On elements of order two in the space Cremona group”

[4] ACC for minimal log discrepancies and termination of flips, May 14 - 18, 2012, American Institute of Mathematics, Palo Alto, California

Talk “BAB and subgroups of Cremona groups”

[5] Affine Algebraic Geometry Meeting, March 1-4, 2012, Osaka

Talk “Subgroups of Cremona groups”

[6] International conference ”KUL!FEST” dedicated to the 60th anniversary of Vik. S. Kulikov, December 3 - 7, 2012, Steklov Mathematical Institute of RAS, Moscow

Talk “On finite Abelian subgroups in the space Cremona group”

[7] “Algebraic and Differential Geometry of Andrei Tyurin”, October 24 - 26, 2012, Steklov Mathematical Institute of RAS, Moscow

Talk “On elements of finite order in the three-dimensional Cremona group”

[8] Visit to Max-Planck-Institut für Mathematik, Bonn (July-August 2012)

Talk “Subgroups of Cremona groups” at MPI-Oberseminar, Bonn

[9] Visit to Kavli Institute for the Physics and Mathematics of the Universe, Kashiwa, Tokyo (February 8 - 9, 2012)

Two Talks “Subgroups of Cremona groups I & II” at “MS Seminar (Mathematics - String Theory)”

[10] Visit to Mathematisches Institut, Universität München (18-19 July 2012)
Talk "Subgroups of Cremona groups" at Oberseminar Algebraische Geometrie

[11] Visit to Fudan University, Shanghai (March 21 – April 3, 2012)
Talk "Fano 3-folds of large Fano index" at the algebraic geometry seminar

[12] Visit to Korea Institute for Advanced Study (KIAS), Seoul (March 17 – 21, 2012)
Talk "Finite subgroups of Cremona groups and singular Fano varieties" at Mathematics seminar of KIAS

[13] Visit to RIMS, Kyoto University (February – March 2012)

Leonid Rybnikov

[1] Conference "Workshop Integrability: Modern Variations ", Bonn, January, 9 – 14
Talk "Laumon spaces and Yangians".
[2] "Japan-Russia Winter School", Moscow, January 15 – February 5
Mini-course "Laumon spaces and Representation Theory".
[3] "XI International school of ITEP-HSE-ITP on Mathematics and Theoretical Physics", Sevastopol, May 1 – 10
Mini-course "Gelfand-Tsetlin bases and MacMahon formula".
[4] "Japan-Russia Summer School", Kyoto, July 15 – August 14
Mini-course "Gelfand-Tsetlin bases and beyond".
[5] Conference "Algebraic Structures in Integrable Systems", Moscow, Decembber, 3 – 7
Talk "Gaudin model and Cactus group".
[6] Visit to MIT (Cambridge, MA, USA) and University of Oregon (Eugene, OR, USA), September-October.
Talk "Gaudin model and piecewise linear transformations" at "Representation Theory and related topics seminar" (Northeastern University).
Talk "Laumon spaces and Yangians" at "Infinite-dimensional algebra seminar" (MIT).
Talk "Gaudin model and piecewise linear transformations" at "Algebra seminar" (University of Oregon).

Timur Sadykov

[1] Russian-German Conference "Multidimensional Complex Analysis", Moscow, Steklov Institute, February, 27 – March, 2
Plenary lecture "Monodromy of hypergeometric systems and analytic complexity of algebraic functions"
[2] Complex Analysis Seminar at Moscow State University

Talk “Monodromy of holonomic systems of partial differential equations with an application in cryptography”

[3] This year I have been hosting visits of my colleagues from abroad rather than traveling myself (as opposed to 2010 when I spent two months in Japan and visited Stockholm University and 2011 when I visited Texas A& M University, University of Utah, Rutgers University and a software company in New York; in 2012, it was time for me to repay the hospitality of my overseas colleagues). The visits that I hosted in 2012 are as follows:

a) Visit by Professor Leon Brenig of Universite Libre de Bruxelles to Russian State University for Trade and Economics with a colloquium talk and a joint research work in progress.

b) Three visits by Professor Susumu Tanabe of Galatasaray University (Istanbul, Turkey) to Russian State University for Trade and Economics with several colloquium talks and a joint research work in progress.

Arkady Skopenkov

[1] The International Alexandroff Conference, Moscow, 2012

Talk “Classification of smooth embeddings of non-simply-connected 4-manifolds into R^7”, May, 2012

[2] European Congress of Mathematicians, Krakow, July, 2012

Invited talk to the section on geometric topology “Classification of embeddings of 4-manifolds into R^7”

Poster “Classification of smooth embeddings of non-simply-connected 4-manifolds into R^7”.

[3] Conference of Moscow Institute of Physics and Technology, November, 2012

Talk “Classification of smooth embeddings of non-simply-connected 4-manifolds into R^7”

[4] Research visit to Bonn, April, 2012

[5] Postnikov Memorial Seminar, Moscow State University,

Talk “Generalizations of Seifert form”

[6] Seminar of Department of Discrete Mathematics, Moscow Institute of Physics and Technology,

Talk “Hardness of recognition of the realizability of hypergraphs”

[7] Seminar on Discrete Mathematics, Moscow Institute of Physics and Technology,

Talk “Algorithms for recognition of the realizability of hypergraphs”

Mikhail Skopenkov

[1] International topological conference "Alexandroff Readings", Moscow, May 21 – 25.

Talk "When the set of embeddings is finite?"

[2] International Conference dedicated to the 65-th anniversary of Askold G. Khovanskii, Moscow, June 04 – 09.

Talk "Surfaces containing several circles through each point".

[3] St. Peterburg school on probability and statistical physics, St. Peterburg, June 18–29.

Without talk.

[4] Oberwolfach workshop "Discrete differential geometry", Oberwolfach, July 8–14.

Talk "Discrete analytic functions: convergence results".

[5] Visit to King Abdullah University of Science and Technology, Thuwal, Saudi Arabia, February.

[6] Visit to Berlin Technical University, March.

Talk "Discrete Riemann surfaces: convergence results".

[7] "Geometry and Topology". International Conference dedicated to the 75-th birthday of Alexei B. Sossinsky. Moscow, October 8–9.

Member of the organizing committee.

[8] Talks at the seminars of A.S. Mischenko, S.P. Novikov, Ya.G. Sinai.

Evgeni Smirnov

International conferences:

[1] British-Russian winter school on McKay correspondence, Warwick, UK, February 20–25, 2012.

Talk "Schubert calculus and Gelfand–Zetlin polytopes"

[2] Spring school "Geometric representation theory", Bochum, Germany, February 27 – March 2, 2012

[3] Workshop "Cluster Algebras and Combinatorics", Graz, Austria, March 8–10, 2012

Talk "Schubert polynomials and related combinatorial objects"

[4] Conference "Geometrie algébrique complexe", Marseille, France, March 12–16, 2012

[5] Third Workshop on Moduli Spaces, Cluster Algebras, Knots, and Topological Recursion, Moscow, Russia, May 28 – June 2, 2012

Talk: "Schubert polynomials, pipe dreams, and associahedra"

[6] "Algebra and Geometry": International conference in honour of A. Khovanskii's 65-th birthday, Moscow, Russia, June 4–10, 2012

Talk: "Schubert polynomials, pipe dreams, and associahedra"

[7] International conference on Cremona groups and essential dimension, Tianjin, China, June 11–15, 2012

Talk "Schubert calculus and Gelfand–Zetlin polytopes"

[8] Kul!Fest. International conference dedicated to Viktor Kulikov's 60th birthday. Moscow, Russia, December 3–7, 2012

National conferences:

[8] Third summer school in Lie algebras and invariant theory, Togliatti, Russia, June 24–29, 2012 (invited lecturer)

Mini-course "Flag varieties" (3 lectures)

[9] Cambridge-Oxford-Warwick seminar (COW), Warwick, UK, February 22, 2012

Talk: "B-orbits on double Grassmannians"

Andrei Sobolevski

[1] Workshop "Mathematics of particles and flows", Vienna, May 28–June 2, 2012

Talk "From particles to Burgers and beyond: some new random growth models"

[2] Conference "Optimal transport (to) Orsay", Orsay, June 18–22, 2012

Talk "Dynamics inside singularities of viscosity solutions to the Hamilton-Jacobi equation"

[3] Conference "Monge-Kantorovich optimal transportation problem, transport metrics and their applications", St Petersburg, June 4–7, 2012

Talk "On minimum-weight perfect matchings on the line"

[4] Workshop "Tropical and Idempotent Mathematics", Moscow, August 26–31, 2012

Talk "A minimum-weight perfect matching process for cost functions of concave type in 1D"

Vladlen Timorin

[1] Christmas meetings with Pierre Deligne, the "Dynasty" foundation meeting dedicated to the 20th Anniversary of the IUM, Moscow, January, 8 – January, 10.

Talk "Matings, captures and regluings"

[2] Holomophic foliations and complex dynamics, Moscow, June, 11 — June, 15.

Talk "Matings, captures and regluings"

[3] The eighteenth International Conference on Difference Equations and Applications, Barcelona (Spain), July, 22 — July, 26.

Talk "Regluing and topological models of rational functions"

[4] INdAM Conference New Trends in Holomorphic Dynamics, Cortona (Italy), September, 2 — September, 7.

Talk "The main cubioid"

Ilya Vyugin

[1] Workshop "Differential and difference equations in the complex domain", Warsaw, 10.09.2012-13.09.2012,

Talk "Isomonodromic confluence of singularities and solutions of fifth and sixth Painleve equations"

[2] Conference "Geometric Days in Novosibirsk", Novosibirsk, 30.08.1012-2.09.1012,

Talk "Holomorphic vector bundles and differential equations on the Riemann sphere".

[3] Conference "ITIS-2012", Petrozavodsk, 19.08.2012-25.08.2012.

Talk "On additive shifts of multiplicative subgroups".

[4] Conference "Differential equations and optimal control", Moscow, 16.04.2012-17.04.2012.

Talk (joined with R.R. Gontsov) "On the question of solubility of some Fuchsian systems".

[5] Seminar "Analitical theory of differential equations" (D.V Anosov, V.P. Lexin), MIAN RAS

Talk "On solutions of fifth Painleve equation".

[6] Seminar "Dinamical systems" (Yu. S. Ilyashenko, I. Schurov), MSU,

Talk "On additive shifts of multiplicative subgroups".

[7] Seminar "Coding theory" (L.A. Bassalygo), IITP RAS,

Talk "On additive shifts of multiplicative subgroups".

Alexei Zykin

[1] Visit to Institut Fourier, Grenoble, 06.2012
Talk: "Propriétés asymptotiques des fonctions zêta"

2.3 Teaching

Ivan Arzhantsev

[1] Graded Algebras and Invariant Theory, Independent University of Moscow, II-V years students, September-December 2012, 2 hours per week.

Program. In this course we study Poincare series and Hilbert polynomials of graded algebras, homogeneous systems of parameters, regular sequences and Cohen-Macauley algebras. It is natural to illustrate these rather abstract notions of commutative algebra by means of algebras of invariants. The course is based on the survey paper by Richard Stanley (1979) on invariants of finite groups and their applications in combinatorics.

[2] Basic Algebra, Moscow State University, I year students, September-December 2012, 5 hours per week.

Program. Systems of linear equations, matrix algebra, groups, rings, fields, polynomials.

[3] Algebra, Moscow State University, II year students, September-December 2012, 2 hours per week.

Program. Finite groups, abelian groups, actions of groups, representation theory, rings and fields.

[4] Introduction to Lie Algebras, Moscow State University, II year students, September-December 2012, 2 hours per week.

Program. The aim of this special seminar is to introduce basic notions and results on Lie algebras. We discuss explicit examples, structure theorems and representations, and pay special attention to derivations and finite dimensional simple Lie algebras.

[5] Locally nilpotent derivations. Moscow State University, III-V year students, September-December 2012, 2 hours per week.

Program. We study algebraic theory of locally nilpotent derivations on finitely generated commutative algebras and its applications in Algebraic Geometry and Invariant Theory.

Alexander Bufetov

[1] Geometry. Independent University of Moscow, I year students, September-December 2012, 2 hours per week, joint with Vladlen Timorin.

Program: Volumes and determinants, complex numbers, projective plane, SL(2,R) and Lobachevsky geometry, projective classification of conics and quadrics, geometry of discrete groups, convex geometry. Literature: Prasolov, Tikhomirov, Geometry.

[2] Problems in Asymptotic Representation Theory, special course, undergraduate and master students, September-December 2012, 2 hours per week, joint with Grigori Olshanski.

Program: Radon-Nikodym theorem, Kakutani theorem, spaces of configurations, correlation functions, determinantal measures, the Macchi-Soshnikov theorem, scaling limits, multiplicative functionals, algebras of anticommutation relations, infinite determinantal measures.

Evgeny Feigin

[1] Algebra. Independent University of Moscow, I year students, September-December 2012, 2 hours per week.

Program:

1. Groups, rings, fields: definitions, examples, properties.
2. Vector spaces and linear maps: bases, matrices, quotient spaces, linear equations, determinants.
3. Polynomials: divisibility, irreducibility, roots, the fundamental theorem of algebra, symmetric polynomials.
4. Group theory: homomorphisms, quotient groups, abelian groups, representations.

5. Linear maps: eigenvectors, Jordan normal form, exponent.
6. Dual spaces, bilinear and quadratic forms, euclidean spaces.
7. Tensor algebra: tensor product, exterior and symmetric algebras, determinant, Grassmann algebra.

Sabir Gusein-Zade

[1] Calculus on manifolds, Independent University of Moscow, II year students, September-December 2012, 2 hours per week.

Program.
1. Submanifolds of an affine space (the implicit function theorem, the inverse function theorem, the theorem about the image).
2. Manifolds, tangent vectors, the tangent space, the differential of a map.
3. Vector fields and one-parameter groups of diffeomorphisms.
4. Partition of unity.
5. Riemann metrics on manifolds, volume, the integral of the first kind.
6. Tensors on manifolds, tensor operations.
7. Differential forms on manifolds, the differential operator.
8. Manifolds with boundaries, orientations of manifolds.
9. Integration of differential forms. Stokes theorem.
10. De Rham cohomologies. Their homotopy invariance.
11. Lie derivative.
12. Lie groups. Actions of groups on manifolds.
13. Linearization of finite and compact group actions.
14. Sard's theorem. Transversality. Thom transversality theorem (weak).
15. Reduction of analytic and geometric object to normal forms. Morse Lemma. Darboux Lemma.

[2] Analytic geometry, Moscow State University, I year students, September-December 2011, 4 hours per week.

Program.
1. Coordinates on a plane and in the space. Coordinates of points and coordinates of vectors.
2. Coordinate change on a plane and in the space.
3. Scalar product of vectors. Scalar product in coordinates (in orthogonal and in arbitrary affine coordinates).
4. Orthogonal coordinate changes on a plane.
5. Division of a segment in a given ration.
6. Lines on a plane. Parametric definition and definition by an equation.
7. Definition of a semi-plane by an inequality. Systems of linear inequalities on a plane.
8. Distance from a point to a line on a plane. Normal equation of a line.
9. Pencils of lines on a plane. Proper and non-proper pencils.

10. A plane in the space. Parametric definition and definition by an equation.
11. Distance from a point to a plane. Normal equation of a plane.
12. Lines in the space. Parametric definition and definition by equations.
13. Pencils of planes in the space. The condition of belonging of a plane to the pencil defined by two planes.
14. Bunches of planes in the space. The condition of belonging of a plane to the bunch defined by three planes.
15. Vector product of vectors. Definition and main properties. Computation of the vector product in orthogonal coordinates.
16. The oriented area of a parallelogram on a plane and the oriented volume of a parallelepiped in the space. Expression of the oriented area and the oriented volume in terms of a determinant.
17. Expression of the volume of a parallelepiped through the scalar and the vector products (the mixed product).
18. Matrix expression of a coordinate change on a plane or in the space.
19. Orthogonal coordinate changes and orthogonal matrices.
20. Gram matrix of a system of vectors. Its connection with the area and with the volume.
21. Orthogonal coordinate changes in the space Euler angles.
22. Algebraic curves on a plane. Theorem on "splitting-off of a line".
23. Plane curves of degree two. The affine classification.
24. The orthogonal classification of the curves of degree two. Reduction of a curve equation to the canonical form.
25. Quadratic forms in two and three variables. The matrix of a quadratic form and its change under a coordinate change.
26. The invariants of curves of degree two.
27. The semi-invariant of curves of degree two.
28. Determination of the canonical equation of a curve of degree two through the values of the invariants and of the semi-invariant.
29. Conjugate diameters of a curve of degree two. Tangents to a curve of degree two.
30. The ellipse and its geometric properties.
31. The hyperbola and its geometric properties.
32. The parabola and its geometric properties.
33. Definition of a curve of degree two in the polar coordinates. Rational parametrization of a curve of degree two.
34. Surfaces of degree two. The affine classification.
35. The orthogonal classification of the surfaces of degree two. Reduction of a surface equation to the canonical form.
36. The invariants of surfaces of degree two. Partial classification of the surfaces of degree two with the help of the invariants.
37. The plane conjugate to a direction for a surface of degree two. Tangent planes to a surface of degree two. Straight line generators of a surface of degree two.

38. Curves of degree two as conical sections.
39. Affine transformations. Matrix definition of an affine transformation. Orthogonal affine transformations.
40. The change of the matrix of an affine transformation under a coordinate change. The determinant of an affine transformation and its geometric meaning.
41. Orthogonal transformations (isometries) of a plane.
42. Orthogonal transformations (isometries) of the space.
43. Representability of an arbitrary affine transformation as the composition of an isometry and tensions.
44. The correspondence between points and pencils of lines on a plane. The completion of an affine plane. The projective plane.
45. Projective transformations of a plane. Their connection with projections in the space.
46. The projective invariant of four points on a projective line (the double ratio).
47. Projective (homogeneous) coordinates on a projective plane. A line on a projective plane.
48. Duality between points and lines on a projective plane. Dual statements.
49. Curves of degree two on a projective plane. The classification.
50. Curves of degree two passing through five and through four points on a projective plane.
51. The double ratio of four points on a curve of degree two.
52. Duality of curves of degree two on a projective plane. Pascal and Brianchon theorems.
53. The projective space.
54. Lines in the projective space. Plücker equations.

[3] Topological invariants of singularities, Moscow State University, III-V year students, September-December 2012, 2 hours per week.

Program.
1. The Euler characteristic as an additive function on algebras of sets.
2. The Euler characteristic as a measure for a notion of an integral (with respect to the Euler characteristic).
3. Quasi-projective varieties and their properties.
4. Properties of the integral with respect to the Euler characteristic over constructible sets. Fubini formula.
5. The integral with respect to the Euler characteristic and the Riemann-Hurwitz formula.
6. The integral with respect to the Euler characteristic and the (co)homology groups of algebraic varieties.
7. Classical monodromy transformation of a function germ and its zeta function.
8. A'Campo formula as an integral integral with respect to the Euler characteristic.
9. Order functions and the Poincaré series of filtrations.
10. The Poincaré series of a filtration as an integral with respect to the Euler characteristic over the projectivization of the space of function-germs.
11. Computation of the Poincaré series of the filtration defined by the natural valuation

corresponding to a plane curve singularity.
12. Multi-index filtrations. Different definitions of Poincaré series.
13. Computation of the Poincaré series of the multi-index filtration corresponding to a reducible plane curve singularity.

Alexander Kuznetsov

[1] "Derived categories of coherent sheaves", Science-educational center, Steklov Mathematical Institute, Spring 2012, 3 hours per week.

Program

- Exceptioanl objects, exceptional collections, Mutation functors and braid group action.
- Semiorthogonal decompositions, admissible subcategories, saturated categories.
- Saturated categories, Serre functor.
- Resolution of the diagonal on a projective space and Beilinson spectral sequence. Fourier–Mukai functors. Resolutions of the diagonal associated with an exceptional collection or a semiorthogonal decomposition. Exceptional collections on Grassmannians.
- Semiorthogonal decompositions for a flat morphism. Product of varieties. Projectivization of a vector bundle. Severi–Brauer varieties, Azumaya algebras and twisted derived categories.
- Semiorthogonal decompositions for a blowup. Relation of derived categories under a flop and a flip.
- Spinor bundles an exceptional collections on quadrics. Derived category of a fibration in quadrics and the sheaf of even parts of Clifford algebras.
- Lefschetz decompositions and homological projective duality.
- Homological projective duality for the double Veronese embedding and some Grassmannians. Autoequivalences, spherical twists, stability conditions and mirror symmetry.

More detailed information is on the course web-page

`http://www.mi.ras.ru/~akuznet/dercat/index-dercat.htm`

[2] "Root systems and Dynkin diagrams", summer school "Modern Mathematics", Dubna, July 2012, 3 lectures.

A root system is a finite set of vectors in a Euclidian vector space such that for any of these vectors v the reflection s_v with respect to the hyperplane H_v orthogonal to v preserves the system, and moreover, for any vector v' from the system the difference $s_v(v') - v'$ is an integer multiple of v.

In dimension 2 there are only three (reduced and irreducible) root systems: A_2, B_2 and G_2.

It turns out that one can classify all root systems. There are several infinite series and several exceptional systems.

We will discuss root systems in spaces of arbitrary dimension, classification, and Dynkin diagrams appearing in this relation. We also will consider an important generalization — affine systems and will discuss the areas of mathematics where these notions appear.

More detailed information is on the course web-page

`http://www.mccme.ru/dubna/2012/courses/kuznetsov.htm`

Sergei Natanzon

[1] Calculus. National research university 'Higher school of economics'. I year students, January-June 2012, 2 hours per week.

Program

1. Topology of vector spaces. 1.1. Open and closed sets. 1.2. Compacts in vector spaces. 1.3. Continuous maps. 2. Differentiable functions of many variables. 2.1. Differential of function. 2.2 Lagrange theorem. 2.3. Taylor's formula. 3. Differentiable maps. 3.1. Differential of maps. 3.2. Local geometry of curves. 3.3. Global geometry of curves on the plane. 4. Submanifolds. 4.1. Implicit Function Theorem. 4.2. Implicit Map Theorem. 4.3. Tangent space. 4.4. Lagrange multipliers and conditional extreme. 10. The canonical form of maps. 5.1. Expansion of diffeomorphisms on elementary. 5.2. The maps of constant rank. 5.3. Morse Lemma.

[2] Frobenius manifolds. Independent University of Moscow, 3-4 year students, January-June 2012, 2 hours per week.

Program

1. Frobenius pair. 2. Dubrovin structure: definition and two examples. 3. Frobenius structure. 4. Darboux-Egorov Equation. 5. Eyler field. 6. Potential. 7. WDVV Equations. 8. The simplest solutions of WDVV. 9. Analytic and algebraic solutions of WDVV. 10. Cohomological field theory. 11. Gromov-Witten invariants. 12. A bunch of flat cometrik.

[3] Sheafs and homological algebra. National research university 'Higher school of economics'. 3-5 year students, September-December 2012, 2 hours per week.

Program

1. Introduction. 2. Sheafs. 2.1. Basic definitions. 2.2. Covers. 3. Cohomology with coefficients in the sheaf. 3.1. Canonical resolvent of the beam. 3.2. Cohomology. 4. Exact sequences. 4.1. Soft sheaves. 4.2. Long exact sequence. 5. Axiomatic theory of cohomology. 5.1. Acyclic resolvents. 5.2. Axiomatic approach. 6. Cech cohomology. 6.1. Cohomology of coverings. 6.2. Leret.Theorem 7. De Rham cohomology. 7.1. Sheafs

of modules. 7.2. De Rham's theorem. 8. Vector bundles. 8.1. Definitions and examples. 8.2. Universal bundle. 9. Complex manifolds. 9.1. Differential forms. 9.2. Dolbeault cohomology. 10. Line bundles.

Grigori Olshanski

[1] With A. Bufetov
"Around asymptotic representation theory"
Independent University of Moscow, September-December 2012, 2 hours per week.
Program
Reminders from measure theory. The Radon–Nikodým theorem.
Direct products of measures. The Kakutani theorem.
Probability measures on a space of point configurations. Correlation functions.
Determinantal measures and their correlation kernels. The Macchi–Soshnikov theorem.
Examples of determinantal measures from asymptotic representation theory and random matrix theory.
Limit transitions in correlation kernels.
Multiplicative functionals on a space of point configurations. Infinite determinantal measures.
Representations of the algebra of anticommutation relations and applications to determinantal measures.
Approximatively finite-dimensional algebras and their representations.

[2] "Non-colliding processes with infinitely many particles".
Mini-course (3 lectures). St. Petersburg School in Probability and Statistical Physics (International Summer School), June 18, 2012 – June 29, 2012.
Program
Models of Markov dynamics for N interacting non-colliding particles ($N = 1, 2, \dots$) have been studied in the Random Matrix literature since Dyson's paper (1962) on the matrix-valued Brownian motion. However, extension of the theory to the case of infinitely many particles presents substantial difficulties. For instance, one wants to take an appropriate large-N limit, but it is difficult to justify it, to prove that the Markov property is not destroyed, or to make sense of a formal expression for the hypothetical infinite-particle generator. I will describe a new and relatively elementary method of constructing models of infinite-dimensional Markov dynamics based on some ideas from representation theory of infinite-dimensional groups.

Alexei Penskoi

[1] Differential Geometry. Independent University of Moscow, II year students, February-May 2012, 4 hours per week (lecture 2 hours + exersise class 2 hours).

1. Curves and surfaces in the plane and the three-dimensional space. Curvature, torsion, Frenet frame. First and second fundamental forms. Principal curvatures, mean curvature and Gauß curvature. Mean curvature normal vector. Euler formula for the normal section curvature.

2. Surfaces in n-dimensional space. First and second fundamental forms. Connections in the tangent and normals bundles on a surface. Second fundamental form and Weingarten operator. Gauß-Weingarten derivational equations. Gauß-Bonnet theorem for surfaces.

3. Basic theory of Lie groups ans algebras.

4. Vector bundles and gluing cocycles. Structure group. Euclidean and hermitian bundles. Natural operations with bundles. Orientable bundles.

5. Connections in vector bundles. Connection local form, Christoffel symbols. Connections in euclidean and hermitian bundles. Connections compatible with metrics and their curvature.

6. Riemannian manifolds. Curvature, torsion. Levi-Civita connection. Symmetries of curvature tensor. Ricci tensor. Scalar curvature.

7. Riemannian manifolds II. Geodesics. Geodesic coordinates. Lagrangian approach to geodesics. Second variation.

8. Submanifolds of Riemannian manifolds. First and second fundamental forms.

9. Characteristic classes. Chern-Weil construction of characteristic classes. Chern, Pontryagin and Euler classes and their properties.

10. Vector bundles and their cohomologies. Thom class. Mathai-Quillen construction of the Thom class. The connection between the Thom class and the Euler class.

11. Elements of K-theory: K-groups and K-groups with a compact support.

12. Differential operators in vector bundles.

13. Atiyah-Singer theorem.

[2] Calculus on Manifolds. Math in Moscow program of the Independent University of Moscow for undergraduate students from the U.S. and Canada, February-May 2012, 4 hours per week (lecture 2 hours + exersise class 2 hours).

1. Definition and examples of smooth manifolds.

2. Orientability and orientation.

3. Tangent vectors and tangent space to a manifold at a point. Tangent bundles. Vector fields.

4. Skew-symmetric forms on linear spaces. Wedge product.

5. Differential forms on manifolds. Exterior differential.

6. Smooth maps of manifolds. Diffeomorphisms. The transformation rule under coordinate change for functions, vector fields and differential forms.

7. Integration. Coordinate change in the integral. Integration of differential forms. Stokes theorem. Green's formula, Gauss-Ostrogradskii divergence theorem, Stokes formula for a surface in $\mathbb{R}^3$.

8. Closed and exact forms. The Poincare lemma. De Rham cohomology.

[3] Differential Geometry. Math in Moscow program of the Independent University of Moscow for undergraduate students from the U.S. and Canada, September-December 2012, 4 hours per week (lecture 2 hours + exersise class 2 hours).

1. Plane and space curves. Curvature, torsion, Frenet frame.

2. Surfaces in 3-space. Metrics and the second quadratic form. Curvature.

3. Connections in tangent and normal bundles to a k-surfaces in $\mathbb{R}^n$.

4. Parallel translations.

5. Geodesics.

6. Gauß and Codazzi formulas. "Theorema egregium" of Gauß.

7. Gauß-Bonnet theorem.

8. Extremal properties of geodesics. Minimal surfaces.

[4] Exercise classes for various courses at Moscow State University: Classical Differential Geometry, February-May 2012, 2 hours per week; Analytic Geometry, September-December 2012, 4 hours per week; Differential Geometry, September-December 2012, 2 hours per week.

Yuri Prokhorov

[1] Algebra-1. Moscow State University, I year students, September-December 2012, 3 hours per week (plus seminars).

Program. Matrices and vectors. Solutions of linear equations (Gaussian elimination). Determinants. Linear independence and rank. Invertible and singular matrices. Vector spaces. Dimension. Rings, fields, algebras. Isomorphisms and homomorphisms. Polynomials. Field of fractions. Unique factorization domains. Complex numbers. Discriminant. Resultant. Cyclic groups. Lagrange's Theorem. Fundamental homomorphism theorem. Rings: ideals, homomorphisms and quotient rings. Field extensions.

[2] Elliptic curves and cryptography. Moscow State University, V year students, September-December 2012, 2 hours per week.

Program. Fields. Finite fields. Algebraic varieties (affine and projective). Field of rational functions. Local ring. Dimension. Singular points. Elliptic curves. Weierstrass equation. j-invariant. Hessian. Group law. Inflection points. Torsion group. Elliptic curves over non-closed fields. Non-rationality of elliptic curves. Elliptic curves over the complex numbers. Riemann surfaces. Elliptic functions. Modular group.

[3] Birational algebraic geometry. Independent University of Moscow, III year students, September-December 2012, 2 hours per week.

Program. Introduction: rational maps, divisors, linear systems, rational and unirational varieties. Divisorial algebra, its finite generation, Zariski decomposition. Case of surfaces and difficulties in higher dimensions. Cohomological birational invariants. Rationally connected and uniruled varieties. Deformations of rational curves. On existence of rational curves on varieties. Criterion of uniruledness. Birational classification of surfaces. Difficulties in higher dimensions. Non-rationality of hypersurfaces of large degree. Castelnuovo rationality criterion. Lüroth problem. Different approaches to establish non-rationality. Groups of birational automorphisms. Cremona transformations.

[4] Fano varieties and Cremona groups. 35th Autumn School in Algebraic Geometry, Lukecin, Poland, September 23-29, 2012 totally 5 lectures for graduate students (as a senior teacher).

Program. Basic properties of Fano varieties. Examples. Del Pezzo surfaces. Introduction to Mori theory, Fano varieties in the framework of the MMP. Three-dimensional MMP. Outline of Mori-Mukai classification. Introduction to Sarkisov links. Examples. Outline of Iskovskikh classification (three-dimensional case). Application to the classification of finite subgroups of Cremona groups.

Leonid Rybnikov

[1] Lie Groups and Lie Algebras, Higher School of Economics, Department of Mathematics, III and IV year students, January - June 2012, 2 hours per week.
Program.

1. Lie groups: definitions and examples.
2. Action of a Lie group on a manifold.
3. Tangent Lie algebra. Exponential map.
4. Linear representations of Lie groups and Lie algebras.
5. Finite-dimensional representations of $\mathfrak{sl}_2(\mathbb{C})$ and SU_2.
6. Invariant tensors on Lie groups: Poincare-Birkhof-Witt theorem.
7. Invariant tensors on Lie groups: Chevalley complex.
8. Solvable and nilpotent Lie algebras: theorems of Lie and Engel.
9. Compact Lie groups: invariant integrating.
10. Maximal tori and root systems.
11. Compact Lie groups and complex semisimple Lie algebras.
12. Representations of compact Lie groups and the Weyl formula.

[2] Geometry, Higher School of Economics, Department of Mathematics, I year students, September 2012 - March 2013, 3 hours per week.
Program.

1. Geometric transformations.
2. Affine plane and affine transformations.
3. Projective plane and projective transformations.
4. Affine and projective spaces.
5. Bilinear forms and quadrics.
6. Conics and projective duality.
7. Hyperbola, parabola, and ellipse.

8. Transformation groups: examples.

9. Discrete groups of affine transformations.

10. Lobachevsky plane: models.

11. Lobachevsky plane: symmetries.

12. Lobachevsky plane: discrete groups.

[3] Algebra (exercises, after E. Smirnov). Independent University of Moscow, I year students, February-May 2012, 2 hours per week.

[4] (future) Introduction to Geometric Representation Theory. Independent University of Moscow, February-May 2013, 2 hours per week.

Program.

1. Complex semisimple Lie groups and Lie algebras.

2. Hamiltonian actions of Lie groups and the Moment map.

3. Flag variety.

4. Springer resolution of the nilcone.

5. Borel-Moore homology.

6. Homological action of the Weyl group.

7. (if time allows) Equivariant cohomology and localization.

8. (if time allows) Homological actions of Lie algebras.

Timur Sadykov

[1] Complex analysis. Independent University of Moscow, II year students, February-May 2012, 4 hours per week.

Program

1. Complex numbers. The Riemann sphere and the stereographic projection. Holomorphic functions. Power series and their properties (the Abel lemma, the disk of convergence, analyticity of the sum of a power series). Exponential and trigonometric functions.

2. The Cauchy-Riemann equations. The Cauchy theorem and the cauchy integral. The Taylor series development of a holomorphic function. The Cauchy inequalities. Liouville's and Morera's theorems. The Weierstrass theorem on the derivative of a series with holomorphic elements.

3. The Riemann theorem on removable singularities. Analytic continuation across a smooth curve. The Sokhotski theorem. Laurent series. Classification of isolated singularities in terms of the coefficients of Laurent series. Meromorphic functions. The multiplicities of a zero and a pole of a meromorphic function.

4. Analytic continuation. The uniqueness theorem. The sheaf of germs of holomorphic functions.

5. The Riemann surface of a multi-valued analytic function. Analytic continuation and coverings. Monodromy theorem for simply connected domains. The analytic continuation of the antiderivative of a holomorphic function.

6. The divisor of a meromorphic function. Mittag-Leffler's theorem on meromorphic functions with prescribed divisors.

7. The Weierstrass theorem on the existence of an entire functions with given poles and their multiplicities. The order and the type of an entire function.

8. Cauchy's residue theorem. Computing definite integrals through residues. Logarithmic residues. The Rouchet theorem. The principle of conservation of a domain. The maximum principle. Univalent functions. The Hurwitz theorem on sequences of univalent functions.

9. Montelle's theorem. The Riemann mapping theorem.

10. Caratheodory's theorem for domains with a "simple" boundary. Reflection principle. Picard's theorems. The Scwartz lemma.

11. The sheaf of holomorphic solutions to a linear differential equation with holomorphic coefficients. A survey of the solution to Hilert's 21st problem by A.A. Bolibrukh.

12. Elliptic functions, definition and basic properties. The Weierstrass function. Zeta function and the Riemann hypothesis.

Recommended textbooks

1. B.V. Shabat, Introduction to Complex Analysis. Moscow, Nauka Publishing Company, 1985.

2. O. Forster, Lectures on Riemann Surfaces. Springer, 1980.

[2] In 2012, I have served as the supervisor of three master theses and as scientific advisor of three PhD students. One of them, Vitalij Krasikov, has finished his graduate studies in October 2012 and is scheduled to defend his thesis (attached to the present report) in Spring 2013.

Arkady Skopenkov

[1] Classical problems of topology of 3-manifolds.

Independent University of Moscow, September-December 2012, 2 hours per week.

Program. We attempt to present some results, ideas and methods of 3-dimensional topology in a way which makes clear the visual and intuitive part of the constructions and the arguments. In particular, we show how abstract algebraic constructions appear naturally in the study of geometric problems. Before giving a general construction, we illustrate the main ideas in simple but important particular cases, in which the essence is not veiled by technicalities. More specifically, we present several classical and modern results on the homeomorphism, the embedding and the knotting problems.

More detailed version in Russian: http://ium.mccme.ru/f12/askopenkov-f12.pdf

[2] Algebraic Topology From Geometric Viewpoint, Moscow State University, September-December 2012, 2 hours per week.

Program. We attempt to present some results, ideas and methods of algebraic topology in a way which makes clear the visual and intuitive part of the constructions and the arguments. In particular, we show how abstract algebraic constructions (cohomology, characteristic classes and equivariant maps) appear naturally in the study of geometric problems. Before giving a general construction, we illustrate the main ideas in simple but important particular cases, in which the essence is not veiled by technicalities. More specifically, we present several classical and modern results on graphs, low-dimensional manifolds and vector fields.

More detailed version in Russian: http://www.mccme.ru/circles/oim/SPECKURS.pdf

[3] Discrete analysis (exercises), II year students, September-December 2012, 2 hours per week. Moscow Institute of Physics and Technology

Program. We study certain topics in combinatorics and graph theory (including random graphs).

More detailed version in Russian: http://dm.fizteh.ru/materials/year2/discretean_fall2012

[4] Some more proofs from the Book: solvability and insolvability of equations in radicals (minicourse), Summer School 'Modern Mathematics', July 2012, 10 hours.

Program. We present short elementary proofs of

the Gauss Theorem on constructibility of regular polygons;

the existence of a cubic equation unsolvable in real radicals;

the existence of a quintic equation unsolvable in complex radicals (Galois Theorem).

We do not use the terms 'Galois group' or even 'group'. However, our presentation is a good way to learn (or recall) starting idea of the Galois theory. The minicourse is accessible for students familiar with elementary algebra (including complex numbers), and could be an interesting easy reading for mature mathematicians. The material is presented as a sequence of problems, which is peculiar not only to Zen monasteries but also to serious mathematical education; most problems are presented with hints or solutions.

More detailed version in Russian: http://www.mccme.ru/dubna/2012/courses/skopenkov.htm

Mikhail Skopenkov

[1] Dissections of rectangles, random walks, and electric networks, Independent University of Moscow, I-III year students, September-December 2012, 2 hours per week.

Program.

1. Definition of a random walk. Definition of an electric network. Physical interpretation of the hitting probability. The 1-dimensional random walk is recurrent.

2. The existence and uniqueness of a potential in an electric network. The maximum principle. Conductance and its probabilistic interpretation. Energy conservation. The variational principle. The short-cut principle. Conductance between the center and the boundary of a square lattice $n \times n$. The 2-dimensional random walk is recurrent.

3. Conductances of trees. The 3-dimensional random walk is not recurrent.

4. Physical interpretation of dissections of a rectangle into squares. The Dehn theorem on tiling of a rectangle.

5.* Alternating-current networks. The Laszkovich–Freiling–Rinne-Szekeres theorem on dissections into similar rectangles.

6. Conductance of regular graphs. Infinite electric networks. Conductance between adjacent nodes of an infinite square lattice.

[2] Distance courses for mathematical olympiads winners in Moscow Institute for Open Education (http://math.olymp.mioo.ru) and Higher School of Economics (http://lms.hse.ru).

Evgeni Smirnov

[1] Algebra, Independent University of Moscow, 1st-year students, February–June 2012, 2 hours of lectures and 2 hours of exercise sessions per week.

Course outline:

1. Multilinear algebra

1.1. Tensor product of vector spaces. Linear operators and bilinear forms as tensors.

1.2. Bilinear and quadratic forms. Geometry of a vector space with a quadratic form. Positive definite and Hermitian forms.

2. Notions of group theory

2.1. Groups, subgroups, normal subgroups, quotient groups. Action of a group on a set. Orbits, isotropy subgroups, conjugacy classes, Burnside formula.

2.2. Sylow theorems. Simple groups. Semidirect products. Symmetry groups of geometric objects.

3. Representation theory of finite groups

3.1. Representations, main definitions, Maschke theorem, Schur lemma

3.2. Characters, orthogonality relations.

3.3. Regular representation. Burnside formula, group algebra.

3.4. Induced representations. Frobenius reciprocity. Representations of symmetric groups.

[2] Algebra, 1st year, 4th quarter. Department of Mathematics, Higher School of Economics, April–June 2012, 3 hours of lectures and 2 hours of exercise classes per week

Course outline:

1. Rings, fields, ring homomorphisms. Ideals, quotient rings, the homomorphism theorem. Eucledian domains, principal ideal domains. GCD and LCD, associated elements. Chinese remainder theorem for principal ideal domains.

2. Unique factorization domains. Prime and irreducible elements. Integral principal ideal domains are unique factorization domains. Polynomials. Gauss's Lemma. The polynomial ring over a UFD is a UFD.

3. Modules over rings. Examples. Vector space with an operator is a $k[t]$-module. Homomorphisms. System of generators, basis. Free modules, rank. Lattices and sublattices. Submodule of a free module is free.

4. Description of finitely generated modules over principal ideal domains. Applications: classification of finitely generated abelian groups, Jordan normal form of an operator. Jordan normal form theorem.

Andrei Sobolevski

[1] Calculus. Independent University of Moscow, I year students, January–May 2012, 4 hours per week.

1. Normed spaces, metrics, topologies
2. Continuous functions, convergence and series
3. Measure and Lebesgue integral
4. Improper integrals. Integrals along curves
5. Differentiation of multivariate functions
6. Classical vector analysis and differential forms
7. The Stokes theorem in general form
8. Multivariate extremal problems, Langange multipliers
9. Implicit function theorem
10. A glimpse on differential equations

[2] Concrete probability theory. Independent University of Moscow, II year students, September–December 2012, 2 hours per week.

1. Random integers, generating functions
2. Random scalars and vectors, characteristic functions
3. I.i.d. random variables, the law of large numbers and weak convergence
4. Central limit theorem and its generalization for stable laws
5. Stein's method for central limit theorem
6. Distributions of extremal values
7. Large deviations in descrete and continuous setting

[3] Topics in probability theory, Moscow Institute of Physics and Technology, III year students, September–December 2012, 2 hours per week.

1. Probability spaces, algebras of events, filtrations of algebras
2. A simple random walk and the Wiener process as its limit
3. Description of a random process in terms of correlation functions. Kolmogorov's theorem (without proof)
4. Markov processes. The Smoluchowski equation. Diffusion processes and the Fokker–Planck equation
5. Boundary conditions for Fokker–Planck equation. Distribution of the first exit time. Equilibrium probability distributions, reversibility, the Gibbs distribution
6. Stochastic integrals and stochastic differential equations. Itô's calculus
7. Spectral decomposition of random functions
8. Stationary processes and ergodic theory

Vladlen Timorin

[1] Geometry. Independent University of Moscow, I year students, September–December 2012, 2 hours per week.

Program:

- Complex numbers, fractional linear transformations.
- The cross-ratio.
- The Poincare upper half-plane.
- Groups of isometries, Fuchsian groups.
- Dot and cross products in $\mathbb{R}^3$.
- Quaternions and $SO(3)$.
- Periodic tessellations of the plane.
- Finite subgroups of $SO(3)$, regular polyhedra.
- Schläfli symbols.
- The Euler formula for convex polytopes.
- The projective plane, homogeneous coordinates.
- Projective transformations, cross ratios, the Möbis–von Staudt theorem.
- Theorems of Desuargues, Pascale, Pappus and Brianchon.
- Conics, classification.
- Rational parameterizations, the Bezout's theorem for conics.

[2] Basic Representation Theory, September–December 2012, Independent University of Moscow and the National Research University Higher School of Economics, M.Sc. students in Mathematics and Math in Moscow program students, 4 hours per week.

Program

- Introduction. Basic concepts of Representation Theory.
- Invariant subspaces and complete reducibility.
- Basic operations on representations.
- Properties of complex irreducible representations.
- The character theory.
- Representations of $SU(2)$ and $SO(3)$.
- Spherical harmonics.

Ilya Vyugin

[1] Calculus (lectrures and seminars). Independent University of Moscow, I year students, September-December 2012, 2+2 hours per week.

Program

1. Elements of set theory.
2. The theory of the real numbers.
3. The topology of the line compact.
4. Sequences.
5. Continuous functions.
6. Differential calculus. Taylor's formula.
7. Integral calculus.
8. Series, numerical and functional.
9. Differential calculus of functions of several variables.

[2] Calculus (seminars). High School Economics, II year students, September-December 2012, 2 hours per week.

Program

1. Improper integrals.
2. Integrals depending on a parameter.
3. Special functions.
4. Measure and the Lebesgue integral.

[3] Applied linear algebra (lectures), Moscow Institute of Phisics and Technology, III year students, Feburary-May 2012, 2 hours per week.

Program.

1. Solving of systems of linear equations by Gaussian elimination (LU-decomposition).

2. Estimation of condition numbers of linear systems.

3. Systems with symmetric matrix. Cholesky decomposition.

4. The linear least squares problem. Methods Householder reflections and Givens rotations. QR-decomposition.

5. The linear least squares problem. Singular decomposition.

6. The problem of finding the eigenvalues. Hessenberg form and Schur form. Finding the eigenvectors using the Shur form.

7. Power method and the inverse iteration method. Finding Schur form an orthogonal iteration and QR-iterations.

8. Finding the eigenvalues and eigenvectors of a symmetric matrix.

9. Finding the singular value decomposition using algorithms for finding the eigenvalues and eigenvectors of symmetric matrices.

10. System with implicit matrix. Krylov subspace. Arnoldi and Lanczos methods. The decision system using Krylov subspaces.

11. Iterative methods.

Alexei Zykin

[1] Introduction to Algebraic Number Theory. Independent University of Moscow and Mathematical Department of the Higher School of Economics, 2–4 year students, September–May 2012–2013, 2 hours per week.

Program

1. **Introduction.** Diophantine equations (Fermat's last theorem, congruent numbers and elliptic curves).

2. **Galois theory and finite fields.** Basic facts from Galois theory. The structure of finite fields. Equations over finite fields. Quadratic reciprocity law.

3. ***p*-adic numbers.** Congruences and p-adic numbers. Hensel's lemma. Ostrowski's theorem.

4. **Quadratic forms.** Representation of numbers by quadratic forms over $\mathbb{Q}_p$ and $\mathbb{Q}$. Minkowski–Hasse theorem.

5. **Algebraic number fields.** Prime ideal decomposition, ramification, discriminant, class number, units.

6. **Elliptic curves.** Basic properties. Mordell-Weil theorem.

7. **Zeta-functions.** Distribution of primes and Riemann zeta function. Dirichlet theorem on primes in arithmetic progressions. Functional equation for Dedekind zeta-function and residue formula.

[2] Advanced Topics in Number Theory. Independent University of Moscow and Mathematical Department of the Higher School of Economics, 3–5 year students, September–May 2011–2012, 2 hours per week.

1. **Elliptic curves.** Basic notions of the theory of algebraic curves. Geometric properties of elliptic curves. Isogenies, Tate module, Weil pairing. Elliptic curves over $\mathbb{C}$ and lattices. Elliptic curves over finite fields, supersingular elliptic curves. Formal groups and elliptic curves. Galois cohomology, heights and Mordell–Weil theorem. L-functions of elliptic curves, Taniyama–Shimura–Weil conjecture.

2. **Modular forms — basic properties.** Elliptic functions, j-invariant. The algebra of modular forms for $SL_2(\mathbb{Z})$. Delta function, partition number, the sum of four squares. L-functions of modular forms, functional equation. Hecke operators and Euler product. Modular forms for congruence subgroups: a sketch.

3. **Analytic methods in algebraic number theory.** Distribution of primes and Riemann zeta function. Dirichlet theorem on primes in arithmetic progressions. Functional equation for Dedekind zeta-function and residue formula.

4. **Local and global fields.** Valuations of algebraic number fields. p-adic fields. Prime ideal decomposition and complete local rings. Different and ramification. Cyclotomic fields. Ideles and adeles. Modular forms and adeles.

5. **Basic notions of the class field theory.** Statements of the main results. Applications: Hilbert class field and prime ideal decomposition, Hasse principle for quadratic forms, central simple algebras. Elliptic curves with complex multiplication: statements of the main results.

6. **Curves over finite fields.** The analogy between number fields and function fields. Geometry of curves over finite fields. Riemann hypothesis for curves over finite fields. The maximal number of points problem for curves over finite fields.