The IUM report to the Simons foundation, 2016

Contents

1 Introduction: list of awardees

The Simons foundation supported two programs launched by the IUM:
Simons stipends for students and graduate students;
Simons IUM fellowships.

11 applications were received for the Simons stipends contest. The selection committee consisting of *Yu.Ilyashenko (Chair), G.Dobrushina, G.Kabatyanski, S.Lando, I.Paramonova (Academic Secretary), A.Sossinsky, M.Tsfasman* awarded Simons stipends for 2016 year to the following students and graduate students:

1. Egorova, Elena Evgenevna
2. Kononov, Yakov Alexandrovich
3. Loginov, Konstantin Valerevich
4. Matushko, Maria Georgievna
5. Oganesyan, Dmitry Alexeevich
6. Oganesyan, Vardan Spartakovich
7. Sechin, Pavel Andreevich
8. Zaev, Danila Andreevich
9. Zubov, Dmitry Igorevich

14 applications were received for the Simons IUM fellowships contest for the first half year of 2016 and 16 applications were received for the second half year. The selection committee consisting of *Yu.Ilyashenko (Chair), G.Dobrushina, B.Feigin, I.Paramonova (Academic Secretary), A.Sossinsky, M.Tsfasman, V.Vassiliev* awarded

Simons IUM-fellowships for the first half year of 2016 to the following researches:

1. Aizenberg, Anton Andreevich
2. Elagin, Alexei Dmitrievci
3. Kazarian, Maxim Eduardovich
4. Khoroshkin, Anton Sergeevich
5. Krasilshchik, Iosif Semenovich

6. Kuznetsov, Alexander Gennadevich
7. Olshanski, Grigory Iosifovich
8. Penskoi, Alexei Victorovich
9. Pushkar, Petr Evgenevich
10. Shabat, George Borisovich
11. Smirnov, Evgeni Yurevich
12. Verbitsky, Mikhail Sergeevich

Simons IUM-fellowships for the second half year of 2016 to the following researches:

1. Aizenberg, Anton Andreevich
2. Belavin, Alexander Abramovich
3. Gorodentsev, Alexei Lvovich
4. Kazarian, Maxim Eduardovich
5. Leyenson, Maxim Ilyich
6. Penskoi, Alexei Victorovich
7. Pushkar, Petr Evgenevich
8. Rybakov, Sergey Yurevich
9. Shabat, George Borisovich
10. Skopenkov, Arkady Borisovich
11. Skopenkov, Mikhail Borisovich
12. Smirnov, Evgeni Yurevich
13. Vyugin, Ilya Vladimirovich

The report below is split in two sections corresponding to the two programs above. The first subsection in each section is a report on the research activities. It consists of the titles of the papers published or submitted in the year of 2016, together with the corresponding abstracts. The second subsection of each section is devoted to conference and some most important seminar talks. The last subsection of the second section is devoted to the syllabi of the courses given by the winners of the Simons IUM fellowships. Most of these courses are innovative, as required by the rules of the contest for the Simons IUM fellowships.

The support of the Simons foundation have drastically improved the financial situation at the IUM, and the whole atmosphere as well. On behalf of the IUM, I send my deep gratitude and the best New year wishes to Jim Simons, Yuri Tschinkel, and the whole team of the Simons foundation.

Yulij Ilyashenko

President of the Independent University of Moscow

2 Program: Simons stipends for students and graduate students

2.1 Research

2.1.1 Elena Egorova

[1] Elena Egorova, Grigory Kabatiansky, Marcel Fernndez, Moon Ho Lee, Signature codes for the A-channel and collusion-secure multimedia fingerprinting codes, IEEE International Symposium on Information Theory. IEEE, 2016. p. 3043-3047.

We consider collusion-resistant fingerprinting codes for multimedia content. We show that the corresponding IPPcodes may trace all guilty users and at the same time have exponentially many code words. We also establish an equivalence between signature codes for the A-channel and multimedia fingerprinting codes and prove that the rate of the best t-signature codes for A-channel is at least $\Theta(t^{-2})$. Finally, we construct a family of t-signature codes for the A-channel with polynomial decoding complexity and rate $\Theta(t^{-3})$

[2] Egorova E., Fernandez M., Kaatiansky G., Multimedia Fingerprinting Codes Resistant Against Colluders and Noise, Proceedings of 2016 8th IEEE International Workshop on Information Forensics and Security (WIFS), 4-7 December, 2016

Coding schemes for multimedia fingerprinting in the presence of noise and colluders are investigated. We prove that best such codes have nonvanishing rate, i.e., have exponentially many codewords (users) and can trace the entire coalition of pirates and do it either with zero error probability or w.h.p. depending on the corresponding model of errors.

[3] Egorova E., Potapova V. Signature Codes for a Special Class of Multiple Access Channel, Proceedings of 2016 XV International Symposium "Problems of Redundancy

in Information and Control Systems" (REDUNDANCY), 26-29 September 2016, Saint-Petersburg, Russia,ISBN 978-1-5090-4230-2, pages: 38-42

A specific noiseless multiple-access channel model is studied in this paper. Proofs for lower and upper bounds on the achievable-rate of corresponding codes are given.

[4] E.Egorova, V. Potapova, "Signature codes for compositional multiple-access channel *to appear in Journal Problems of Information Transmission.*

In this paper the notion of q-ary s-compositional code is introduced and it is proved that the rate of such code is

$$\frac{q-1}{4}\frac{\log s}{s} < R < \frac{q-1}{2}\frac{\log s}{s}.$$

2.1.2 Yakov Kononov

[1] Ya.Kononov, A.Morozov, *On Factorization of Generalized Macdonald Polynomials*, Eur.Phys.J. C76 (2016) no.8, 424,
http://arxiv.org/abs/arXiv:1607.00615

A remarkable feature of Schur functions – the common eigenfunctions of cut-and-join operators from W_∞ – is that they factorize at the peculiar two-parametric topological locus in the space of time-variables, what is known as the hook formula for quantum dimensions of representations of $U_q(SL_N)$ and plays a big role in various applications. This factorization survives at the level of Macdonald polynomials. We look for its further generalization to *generalized* Macdonald polynomials (GMP), associated in the same way with the toroidal Ding-Iohara-Miki algebras, which play the central role in modern studies in Seiberg-Witten-Nekrasov theory. In the simplest case of the first-coproduct eigenfunctions, where GMP depend on just two sets of time-variables, we discover a weak factorization – on a codimension-one slice of the topological locus, what is already a very non-trivial property, calling for proof and better understanding.

[2] D.Gepner, A.Belavin, Ya.Kononov, *Flat coordinates for Saito Frobenius manifolds and String theory*,
TMF(2016),
http://arxiv.org/abs/1510.06970

It was shown in [DVV] for 2d topological Conformal field theory (TCFT) [EY, W] and more recently in [BSZ]-[BB2] for the non-critical String theory [P]-[BAlZ] that a number of models of these two types can be exactly solved using their connection with the Frobenius manifold (FM) structure introduced by Dubrovin. More precisely these models are connected with a special case of FMs, so called Saito Frobenius manifolds (SFM) (originally called Flat structure together with the Flat coordinate system), which arise on the space of the versal deformations of the isolated Singularities after choosing of a suitabe so-called Primitive form, and which also arises on the quotient spaces by reflection groups. In this paper we explore the connection of the models of TCFT and non-critical String theory with SFM. The crucial point for obtaining an explicit expression for the correlators is finding the flat coordinates of SFMs as functions of the parameters of the deformed singularity. We suggest a direct way to find the flat coordinates, using the integral representation for the solutions of Gauss-Manin system connected with the corresponding SFM for a simple singularity. Also, we address the possible generalization of our approach for the models investigated in [Gep] which are $SU(N)_k/(SU(N1)_{k+1} \times U(1))$ Kazama-Suzuki theories [KS].

[3] Ya.Kononov, A.Morozov, *On rectangular HOMFLY for twist knots*,
Mod.Phys.Lett. A Vol. 31, No. 38 (2016) 1650223,
http://arxiv.org/abs/arXiv:1610.04778

As a new step in the study of rectangularly-colored knot polynomials, we reformulate the prescription of arXiv:1606.06015 for twist knots in the double-column representations $R = [rr]$ in terms of skew Schur polynomials. These, however, are mysteriously shifted from the standard topological locus, what makes further generalization to arbitrary $R = [rs]$ not quite straightforward.

[4] Ya.Kononov, A.Morozov, *Rectangular superpolynomials for the figure-eight knot*,
preprint,
http://arxiv.org/abs/arXiv:1609.00143

We rewrite the recently proposed differential expansion formula for HOMFLY polynomials of the knot 4_1 in arbitrary rectangular representation $R = [rs]$ as a sum over all Young sub-diagrams of R with extraordinary simple coefficients $D_{\lambda^{tr}}(r)D_\lambda(s)$ in front of the Z-factors. Somewhat miraculously, these coefficients are made from quantum dimensions of symmetric representations of the groups $SL(r)$ and $SL(s)$ and restrict summation to diagrams with no more than s rows and r columns. They possess a natural β-deformation to Macdonald dimensions and produces positive Laurent polynomials, which can be considered as plausible candidates for the role of the rectangular superpolynomials. Both polynomiality and positivity are non-evident properties of arising expressions, still they are true. This extends the previous suggestions for symmetric and antisymmetric representations (when $s = 1$ or $r = 1$ respectively) to arbitrary rectangular representations. As usual for differential expansion, there are additional gradings. In the only example, available for comparison – that of the trefoil knot 3_1, to which our results for 4_1 are straightforwardly extended, – one of them reproduces the "fourth grading" for hyperpolynomials. Factorization properties are nicely preserved even in the 5-graded case.

2.1.3 Konstantin Loginov

[1] Standard models of terminal threefolds admitting a del Pezzo fibration structure.
(In preparation)

We start with a three-dimensional projective variety that has at worst terminal singularities and admits a structure of a Mori fiber space such that the general fiber is a smooth del Pezzo surface and the base is a curve. We prove that such a variety has a modification with at worst Gorenstein singularities. Then we show that under some extra assumptions it can be embedded into the projectivization of a vector bundle on the base.

2.1.4 Maria Matushko

[1] With S. Khoroshkin, E. Sklyanin
On spin Calogero-Moser system at infinity
arXiv:1608.00599 *Accepted by Journal of Physics A: Mathmatical and Theoretical*

We present a construction of a new integrable model as an infinite limit of Calogero models of N particles with spin. It is implemented in the multicomponent Fock space. Explicit formulas for Dunkl operators, the Yangian generators in the multicomponent Fock space are presented. The classical limit of the system is examined.

[2] With V. Sokolov
Polynomial forms for quantum elliptic Calogero-Moser Hamiltonans
Submitted to Theoretical and Mathematical Physics

A conjecture on a change of variables that brings the elliptic Calogero-Moser A_N-operator to a differential operator with polynomial coefficients is formulated. In case $N \leq 3$ it is proved and explicit formulas for differential operators are presented.

2.1.5 Dmitry Oganesyan

[1] Abel pairs and modular curves// . 2016. **446**. 165-181.

In this paper we consider rational functions on algebraic curves, which have one zero and one pole (and call pair of such function and curve Abel pair). We investigate moduli spaces of such functions on curves of genus one; the number of Belyi pairs among them is calculated for fields $\mathbb{C}$ and $\overline{\mathbb{F}_p}$. This result could be fruitfully used for investigation of Hurwitz's space and modular curves for fields of finite characteristic

[2] Zolotarev polynomials and reduction of Shabat polynomials in positive characteristic, *to appear in Moscow University Mathematics Bulletin.*

In this paper we consider Shabat polynomials over fields of different characteristic and it's deformations to the polynomials with three finite critical values. Using such technics, we found primes of bad reduction of Shabat polynomials, corresponding to trees of diameter 4.

2.1.6 Vardan Oganesyan

[1] On operators of the form $\partial_x^4 + u(x)$ from a pair of commuting differential operators of rank 2 and genus g,
Russian Mathematical Surveys, 2016, 71:3, 591-593.

In this paper we consider differential operators of the form $L = \partial_x^4 + u(x)$. We find the commutativity condition for the operator L with a differential operator M of order $4g + 2$, where L and M are operators of rank 2.

[2] Explicit characterization of some commuting differential operators of rank 2,
International Mathematics Research Notices (2016), doi:10.1093/imrn/rnw085

In this paper we study self-adjoint commuting differential operators of rank 2. We consider differential operator $L = \partial_x^4 + u(x)$ and find the commutativity condition for the operator L with a differential operator M of order 4g + 2. Some examples are constructed. These examples do not commute with differential operators of odd order. Also eigenfunctions of operator L are studied.

[3] AKNS hierarchy and finite-gap Schrodinger potentials
arXiv:1512.03981 *submitted to Letters in Mathematical Physics*

In this paper we study AKNS hierarchy and nonlinear Schrodinger equation. We find explicit necessary conditions for functions p and q to be solution of some equation of AKNS hierarchy. Using functions p and q we construct finite-gap Schrodinger potentials

2.1.7 Pavel Sechin

[1] The category of flat Hodge-Tate structures
translated from Russian, Mathematical Notes, 2016, Vol. 99, No. 1, pp. 166-171

In this paper we calculate the Hopf algebra of the tannakian category of flat Hodge-Tate structures, as well as the algebra $\oplus_{i=0}^{\infty} Ext^i(\mathbb{Q}(0), \mathbb{Q}(i))$.

[2] Ring of operations from Morava K-theory to Chow groups (*in Russian*)
(to appear) Matematicheskiye zametki, 2017, Vol. 101, No. 1

In this paper we calculate the ring of operations from algebraic Morava K-theories to Chow groups.

[3] Chern classes from Algebraic Morava K-theories to Chow Groups
arXiv:1605.04444 *submitted to International Mathematics Research Notices*

In this paper we calculate the ring of unstable (possibly non-additive) operations from algebraic Morava K-theory $K(n)^*$ to Chow groups with $\mathbb{Z}_{(p)}$-coefficients. More precisely, we prove that it is a formal power series ring on generators $c_i : K(n)^* \to CH^i \otimes \mathbb{Z}_{(p)}$, which satisfy a Cartan-type formula.

2.1.8 Danila Zaev

[1] On some topics of analysis on noncommutative spaces
arXiv:1612.04371 [math.OA]

We consider a conservative Markov semigroup on a semi-finite W^*-algebra. It is known that under some reasonable assumptions it is enough to determine a kind of differential structure on such a "noncommutative space". We construct an analogue of a Riemannian metric in this setting, formulate a Poincaré-type inequality, provide existence and uniqueness results for quasi-linear elliptic and parabolic PDEs defined in terms of the constructed noncommutative calculus.

2.1.9 Dmitry Zubov

[1] With A.I. Bufetov and S. Gouëzel
Finitely-additive measures on the leaves of the invariant foliations of Anosov diffeomorphisms
to appear on arXiv in January 2017

We describe the limit behaviour of the averages of C^2 functions along the iterated unstable balls of a topologically mixing Anosov diffeomorphism in terms finitely-additive measures supported on the leaves of the unstable foliation, invariant under the holonomy in stable direction. The classification of finitely-additive measures is given in terms of the spectrum of the transfer operator acting on the Banach space of currents with a weak Sobolev norm.

2.2 Scientific conferences and seminar talks

2.2.1 Elena Egorova

[1] 2016 IEEE International Symposium on Information Theory, July 10-15, 2016. Barcelona, Spain

Session talk "Signature codes for the A-channel and collusion-secure multimedia fingerprinting codes"

[2] Tutorial Week at CIRM in context of the conference Nexus of Information and Computation Theories, January 25 29, 2016, Marseille, France

Poster "On multimedia digital fingerprinting codes "

[3] Nexus of Information and Computation Theories, Secrecy and Privacy Theme March 21 - April 1, 2016, Paris, France, participation.

[4] 8th IEEE International Workshop on Information Forensics and Security (WIFS) 2016, 4-7 December, 2016

Session talk "Multimedia Fingerprinting Codes Resistant Against Colluders and Noise"

[5] 1st Student Conference of CS faculty, 5-6, April, 2016, Higher School of Economics, Moscow, Russia

Session talk "Compositional codes for the multiple access channel with partial activity"

[6] The Alan Turing Contest in Theoretical Computer Science and Discreate Mathematics for Russian-speaking students (TuCo 2016), 1st of July, 2016, St. Petersburg Academic University, St. Petersburg, Russia

Session talk "Signature codes for a special form of multiple access channel", **received third prize**

[7] Kolmogorov Seminar on Descriptive and Computational Complexity of the Department of Mathematical Logic and Theory of Algorithms at Faculty of Mechanics and Mathematics of Moscow State University, Spring 2016

Talk "Digital fingeprint codes and IPP codes"

[8] Kolmogorov Seminar on Descriptive and Computational Complexity of the Department of Mathematical Logic and Theory of Algorithms at Faculty of Mechanics and Mathematics of Moscow State University, Spring 2016

Talk “Collusion secure digital fingerprinting codes”

[9] Seminar on Coding Theory at IITP RAS, fall 2016

Talk “Signature codes for compositional multiple-access channel”

2.2.2 Yakov Kononov

[1] Feigin’s seminar (HSE) on Mathematical Physics. Talks: The equivariant vertex and Nekrasov formula.

[2] Morozov’s seminar (ITEP). Talk: Generalized Macdonald polynomials.

2.2.3 Konstantin Loginov

[1] Conference “Cremona Conference 2016”, Basel, Switzerland, September 5—16

[2] Summer school “Algebra and Geometry”, Yaroslavl, Russia, July 25—31

[3] Talk “p-elementary subgroups of the plane Cremona group” at “Geometric structures on varieties” seminar (Higher School of Economics)

[4] Talk “Measures of irrationality of surfaces in $\mathbb{P}^3$” at Iskovskikh Seminar (Steklov Mathematical Institute)

[5] Talk “Picard-Lefshetz Theorem” at Variety Seminar (Independent University of Moscow)

[6] Talk “Abelian varieties and theta functions” at Introduction to Algebraic Geometry Seminar (Steklov Mathematical Institute)

[7] Talk “Hasse-Weil bound” at Student Geometric Seminar (Independent University of Moscow)

[8] Talk “Algebraic Codes on Del Pezzo Surfaces” at Algebraic Codes Seminar (Institute for Information Transmission Problems)

2.2.4 Maria Matushko

[1] Conference “Recent Advances in Quantum Integrable Systems”, Geneva, Switzerland, August 22 – 26

[2] Workshop on Classical and Quantum Integrable Systems, Euler International Mathematical Institute, St. Petersburg, July 11 – 15

[3] Weekly seminar “Problems of Mathematical Physics”, Higher School of Economics

Talk “Spin Calogero-Moser system at infinity”

2.2.5 Dmitry Oganesyan

[1] March-April, Moscow.

Talk "The Fried families and reduction of Belyi pairs to the field with positive characteristic" (HSE)

Talk "An amazing formula and its consequences" (MSU)

[2] September-December, Moscow. Talk " Toric unicellular dessins and propellers" (MSU)

Talk "Abel-Pell equations" (MSU)

Talk "Abel pairs of genus 1 and their j-invariants." seminar (MSU)

2.2.6 Vardan Oganesyan

[1] Conference "Recent Advances in Complex Differential Geometry", Toulouse, France, July 1322

Poster "Commuting differential operators"

[2] Conference "4-th Hedelberg Laureate Forum",Heidelberg University, Heidelberg, Germany, September 18-23

Poster "Commuting differential operators"

[3] Conference "Lomonosov-2016", Moscow State University, Moscow, April 11-15

Talk "Matrix commuting differential operators""

[4] Conference "Alexandroff Readings" Moscow State University, Moscow, May 21-25

Talk "AKNS hierarchy and finite gap Schrodinger potential"

2.2.7 Pavel Sechin

[1] Workshop "Algebraic Cobordism and Projective Homogeneous Varieties", Oberwolfach, January, 31 – February, 6

Talk "Chern classes for Morava K-theories"

[2] Visit to Munich, May

Talk "Chern classes for Morava K-theories" at "Oberseminar: Motivische Algebraische Topologie" (Ludwig Maximilian University of Munich)

[3] Visit to Saint-Petersburg, November

Talk "Morava-orientable orientable theories" at "A1-homotopy and K-theory Seminar" (Chebyshev Laboratory at Saint-Petersburg State University)

2.2.8 Danila Zaev

[1] Talk “C^*-Dirichlet forms” at “Algebras in analysis” (Moscow State University), Moscow, April, 1

[2] Talk “Ergodic decomposition of Kantorovich problem” “Seminar on applied mathematics” (Moscow Institute of Physics and Technology), Moscow, September, 23

[3] Talk “Noncommutative Lipschitz algebras” at “Noncommutative geometry and functional analysis” (Higher School of Economics), Moscow, October, 7

2.2.9 Dmitry Zubov

[1] Conference “School on Algebraic, Geometric and Probabilistic Aspect of Dynamical Systems and Control Theory” at The Abdus Salam International Centre for Theoretical Physics (ICTP), Trieste, Italy, 4-15 July 2016

Talk: “On cohomological equations for suspension flows over Vershik automorphisms”

Conference “Geometric Analysis and Control Theory” at The Sobolev Institute of Mathematics, Novosibirsk, Russia, 8-12 December 2016

Talk: “Finitely-additive measures on the invariant foliations of Anosov diffeomorphisms”

Confrerence “Traditional Winter Session PDMI-MI RAS Devoted to the Topic ’Probability Theory’ ”, Saint-Petersburg, Russia, 13-15 December 2016

Talk: “Projections of orbital measures for the classical Lie groups”

[2] Visit to Marseille, France, in March 2016

Talk: “On cohomological equations for suspension flows over Vershik automorphisms” at “Séminaire TEICH” (Institut de Mathématiques de Marseille), 18 March 2016

[3] Series of lectures “Palm measures” (4 lectures, joint with Alexander I. Bufetov) at the Summer School “Contemporary Mathematics”, Dubna, Russia, 19-30 July 2016

Talk “On cohomological equations for suspension flows over Vershik automorphisms” at the Weekly seminar of the Laboratory of Algebraic Geometry and its Applications (Higher School of Economics, Moscow, Russia), 5 February 2016

Talk “Dynamics on flat surfaces” at the PhD students’ seminar (Higher School of Economics, Moscow, Russia), 12 April 2016

Talk “Finitely-additive measures on the invariant foliations of Anosov diffeomorphisms” at the Seminar of the Research Group “Dynamical Systems” (Higher School of Economics, Moscow, Russia), 27 April 2016

Talk “Finitely-additive measures in parabolic dynamics” at the Seminar “Ergodic Theory and Mathematical Physics” (Moscow State University, Moscow, Russia), 30 November 2016

3 Program: Simons IUM fellowships

3.1 Research

3.1.1 Anton Aizenberg

[1] Homology cycles in manifolds with locally standard torus actions.
Homology, Homotopy Appl. 18:1 (2016), 1–23.

We describe the homology of a closed manifold X with a locally standard action of a half-dimensional torus under the assumption that proper faces of its orbit space Q are acyclic and the free part of action is trivial. There are three types of homology classes in X: (1) classes of face submanifolds; (2) k-dimensional classes of Q lifted to X and swept by actions of subtori of dimensions $< k$; (3) relative k-classes of Q modulo ∂Q lifted, in appropriate way, to X and swept by actions of subtori of dimensions $\geq k$. The submodule spanned by face classes is an ideal in $H_*(X)$ with respect to the intersection product. As a ring it is isomorphic to $(\mathbb{Z}[S_Q]/\Theta)/W$, where $\mathbb{Z}[S_Q]$ is the face ring of the Buchsbaum simplicial poset dual to Q; Θ is an ideal generated by a linear system of parameters; and W is a submodule lying in the socle of $\mathbb{Z}[S_Q]/\Theta$. The intersection product in homology is described in terms of the product in the face ring and intersection products on the orbit space and on the torus. Manifolds with torus actions provide a topological interpretation for the results of Novik and Swartz concerning socles of Buchsbaum face rings.

[2] Topological model for h"-vectors of simplicial manifolds.
Bol. Soc. Mat. Mexicana (2016), 1–9.

Given a simplicial poset S whose geometrical realization is a closed orientable homology manifold, Novik and Swartz introduced a Poincare duality algebra $(\mathcal{R}[S]/(l.s.o.p.))/I_{NS}$, which is a quotient of the face ring of the poset S. The ranks of graded components of this algebra are now called h''-numbers of S and can be computed from face-numbers and Betti numbers of S. We introduce a topological model for this Poincare duality algebra. Given an $(n-1)$-dimensional simplicial homology manifold S we construct a $2n$-dimensional homology manifold with boundary $\widehat{X}$ carrying the action of a compact n-torus. The Poincare–Lefschetz duality on $\widehat{X}$ is used to reconstruct the algebra $(\mathcal{R}[S]/(l.s.o.p.))/I_{NS}$.

[3] With M. Masuda
Volume polynomials and duality algebras of multi-fans.
Arnold Math J. (2016) 2: 329–381.

We introduce a theory of volume polynomials and corresponding duality algebras of multi-fans. Any complete simplicial multi-fan Δ determines a volume polynomial V_Δ whose values are the volumes of multi-polytopes based on Δ. This homogeneous polynomial is further used to construct a Poincare duality algebra $\mathcal{A}^*(\Delta)$. We study the structure and properties of V_Δ and $\mathcal{A}^*(\Delta)$ and give applications and connections to other subjects, such as Macaulay duality, Novik–Swartz theory of face rings of simplicial manifolds, generalizations of Minkowski's theorem on convex polytopes, cohomology of torus manifolds, computations of volumes, and linear relations on the powers of linear forms. In particular, we prove that the analogue of the g-theorem does not hold for multi-polytopes.

[4] Locally standard torus actions and h'-vectors of simplicial posets.
J. Math. Soc. Japan 68:4 (2016), 1–21.

We consider the orbit type filtration on a manifold with a locally standard torus action and study the corresponding spectral sequence in homology. When all proper faces of the orbit space are acyclic and the free part of the action is trivial, this spectral sequence can be described in full. The ranks of diagonal terms of its second page are equal to h'-numbers of a simplicial poset dual to the orbit space. Betti numbers of a manifold with a locally standard torus action are computed: they depend on the combinatorics and topology of the orbit space but not on the characteristic function.

A toric space whose orbit space is the cone over a Buchsbaum simplicial poset is studied by the same homological method. In this case the ranks of the diagonal terms of the spectral sequence at infinity are the h"-numbers of the simplicial poset. This fact provides a topological evidence for the nonnegativity of h"-numbers of Buchsbaum simplicial posets and links toric topology to some recent developments in enumerative combinatorics.

[5] Toric manifolds over 3-polytopes.
arXiv:1607.03377 *preprint.*

In this note we gather and review some facts about existence of toric spaces over 3-dimensional simple polytopes. First, over every combinatorial 3-polytope there exists a quasitoric manifold. Second, there exist combinatorial 3-polytopes, that do not correspond to any smooth projective toric variety. We restate the proof of the second claim which does not refer to complicated algebro-geometrical technique. If follows from these results that any fullerene supports quasitoric manifolds but does not support smooth projective toric varieties.

[6] Dimensions of multi-fan algebras.
arXiv:1607.03889 *preprint.*

Given an arbitrary non-zero simplicial cycle and a generic vector coloring of its vertices, there is a way to produce a graded Poincare duality algebra associated to these data. The procedure relies on the theory of volume polynomials and multi-fans. This construction includes many important examples, such as cohomology of toric varieties and quasitoric manifolds, and Gorenstein algebras of triangulated homology manifolds, introduced by Novik and Swartz. In all these examples the dimensions of graded components of such duality algebras do not depend on the vector coloring. It was conjectured that the same holds for any simplicial cycle. We disprove this conjecture by showing that the colors of singular points of the cycle may affect the dimensions. However, the colors of smooth points are irrelevant. By using bistellar moves we show that the number of different dimension vectors arising on a given 3-dimensional pseudomanifold with isolated singularities is a topological invariant. This invariant is trivial on manifolds, but nontrivial in general.

3.1.2 Alexander Belavin

[1] With V. Belavin
On exact solution of topological CFT models based on Kazama-Suzuki cosets
JHEP 1610 (2016) 128

This paper is dedicated to a new approach to computations of the flat coordinates on Frobenius manifolds connected with isolated singularity deformed by relevant and marginal perturbation. Our method is based on the conjecture about an integral representation for the flat coordinates.

[2] With V. Belavin
Flat structures on the deformations of Gepner chiral rings
J.Phys. A49 (2016) no.41, 41LT02, 2016

This paper is dedicated to a new approach to computations of the flat coordinates on Frobenius manifolds connected with isolated singularity deformed not only by relevant and marginal but also by irrelevant perturbations. Our method is based on the conjecture about an integral representation for the flat coordinates.

3.1.3 Alexei Elagin

[1] A.Elagin, V.Lunts, "On full exceptional collections on del Pezzo surfaces", Moscow Mathematical Journal, to appear in 2016.

We prove that any numerically exceptional collection of maximal length, consisting of line bundles, on a smooth del Pezzo surface is a standard augmentation in the sense of L. Hille and M. Perling. We deduce that any such collection is exceptional and full.

[2] A. Elagin, "A criterion for left-orthogonality of an effective divisor on a surface", arxiv:1610.02325v2

We find a criterion for an effective divisor D on a smooth surface to be left-orthogonal or strongly left-orthogonal (i.e. for the pair of line bundles $(\mathcal{O}, \mathcal{O}(D))$ to be exceptional or strong exceptional).

3.1.4 Alexei Gorodentsev

[1] Gorodentsev, Alexey L. Algebra I. Textbook for Students of Mathematics. XX+564 pages with 37 b/w illustrations and 42 illustrations in colour. Springer International Publishing, 2016. eBook ISBN 978-3-319-45285-2, Hardcover ISBN 978-3-319-45284-5, DOI 10.1007/978-3-319-45285-2.
See `http://www.springer.com/gp/book/9783319452845`.

This book is the first volume of an intensive "Russian-style" two-year undergraduate course in abstract algebra, and introduces readers to the basic algebraic structures fields, rings, modules, algebras, groups, and categories and explains the main principles of and methods for working with them.

The course covers substantial areas of advanced combinatorics, geometry, linear and multilinear algebra, representation theory, category theory, commutative algebra, Galois theory, and algebraic geometry topics that are often overlooked in standard undergraduate courses.

This textbook is based on courses the author has conducted at the Independent University of Moscow and at the Faculty of Mathematics in the Higher School of Economics. The main content is complemented by a wealth of exercises for class discussion, some of which include comments and hints, as well as problems for independent study.

3.1.5 Maxim Kazarian

[1] Kazarian, M., Zograf, P. Rationality in map and hypermap enumeration by genus, arXiv:1609.05493.

We show that the generating functions for a fixed genus map and hypermap enumeration become rational after a simple explicit change of variables. Their numerators are polynomials with integer coefficients that obey a differential recursion, and denominators are products of powers of explicit linear functions.

3.1.6 Anton Khoroshkin

[1] "Characteristic classes of flags of foliations and Lie algebra cohomology."
Transformation Groups Volume 21, (2016), Issue 2, pp 479–518

We prove the conjecture by Feigin, Fuchs, and Gelfand describing the Lie algebra cohomology of formal vector fields on an n-dimensional space with coefficients in symmetric powers of the coadjoint representation. We also compute the cohomology of the Lie algebra of formal vector fields that preserve a given ag at the origin. The latter encodes characteristic classes of ags of foliations and was used in the formulation of the local Riemann-Roch Theorem by Feigin and Tsygan.

Feigin, Fuchs, and Gelfand described the first symmetric power and to do this they had to make use of a fearsomely complicated computation in invariant theory. By the application of degeneration theorems of appropriate Hochschild-Serre spectral sequences, we avoid the need to use the methods of FFG, and moreover, we are able to describe all the symmetric powers at once.

[2] with S.Merkulov, T. Willwacher
"On Quantizable Odd Lie Bialgebras."
Letter in Mathematical Physics, (2016) 106(9), 1199–1215
Motivated by the obstruction to the deformation quantization of Poisson structures in infinite dimensions we introduce the notion of a quantizable odd Lie bialgebra. The main result of the paper is a construction of the highly non-trivial minimal resolution of the properad governing such Lie bialgebras, and its link with the theory of so called *quantizable* Poisson structures.

[3] with T. Willwacher, M. Živković
"Differentials on graph complexes"
accepted by Advances in Mathematics with T. Willwacher, M. Živković
We study the cohomology of complexes of ordinary (non-decorated) graphs, introduced by M. Kontsevich. We construct spectral sequences converging to zero whose first page contains the graph cohomology. In particular, these spectral sequences may be used to show the existence of an infinite series of previously unknown and provably non-trivial cohomology classes, and put constraints on the structure of the graph cohomology as a whole.

3.1.7 Iosif Krasilshchik

[1] With H. Baran, O.I. Morozov, P. Vojčák
Coverings over Lax integrable equations and their nonlocal symmetries, *Theoretical and Mathematical Physics* Vol. 188, no. 3, pp. 1273–1295 (2016). Russian version: Vol. 188, no 3, pp. 361–385 (2016);
arXiv:1507.00897

Using the Lax representation with non-removable parameter, we construct two hierarchies of nonlocal conservation laws for the 3D rdDym equation $u_{ty} = u_x u_{xy} - u_y u_{xx}$ and describe the algebras of nonlocal symmetries in the corresponding coverings.

[2] With A. Sergyeyev, O.I. Morozov
Infinitely many nonlocal conservation laws for the ABC equation with $A + B + C \neq 0$, *Calculus of Variations and Partial Differential Equations*s, Vol. 55 no. 5, 2016 pp. 1–12; arXiv:1511.09430

We construct an infinite hierarchy of nonlocal conservation laws for the ABC equation $Au_tu_{xy} + Bu_xu_{ty} + Cu_yu_{tx} = 0$, where A, B, C are constants and $A + B + C \neq 0$, using a novel nonisospectral Lax pair. As a byproduct, we present new coverings for the ABC equation. The method of proof of nontriviality of the conservation laws under study is quite general and can be applied to many other integrable multidimensional systems.

[3] With H. Baran, O.I. Morozov, P. Vojčák Nonlocal symmetries of Lax integrable equations: a comparative study, *Submitted to Journal of Physics A*;
arXiv:1611.04938 [nlin.SI]

We continue here the study of Lax integrable equations. We consider four three-dimensional equations: (1) the rdDym equation $u_{ty} = u_xu_{xy} - u_yu_{xx}$, (2) the 3D Pavlov equation $u_{yy} = u_{tx} + u_yu_{xx} - u_xu_{xy}$; (3) the universal hierarchy equation $u_{yy} = u_tu_{xy} - u_yu_{tx}$, and (4) the modified Veronese web equation $u_{ty} = u_tu_{xy} - u_yu_{tx}$. For each equation, using the know Lax pairs and expanding the latter in formal series in spectral parameter, we construct two infinite-dimensional differential coverings and give a full description of nonlocal symmetry algebras associated to these coverings. For all the for pairs of coverings, the obtained Lie algebras of symmetries manifest similar (but not the same) structures: the are (semi) direct sums of the Witt algebra, the algebra of vector fields on the line, and loop algebras; all of them contain a component of finite grading. We also discuss actions of recursion operators on shadows of nonlocal symmetries.

[4] A natural geometric construction underlying a class of Lax pairs, Lobachevskii J. of Math., Vol. 37, no. 1, 60–65 (2016);
arXiv:1401.0612

In the framework of the theory of differential coverings [2], we discuss a general geometric construction that serves the base for the so-called Lax pairs containing differentiation with respect to the spectral parameter. Such kind of objects arise, for example, when studying integrability properties of equations like the Gibbons-Tsarev one.

3.1.8 Alexander Kuznetsov

[1] With A. Polishchuk
Exceptional collections on isotropic Grassmannians
J. Eur. Math. Soc. (JEMS) 18 (2016), no. 3, 507–574

We introduce a new construction of exceptional objects in the derived category of coherent sheaves on a compact homogeneous space of a semisimple algebraic group and show that it produces exceptional collections of the length equal to the rank of the Grothendieck group on homogeneous spaces of all classical groups.

[2] Küchle fivefolds of type $c5$
Math. Z. 284 (2016), no. 3-4, 1245–1278.

We show that Küchle fivefolds of type ($c5$) — subvarieties of the Grassmannian $Gr(3,7)$ parameterizing 3-subspaces that are isotropic for a given 2-form and are annihilated by a given 4-form — are birational to hyperplane sections of the Lagrangian Grassmannian $LGr(3,6)$ and describe in detail these birational transformations. As an application, we show that the integral Chow motive of a Küchle fivefold of type ($c5$) is of Lefschetz type. We also discuss Küchle fourfolds of type ($c5$) — hyperplane sections of the corresponding Küchle fivefolds — an interesting class of Fano fourfolds, which is expected to be similar to the class of cubic fourfolds in many aspects.

[3] With Yu. Prokhorov and C. Shramov
Hilbert schemes of lines and conics and automorphism groups of Fano threefolds
preprint math.AG/1605.02010

We discuss various results on Hilbert schemes of lines and conics and automorphism groups of smooth Fano threefolds with Picard rank 1. Besides a general review of facts well known to experts, the paper contains some new results, for instance, we give a description of the Hilbert scheme of conics on any smooth Fano threefold of index 1 and genus 10. We also show that the action of the automorphism group of a Fano threefold X of index 2 (respectively, 1) on an irreducible component of its Hilbert scheme of lines (respectively, conics) is faithful if the anticanonical class of X is very ample with a possible exception of several explicit cases. We use these faithfulness results to prove finiteness of the automorphism groups of most Fano threefolds and classify explicitly all Fano threefolds with infinite automorphism group. We also discuss a derived category point of view on the Hilbert schemes of lines and conics, and use this approach to identify some of them.

[4] With O. Debarre
Gushel–Mukai varieties: linear spaces and periods
preprint math.AG/1605.05648

Beauville and Donagi proved in 1985 that the primitive middle cohomology of a smooth complex cubic fourfold and the primitive second cohomology of its variety of lines, a smooth hyperkähler fourfold, are isomorphic as polarized integral Hodge structures. We prove analogous statements for smooth complex Gushel–Mukai varieties of dimension 4 (resp. 6), i.e., smooth dimensionally transverse intersections of the cone over the Grassmannian $Gr(2,5)$, a quadric, and two hyperplanes (resp. of the cone over $Gr(2,5)$ and a quadric). The associated hyperkähler fourfold is in both cases a smooth double cover of a hypersurface in $\mathbb{P}^5$ called an EPW sextic.

[5] With A. Perry

Derived categories of Gushel–Mukai varieties
preprint math.AG/1605.06568

We study the derived categories of coherent sheaves on Gushel–Mukai varieties. In the derived category of such a variety, we isolate a special semiorthogonal component, which is a K3 or Enriques category according to whether the dimension of the variety is even or odd. We analyze the basic properties of this category using Hochschild homology, Hochschild cohomology, and the Grothendieck group. We study the K3 category of a Gushel–Mukai fourfold in more detail. Namely, we show that this category is equivalent to the derived category of a K3 surface for a certain codimension 1 family of rational fourfolds, and to the K3 category of a birational cubic fourfold for a certain codimension 3 family. The first of these results verifies a special case of a duality conjecture which we formulate. We discuss our results in the context of the rationality problem for Gushel–Mukai varieties, which was one of the main motivations for this work.

[6] With O. Debarre
On the cohomology of Gushel–Mukai sixfolds
preprint math.AG/1606.09384

We provide a stable rationality construction for some smooth complex Gushel–Mukai varieties of dimension 6. As a consequence, we compute the integral singular cohomology of any smooth Gushel–Mukai sixfold and in particular, show that it is torsion-free.

[7] Derived equivalence of Ito–Miura–Okawa–Ueda Calabi–Yau 3-folds
preprint math.AG/1611.08386

We prove derived equivalence of Calabi-Yau threefolds constructed by Ito-Miura-Okawa-Ueda as an example of non-birational Calabi-Yau varieties whose difference in the Grothendieck ring of varieties is annihilated by the affine line.

[8] With A. Fonarev
Derived categories of curves as components of Fano manifolds
preprint math.AG/1612.02241

We prove that the derived category $D(C)$ of a generic curve of genus greater than one embeds into the derived category $D(M)$ of the moduli space M of rank two stable bundles on C with fixed determinant of odd degree.

3.1.9 Maxim Leyenson

[1] Projective models of Hilbert modular surfaces: introduction
In progress

We give an introduction to Hirzebruch's study of projective models of Hilbert modular surfaces of small discriminant, filling in some details and computations. We also include some results by Gundlach.

This text is suitable for graduate students.

[2] An introduction to Serre duality for algebraic curves
In progress

In this text, suitable for BA students, we give an elementary geometric proof of Serre duality on algebraic curves using the sheaf-theoretic language. First, we prove the result for a smooth plane curve. Then we use a generic projection of a curve to the plane, and prove the duality for plane curves with ordinary singularities.

Thus we avoid using theory of residues.

[3] How to invent complex multiplication
In progress

In this elementary text, suitable for MA students, we give an argument which suggests how one could construct abelian extensions of number fields by studying geometry of algebraic curves and their Jacobians. (It turns out that the geometric intuition is working, but only for the CM-fields, unfortunately).

3.1.10 Grigory Olshanski

[1] G. Olshanski, The representation ring of the unitary groups and Markov processes of algebraic origin. Advances in Mathematics 300 (2016), 544–615.

The paper consists of two parts. The first part introduces the representation ring for the family of compact unitary groups $U(1)$, $U(2)$, … . This novel object is a commutative graded algebra R with infinite-dimensional homogeneous components. It plays the role of the algebra of symmetric functions, which serves as the representation ring for the family of finite symmetric groups. The purpose of the first part is to elaborate on the basic definitions and prepare the ground for the construction of the second part of the paper.

The second part deals with a family of Markov processes on the dual object to the infinite-dimensional unitary group $U(\infty)$. These processes were defined in a joint work with Alexei Borodin (J. Funct. Anal. 2012). The main result of the present paper consists in the derivation of an explicit expression for their infinitesimal generators. It is shown that the generators are implemented by certain second order partial differential operators with countably many variables, initially defined as operators on R.

[2] G. Olshanski, Markov dynamics on the dual object to the infinite-dimensional unitary group. In: Probability and Statistical Physics in St. Petersburg. Proceedings of Symposia in Pure Mathematics vol. 91, pp. 373– 394. American Mathematical Society, 2016.

This expository paper is an expanded version of a mini-course of 3 lectures given at the Saint Petersburg School in Probability and Statistical Physics. My aim was to explain, on the example of a particular model, how ideas from the representation theory of big groups can be applied in probabilistic problems.

[3] A. Borodin and G. Olshanski, The ASEP and determinantal point processes. Preprint, 47 pp. Submitted.

We introduce a family of discrete determinantal point processes related to orthogonal polynomials on the real line, with correlation kernels defined via spectral projections for the associated Jacobi matrices. For classical weights, we show how such ensembles arise as limits of various hypergeometric orthogonal polynomials ensembles.

We then prove that the q-Laplace transform of the height function of the ASEP with step initial condition is equal to the expectation of a simple multiplicative functional on a discrete Laguerre ensemble — a member of the new family. This allows us to obtain the large time asymptotics of the ASEP in three limit regimes: (a) for finitely many rightmost particles; (b) GUE Tracy-Widom asymptotics of the height function; (c) KPZ asymptotics of the height function for the ASEP with weak asymmetry.

We also give similar results for two instances of the stochastic six vertex model in a quadrant. The proofs are based on limit transitions for the corresponding determinantal point processes.

3.1.11 Alexei Penskoi

[1] With N. S. Nadirashvili

Isoperimetric inequality for the second non-zero eigenvalue of the Laplace-Beltrami operator on the projective plane

arXiv:1608.07334, *submitted to Duke Mathematical Journal*

In this paper an isoperimetric inequality for the second non-zero eigenvalue of the Laplace-Beltrami operator on the real projective plane is proven. For a metric of area 1 this eigenvalue is not greater than 20π. This value could be attained as a limit on a sequence of metrics of area 1 on the projective plane converging to a singular metric on the projective plane and the sphere with standard metrics touching in a point such that the ratio of the areas of the projective plane and the sphere is $3:2$. It is also proven that the multiplicity of the second non-zero eigenvalue on the projective plane is at most 6.

[2] With A. S. Berdnikov and N. S. Nadirashvili
Bounds on Multiplicities of Laplace-Beltrami Operator Eigenvalues on the Real Projective Plane
arXiv:1612.04805

In this paper the known upper bounds for the multiplicities of the Laplace-Beltrami operator eigenvalues on the real projective plane are improved for the eigenvalues with even indexes. Upper bounds for Dirichlet, Neumann and Steklov eigenvalues on the real projective plane with holes are also provided.

3.1.12 Petr Pushkar'

[1] Morse theory on manifolds with boundary. Combinatorial structure on cells and generalized weak Morse inequalities, preprint,
available at: https://www.hse.ru/mirror/pubs/share/198499986

We construct new combinatorial structure on critical points of Morse functions. Results are applyed to estimation of a number of critical points.

3.1.13 Sergey Rybakov

[1] With A. Trepalin
Minimal cubic surfaces over finite fields.
https://arxiv.org/abs/1611.02475

Let X be a minimal cubic surface over a finite field $\mathbb{F}_q$. The image Γ of the Galois group $\mathrm{Gal}(\overline{\mathbb{F}}_q/\mathbb{F}_q)$ in the group $\mathrm{Aut}(\mathrm{Pic}(\overline{X}))$ is a cyclic subgroup of the Weyl group $W(E_6)$. There are 25 conjugacy classes of cyclic subgroups in $W(E_6)$, and 5 of them correspond to minimal cubic surfaces. It is natural to ask which conjugacy classes come from minimal cubic surfaces over a given finite field. In this paper we give a partial answer to this question and present many explicit examples.

3.1.14 George Shabat

[1] Calculating and drawing Belyi pairs.
Proceedingd of sci. sem. POMI, vol, 446 (2016), pp. 181-219.

An overview of the current state of the Grothendieck theory of dessins d'enfants is given and several generalizations are discussed.

[2] On the elliptic time in the adelic gravity.

To appear in the journal FACTA UNIVERSITATIS: Physics, Chemistry and Technology, published by Universty of Nis (Serbia).

In this paper an adelic version of the Newton's inverse square law is suggested.

Papers on the mathematical language in the linguistic journals.

[3] With G. Kreidlin

Natural language and the language of geometric sketches: points of contact.

Znaki czy nie znaki? (red naukowa J. Piatkowska, G. Zeldowicz). Warszawa, 2016, . 197 – 221.

Geometric sketches are analyzed as a formal semiotic system from the viewpoints of mathematical logics and linguistics.

[4] On the numbers and their names (in russian). Vestnik RGGU N 9 (18). Moscow Linguistic Journal, vol. 18. , 2016, pp. 40 – 51.

3.1.15 Arkady Skopenkov

(Abstracts of conference talks are not listed)

[1] A. Skopenkov, How do autodiffeomorphisms act on embeddings, Proceedings A of The Royal Society of Edinburgh, to appear. `http://arxiv.org/abs/1402.1853`

We work in the smooth category. For an n-manifold N denote by $E^m(N)$ the set of isotopy classes of embeddings $N \to \mathbb{R}^m$. The following problem was suggested by E. Rees in 2002: describe the action of self-diffeomorphisms of $S^p \times S^{n-p}$ on $E^m(S^p \times S^{n-p})$.

Let $g : S^p \times S^{n-p} \to \mathbb{R}^m$ be an embedding such that $g|_{a\times S^{n-p}} : a \times S^{n-p} \to \mathbb{R}^m - g(b \times S^{n-p})$ is null-homotopic for some different points $a, b \in S^p$ and $m \geq n+2+\frac{1}{2}\max\{p, n-p\}$.

Theorem. *For a map $\varphi : S^p \to SO_{n-p}$ define an autodiffeomorphism φ' of $S^p \times D^{n-p}$ by $\overline{\varphi}(a, b) := (a, \varphi(a)b)$. Let φ'' be the S^{n-p-1}-symmetric extension of φ to an autodiffeomorphism of $S^p \times S^{n-p}$. Then for each map $\varphi : S^p \to SO_{n-p}$ embedding $g \circ \varphi''$ is isotopic to embedded connected sum $g\#u$ for some embedding $u : S^n \to S^m$.*

Let N be an oriented n-manifold and $f : N \to \mathbb{R}^m$ an embedding. Denote by $E^m(N)/\#$ the quotient set of $E^m(N)$ by embedded connected sum with embeddings $S^n \to \mathbb{R}^m$. As a corollary we obtain that under certain conditions *for orientation-preserving embeddings $s : S^p \times D^{n-p} \to N$ the class of S^p-parametric embedded connected sum $f\#_s g$ in $E^m(N)/\#$ depends only on f, g and the isotopy (the homotopy or the homology) class of $s|_{S^p\times 0}$.*

[2] S. Avvakumov, I. Mabillard, A. Skopenkov, U. Wagner, Eliminating Higher-Multiplicity Intersections, III. Codimension 2. `http://arxiv.org/abs/1511.03501` Submitted to Geometry and Topology.

We study conditions under which a finite simplicial complex K can be mapped to $\mathbb{R}^d$ without higher-multiplicity intersections. An *almost r-embedding* is a map $f\colon K \to \mathbb{R}^d$ such that the images of any r pairwise disjoint simplices of K do not have a common point. We show that if r is not a prime power and $d \geq 2r+1$, then there is a counterexample to the topological Tverberg conjecture, i.e., *there is an almost r-embedding of the $(d+1)(r-1)$-simplex in $\mathbb{R}^d$*. This improves on previous constructions of counterexamples (for $d \geq 3r$) based on a series of papers by M. Özaydin, M. Gromov, P. Blagojević, F. Frick, G. Ziegler, and the second and fourth present author.

The counterexamples are obtained by proving the following algebraic criterion in codimension 2: *If $r \geq 3$ and if K is a finite $2(r-1)$-complex then there exists an almost r-embedding $K \to \mathbb{R}^{2r}$ if and only if there exists a general position PL map $f\colon K \to \mathbb{R}^{2r}$ such that the algebraic intersection number of the f-images of any r pairwise disjoint simplices of K is zero.* This result can be restated in terms of cohomological obstructions or equivariant maps, and extends an analogous codimension 3 criterion by the second and fourth author.

It follows from work of M. Freedman, V. Krushkal, and P. Teichner that the analogous criterion for $r=2$ is false. We prove a beautiful lemma on singular higher-dimensional Borromean rings, yielding an elementary proof of the counterexample. As another application of our methods, we classify *ornaments* $f\colon S^3 \sqcup S^3 \sqcup S^3 \to \mathbb{R}^5$ up to *ornament concordance*.

[3] A. Skopenkov, Stability of intersections of graphs in the plane and the van Kampen obstruction. `http://arxiv.org/abs/1609.03727` Submitted to Topology and its Applications

A map $\varphi : K \to \mathbb{R}^2$ of a graph K is *approximable by embeddings*, if for each $\varepsilon > 0$ there is an ε-close to φ embedding $f : K \to \mathbb{R}^2$. Analogous notions were studied in computer science under the names of *cluster planarity* and *weak simplicity*. This short survey is intended not only for specialists in the area, but also for mathematicians from other areas.

We present criteria for approximability by embeddings (P. Minc, 1997, M. Skopenkov, 2003) and their algorithmic corollaries. We introduce *the van Kampen (or Hanani-Tutte) obstruction* for approximability by embeddings and discuss its completeness. We discuss analogous problems of moving graphs in the plane apart (cf. S. Spież and H. Toruńczyk, 1991) and finding closest embeddings (H. Edelsbrunner). We present higher dimensional generalizations, including completeness of the van Kampen obstruction and its algorithmic corollary (D. Repovš and A. Skopenkov, 1998).

[4] A. Skopenkov, High codimension embeddings: classification, submitted to Bull. Man. Atl.

`http://www.map.mpim-bonn.mpg.de/High_codimension_embeddings`

This page is intended not only for specialists in embeddings, but also for mathematicians from other areas who want to apply or to learn the theory of embeddings.

This article gives a short guide to the Knotting Problem of compact manifolds N in Euclidean spaces and in spheres. After making general remarks we record some of the dimension ranges where no knotting is possible, i.e. where any two embeddings of N are isotopic. We then establish notation and conventions and give references to other pages on the Knotting Problem, to which this page serves as an introduction. We conclude by introducing connected sum and make some comments on codimension 1 and 2 embeddings.

[5] A. Skopenkov, Embeddings just below the stable range: classification, submitted to Bull. Man. Atl. `http://www.map.mpim-bonn.mpg.de/Embeddings_just_below_the_stable_range:_classification`

This page is intended not only for specialists in embeddings, but also for mathematician from other areas who want to apply or to learn the theory of embeddings.

Recall the Whitney-Wu Unknotting Theorem: if N is a connected manifold of dimension $n > 1$, and $m \geq 2n+1$, then every two embeddings $N \to \mathbb{R}^m$ are isotopic. In this page we summarize the situation for $m = 2n \geq 6$ and some more general situations.

[6] A. Skopenkov, 3-manifolds in 6-space, submitted to Bull. Man. Atl.

`http://www.map.mpim-bonn.mpg.de/3-manifolds_in_6-space`

This page is intended not only for specialists in embeddings, but also for mathematicians from other areas who want to apply or to learn the theory of embeddings.

The classification of 3-manifolds in 6-space is of course a particular case of the classification of n-manifolds in $2n$-space. In this page we recall the general results as they apply when $n = 3$ and we discuss examples and invariants peculiar to the case $n = 3$.

[7] A. Skopenkov, 4-manifolds in 7-space, submitted to Bull. Man. Atl.

`http://www.map.mpim-bonn.mpg.de/4-manifolds_in_7-space`

This page is intended not only for specialists in embeddings, but also for mathematician from other areas who want to apply or to learn the theory of embeddings.

Basic results on 4-manifolds in 7-space are particular cases of results on n-manifolds in $(2n-1)$-space for $n = 4$. In this page we concentrate on more advanced results peculiar for $n = 4$.

[8] A. Skopenkov, High codimension links, submitted to Bull. Man. Atl.

`http://www.map.mpim-bonn.mpg.de/High_codimension_links`

This page is intended not only for specialists in embeddings, but also for mathematician from other areas who want to apply or to learn the theory of embeddings. We describe classification of embeddings $S^{n_1} \sqcup \ldots \sqcup S^{n_s} \to S^m$ for $m - 3 \geq n_i$.

[9] A. Skopenkov, Classification of knotted tori, `http://arxiv.org/abs/1502.04470` (the paper is rewritten in 2016, a new version uploaded to arxiv)

We describe the group of (smooth isotopy classes of smooth) embeddings $S^p \times S^q \to R^m$ for $p \leq q$ and $m \geq 2p+q+3$. Earlier such a description was known only for $2m \geq 3p+3q+4$. We use a recent exact sequence of M. Skopenkov.

[10] D. Crowley and A. Skopenkov, Embeddings of non-simply-connected 4-manifolds in 7-space, I. Classification modulo knots.

`http://arxiv.org/abs/1611.04738`

We work in the smooth category. Let N be a closed connected orientable 4-manifold with torsion free H_1, where $H_q := H_q(N;\mathbb{Z})$. The main result is *a complete readily calculable* *classification of embeddings* $N \to \mathbb{R}^7$, up to equivalence which is isotopy and embedded connected sum with embeddings $S^4 \to \mathbb{R}^7$. Such a classification was earlier known only for $H_1 = 0$ by Boéchat-Haefliger-Hudson 1970. Our classification involves Boéchat-Haefliger invariant $\varkappa(f) \in H_2$, Seifert bilinear form $\lambda(f) : H_3 \times H_3 \to \mathbb{Z}$ and β-invariant assuming values in the quotient of H_1 defined by values of $\varkappa(f)$ and $\lambda(f)$.

In particular, for $N = S^1 \times S^3$ we define geometrically a 1–1 correspondence between the set of equivalence classes of embeddings and an explicitly defined quotient of $\mathbb{Z} \oplus \mathbb{Z}$.

[11] D. Crowley and A. Skopenkov, Embeddings of non-simply-connected 4-manifolds in 7-space, II. On the smooth classification. `http://arxiv.org/abs/1612.04776`

We work in the smooth category. Let N be a closed connected orientable 4-manifold with torsion free H_1, where $H_q := H_q(N; \mathbb{Z})$. Our main result is *a readily calculable classification of embeddings* $N \to \mathbb{R}^7$ *up to isotopy*, with an indeterminancy. Such a classification was only known before for $H_1 = 0$ by our earlier work from 2008. Our classification is complete when $H_2 = 0$ or when the signature of N is divisible neither by 64 nor by 9.

The group of knots $S^4 \to S^7$ acts on the set of embeddings $N \to \mathbb{R}^7$ up to isotopy by embedded connected sum. In Part I we classified the quotient of this action. The main novelty of this paper is the description of this action for $H_1 \neq 0$, with an indeterminancy.

Besides the invariants of Part I, the classification involves a refinement of the Kreck invariant from our work of 2008 which detects the action of knots.

For $N = S^1 \times S^3$ we give a geometrically defined 1–1 correspondence between the set of isotopy classes of embeddings and a quotient of the set $\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}_{12}$.

[12] A. Skopenkov, A user's guide to topological Tverberg conjecture.
`http://arxiv.org/abs/1605.05141`

The well-known *topological Tverberg conjecture* was considered a central unsolved problem of topological combinatorics. The conjecture asserts that *for each integers* $r, d > 1$ *and each continuous map* $f \colon \Delta \to \mathbb{R}^d$ *of the* $(d+1)(r-1)$*-dimensional simplex* Δ *there are pairwise disjoint subsimplices* $\sigma_1, \dots, \sigma_r \subset \Delta$ *such that* $f(\sigma_1) \cap \dots \cap f(\sigma_r) \neq \emptyset$*.*

A proof for a prime power r was given by I. Bárány, S. Shlosman, A. Szűcs, M. Özaydin and A. Volovikov in 1981-1996. A counterexample for other r was found in a series of papers by M. Özaydin, M. Gromov, P. Blagojević, F. Frick, G. Ziegler, I. Mabillard and U. Wagner, most of them recent. The arguments form a beautiful and fruitful interplay between combinatorics, algebra and topology. In this expository note we present a simplified explanation of easier parts of the arguments, accessible to non-specialists in the area.

Expository publications for university students.

[13] A. Chernov, A. Daynyak, A. Glibichuk, M. Ilyinskiy, A. Kupavskiy, A. Raigorodskiy and A. Skopenkov, Elements of Discrete mathematics as a sequence of problems. 2016, Moscow, MCCME

`http://www.mccme.ru/circles/oim/discrbook.pdf`

In this book we present sequences of problems on combinatorics and graph theory (including random graphs).

[14] Mathematics via problems: from olympiads and math circles to a profession, editors: A. Zaslavsky, A. Skopenkov, and M. Skopenkov. 2016, Moscow, MCCME, to appear. `http://www.mccme.ru/circles/oim/sturm.pdf`

In this book we present an approach to 'university' mathematics as sequences of 'high-school' problems.

[15] A. Skopenkov, Embeddings into the plane of graphs with vertices of degree 4, Mat. Prosveschenie, 21 (2017), to appear, `http://arxiv.org/abs/1008.4940` (the paper is rewritten in 2016, a new version uploaded to arxiv)

In this expository note we present a proof of the V.A. Vassiliev conjecture on the planarity of graphs with vertices of degree 4 and certain additional structure. Both statement and proof are accessible to high-school students familiar with basic notions of graph theory. The conjecture was first proved by V.O. Manturov (such a proof was one of the main results of his habilitation thesis). In this note the exposition is made clearer and some comments for beginners are added.

[16] A. Volostnov, A. Skopenkov and Yu. Yarovikov, A study on recursive relations, Mat. Prosveschenie, submitted.

In this expository note we present and discuss a short proof of an estimation required for a proof of the Symmetric Local Lovasz Lemma.

[17] A. Skopenkov, How Fermat found extrema,

`http://arxiv.org/abs/1610.05968`

In this expository note we present a short elementary proof of the well-known criterion for a cubic polynomial to have three real roots. The proof is based on Fermat's approach to calculus for polynomials, and illustrates the idea of a derivative rigorously but without technical ε-δ language. The note is accessible to high-school students.

[18] A. Belov, I. Mitrofanov, A. Skopenkov, A. Chilikov, S. Shaposhnikov, 13th Hilbert Problem on superpositions of functions,

`http://www.turgor.ru/lktg/2016/5/index.htm`

In this expository note we present and discuss a structured proof of the Kolmogorov Superposition Theorem.

[19] A. Skopenkov, Algebraic Topology From Algorithmic Viewpoint, draft of a book, `http://www.mccme.ru/circles/oim/algor.pdf` (some sections are added or rewritten in 2016)

In this book we present an 'algorithmic' approach to algebraic topology.

3.1.16 Mikhail Skopenkov

[1] Skopenkov M., Surfaces containing two circles through each point, rejected by 5 journals without referee report, http://arxiv.org/abs/1512.09062

We find all analytic surfaces in space $\mathbb{R}^3$ such that through each point of the surface one can draw two transversal circular arcs fully contained in the surface. The problem of finding such surfaces traces back to the works of Darboux from XIXth century. We prove that such a surface is an image of a subset of one of the following sets under some composition of inversions:

- the set $\{\, p + q : p \in \alpha, q \in \beta \,\}$, where α, β are two circles in $\mathbb{R}^3$;
- the set $\{\, 2\frac{[p\times q]}{|p+q|^2} : p \in \alpha, q \in \beta, p + q \neq 0 \,\}$, where α, β are two circles in S^2;
- the set $\{\, (x, y, z) : Q(x, y, z, x^2 + y^2 + z^2) = 0 \,\}$, where $Q \in \mathbb{R}[x, y, z, t]$ has degree 2 or 1.

The proof uses a new factorization technique for quaternionic polynomials.

[2] Skopenkov M., Krasauskas R., Surfaces containing two circles through each point and Pythagorean 6-tuples, submitted, http://arxiv.org/abs/1503.06481

We study analytic surfaces in 3-dimensional Euclidean space containing two circular arcs through each point. The problem of finding such surfaces traces back to the works of Darboux from XIXth century. We reduce finding all such surfaces to the algebraic problem of finding all Pythagorean 6-tuples of polynomials. The reduction is based on the Schicho parametrization of surfaces containing two conics through each point and a new approach using quaternionic rational parametrization.

[3] Pakharev A., Skopenkov M., Surfaces containing two circles through each point and decomposition of quaternionic matrices, submitted, http://arxiv.org/abs/1510.06510. This paper does not contain any reference to the support of Simons–IUM fellowship because of the requirement of the other funding organization.

We find all analytic surfaces in space $\mathbb{R}^3$ such that through each point of the surface one can draw two circular arcs fully contained in the surface. This paper announces the result and gives the idea of proof using a new decomposition technique for quaternionic matrices.

[4] Elements of mathematics in problems. Through circles and olympiads to profession. Ed. by A. Skopenkov, M. Skopenkov, and A. Zaslavskiy. Moscow Center for Continuous Mathematical Education, in print (in Russian)

In addition to [1]–[4], several talk abstracts have been published in 2016.

3.1.17 Evgeni Smirnov

[1] With G. Merzon
Determinantal identities for flagged Schur and Schubert polynomials
European Journal of Mathematics, 2:1, 227–245 (2016)

We prove new determinantal identities for a family of flagged Schur polynomials. As a corollary of these formulas we obtain determinantal expressions of Schubert polynomials for certain vexillary permutations.

[2] Grassmannians, flag varieties, and Gelfand-Zetlin polytopes
in: Recent Developments in Representation Theory. Proceedings of Maurice Auslander Distinguished Lectures and International Conference, Contemporary Mathematics series, vol. 673, 179–226. American Mathematical Society, 2016.

The aim of these notes is to give an introduction into Schubert calculus on Grassmannians and flag varieties. We discuss various aspects of Schubert calculus, such as applications to enumerative geometry, structure of the cohomology rings of Grassmannians and flag varieties, Schur and Schubert polynomials. We conclude with a survey of results of V. Kiritchenko, V. Timorin and the author on a new approach to Schubert calculus on full flag varieties via combinatorics of Gelfand–Zetlin polytopes.

[3] With V. Kleptsyn
Ribbon graphs and bialgebra of Lagrangian spaces
Journal of Knot Theory and its Ramifications, 26 (2016), 1642006 (26 pages)

To each ribbon graph we assign a so-called L-space, which is a Lagrangian subspace in an even-dimensional vector space with the standard symplectic form. This invariant generalizes the notion of the intersection matrix of a chord diagram. Moreover, the actions of Morse perestroikas (or taking a partial dual) and Vassiliev moves on ribbon graphs are reinterpreted nicely in the language of L-spaces, becoming changes of bases in this vector space. Finally, we define a bialgebra structure on the span of L-spaces, which is analogous to the 4-bialgebra structure on chord diagrams.

[4] Singularities of divisors on flag varieties via Hwang's product theorem
preprint arXiv:1609.07771, 6 pages

We give an alternative proof of a recent result by Pasquier stating that for a generalized flag variety $X = G/P$ and an effective $\mathbb{Q}$-divisor D stable with respect to a Borel subgroup the pair (X, D) is Kawamata log terminal if and only if $\lfloor D \rfloor = 0$.

3.1.18 Mikhail Verbitsky

[1] Liviu Ornea, Misha Verbitsky, *Locally conformally Kahler metrics obtained from pseudoconvex shells*, Proc. Amer. Math. Soc. 144 (2016), 325-335

> A locally conformally Kahler (LCK) manifold is a complex manifold admitting a Kahler covering M, such that its monodromy acts on this covering by homotheties. A compact LCK manifold is called LCK with potential if M admits an authomorphic Kahler potential. It is known that in this case it is an algebraic cone, that is, the set of all non-zero vectors in the total space of an anti-ample line bundle over a projective orbifold. We start with an algebraic cone C, and show that the set of Kahler metrics with potential which could arise from an LCK structure is in bijective correspondence with the set of pseudoconvex shells, that is, pseudoconvex hypersurfaces in C meeting each orbit of the associated R-action exactly once. This is used to produce explicit LCK and Vaisman

metrics on Hopf manifolds, generalizing earlier work by Gauduchon-Ornea and Kamishima-Ornea.

[2] Amerik, Ekaterina; Verbitsky, Misha, *Hyperbolic geometry of the ample cone of a hyperkähler manifold*, Res. Math. Sci. 3 (2016), Paper No. 7, 9 pp

> Let M be a compact hyperkahler manifold with maximal holonomy (IHS). The group $H^2(M, R)$ is equipped with a quadratic form of signature $(3, b_2 - 3)$, called Bogomolov-Beauville-Fujiki (BBF) form. This form restricted to the rational Hodge lattice $H^{1,1}(M, Q)$, has signature $(1, k)$. This gives a hyperbolic Riemannian metric on the projectivisation of the positive cone in $H^{1,1}(M, Q)$, denoted by H. Torelli theorem implies that the Hodge monodromy group Γ acts on H with finite covolume, giving a hyperbolic orbifold $X = H/\Gamma$. We show that there are finitely many geodesic hypersurfaces which cut X into finitely many polyhedral pieces in such a way that each of these pieces is isometric to a quotient $P(M')/Aut(M')$, where $P(M')$ is the projectivization of the ample cone of a birational model M' of M, and $Aut(M')$ the group of its holomorphic automorphisms. This is used to prove the existence of nef isotropic line bundles on a hyperkahler birational model of a simple hyperkahler manifold of Picard number at least 5, and also illustrates the fact that an IHS manifold has only finitely many birational models up to isomorphism, originally deduced by Markman and Yoshioka from the Morrison-Kawamata cone conjecture.

[3] Ornea, Liviu; Verbitsky, Misha, *LCK rank of locally conformally Kähler manifolds with potential*, J. Geom. Phys. 107 (2016), 92-98.

> An LCK manifold with potential is a compact quotient of a Kahler manifold X equipped with a positive Kahler potential f, such that the monodromy group acts on X by holomorphic homotheties and multiplies f by a character. The LCK rank is the rank of the image of this character, considered as a function from the monodromy group to real numbers. We prove that an LCK manifold with potential can have any rank between 1 and $b_1(M)$. Moreover, LCK manifolds with proper potential (ones with rank 1) are dense. Two errata to our previous work are given in the last Section.

[4] Panov, Taras; Ustinovskiy, Yury; Verbitsky, Misha, *Complex geometry of moment-angle manifolds*, Math. Z. 284 (2016), no. 1-2, 309-333.

> Moment-angle manifolds provide a wide class of examples of non-Kaehler compact complex manifolds. A complex moment-angle manifold Z is constructed via certain combinatorial data, called a complete simplicial fan. In the case of rational fans, the manifold Z is the total space of a holomorphic bundle over a

toric variety with fibres compact complex tori. In general, a complex moment-angle manifold Z is equipped with a canonical holomorphic foliation F which is equivariant with respect to the $(C^*)^m$-action. Examples of moment-angle manifolds include Hopf manifolds of Vaisman type, Calabi-Eckmann manifolds, and their deformations.

We construct transversely Kaehler metrics on moment-angle manifolds, under some restriction on the combinatorial data. We prove that any Kaehler submanifold (or, more generally, a Fujiki class C subvariety) in such a moment-angle manifold is contained in a leaf of the foliation F. For a generic moment-angle manifold Z in its combinatorial class, we prove that all subvarieties are moment-angle manifolds of smaller dimension. This implies, in particular, that the algebraic dimension of Z is zero.

[5] Ekaterina Amerik, Misha Verbitsky, *Collections of parabolic orbits in homogeneous spaces, homogeneous dynamics and hyperkähler geometry*, arXiv:1604.03927

Let M be a hyperkähler manifold with $b_2(M) \geq 5$. We improve our earlier results on the Morrison-Kawamata cone conjecture by showing that the Beauville-Bogomolov square of the primitive MBM classes (i.e. the classes whose orthogonal hyperplanes bound the Kähler cone in the positive cone, or, in other words, the classes of negative extremal rational curves on deformations of M) is bounded in absolute value by a number depending only on the deformation class of M. The proof uses ergodic theory on homogeneous spaces.

[6] Ekaterina Amerik, Misha Verbitsky, *Construction of automorphisms of hyperkähler manifolds*, arXiv:1604.03079

Let M be an irreducible holomorphic symplectic (hyperkähler) manifold. If $b_2(M) \geq 5$, we construct a deformation M' of M which admits a symplectic automorphism of infinite order. This automorphism is hyperbolic, that is, its action on the space of real (1,1)-classes is hyperbolic. If $b_2(M) \geq 14$, we construct a deformation which admits a parabolic automorphism.

[7] Ljudmila Kamenova, Misha Verbitsky, *Algebraic non-hyperbolicity of hyperkahler manifolds with Picard rank greater than one*, arXiv:1604.02601

A projective manifold is algebraically hyperbolic if the degree of any curve is bounded from above by its genus times a constant, which is independent from the curve. This is a property which follows from Kobayashi hyperbolicity. We prove that hyperkahler manifolds are non algebraically hyperbolic when the Picard rank is at least 3, or if the Picard rank is 2 and the SYZ conjecture on existence of Lagrangian fibrations is true. We also prove that if the automorphism group of a hyperkahler manifold is infinite then it is algebraically non-hyperbolic.

[8] Anna Fino, Gueo Grantcharov, Misha Verbitsky, *Algebraic dimension of complex nilmanifolds*, arXiv:1603.01877

Let M be a complex nilmanifold, that is, a compact quotient of a nilpotent Lie group endowed with an invariant complex structure by a discrete lattice. A holomorphic differential on M is a closed, holomorphic 1-form. We show that $a(M) \leq k$, where a(M) is the algebraic dimension a(M) (i.e. the transcendence degree of the field of meromorphic functions) and k is the dimension of the space of holomorphic differentials. We prove a similar result about meromorphic maps to Kahler manifolds.

[9] Liviu Ornea, Misha Verbitsky, *Hopf surfaces in locally conformally Kahler manifolds with potential*, arXiv:1601.07421

An LCK manifold with potential is a compact quotient M of a Kahler manifold X equipped with a positive plurisubharmonic function f, such that the monodromy group acts on X by holomorphic homotheties and maps f to a function proportional to f. It is known that M admits an LCK potential if and only if it can be holomorphically embedded to a Hopf manifold. We prove that any non-Vaisman LCK manifold with potential contains a Hopf surface H. Moreover, H can be chosen non-diagonal, hence, also not admitting a Vaisman structure.

[10] Fedor Bogomolov, Ljudmila Kamenova, Steven Lu, Misha Verbitsky, *On the Kobayashi pseudometric, complex automorphisms and hyperkähler manifolds*, arXiv:1601.04333

We define the Kobayashi quotient of a complex variety by identifying points with vanishing Kobayashi pseudodistance between them and show that if a compact complex manifold has an automorphism whose order is infinite, then the fibers of this quotient map are nontrivial. We prove that the Kobayashi quotients associated to ergodic complex structures on a compact manifold are isomorphic. We also give a proof of Kobayashi's conjecture on the vanishing of the pseudodistance for hyperkähler manifolds having Lagrangian fibrations without multiple fibers in codimension one. For a hyperbolic automorphism of a hyperkähler manifold, we prove that its cohomology eigenvalues are determined by its Hodge numbers, compute its dynamical degree and show that its cohomological trace grows exponentially, giving estimates on the number of its periodic points.

3.1.19 Ilya Vyugin

[1] I. V. Vyugin, E. V. Solodkova, I. D. Shkredov, Intersections of Shifts of Multiplicative Subgroups, Mat. Zametki, 100:2 (2016), 185-195

Using Stepanov's method, we obtain an upper bound for the cardinality of the intersection of additive shifts of several multiplicative subgroups of a finite field. The resulting inequality is applied to a question dealing with the additive decomposability of subgroups.

[2] I. V. Vyugin, E. V. Solodkova, I. D. Shkredov, "Additive Energy of Heilbronn Subgroup", Mat. Zametki, 101:1 (2017), 37-49

A new upper bound for the additive energy of the Heilbronn subgroup is determined and some applications in the case of distribution of Fermat quotients are obtained.

3.2 Scientific conferences and seminar talks

3.2.1 Anton Aizenberg

[1] Conference "Toric topology 2016 in Kagoshima", Kagoshima, Japan, 2016, April, 19 – April, 22.

Talk "Volume polynomials and duality algebras of multi-fans".

[2] Conference "7-th European Congress of Mathematics", Berlin, Germany, 2016, July, 18 – July, 22.

Contributed talk "Volume polynomial and Poincare duality algebra of multi-polytope".

[3] Conference "Alexandrov readings", Moscow, Russia, 2016, May, 22 – May, 26.

Talk "Toric origami manifolds and metric properties of planar graphs".

[4] Session of Moscow Mathematical Society, Moscow State University, 2016, April, 26.

Talk "Combinatorics of triangulated manifolds and volume polynomials".

[5] Weekly seminar, Laboratory of Algebraic Geometry and its Applications, Higher School of Economics, Moscow, May, 13.

Talk: "Stanley–Reisner algebras of simplicial manifolds".

[6] Seminar "Knots and representation theory", Moscow State University, 2016, May, 17.

Talk "Combinatorics of simplicial manifolds".

[7] Seminar "Algebraic topology and its applications", Moscow State University, 2016, November, 15.

Talk "For which reason there are no symplectic toric manifolds over a dodecahedron?"

3.2.2 Alexander Belavin

[1] Workshop on Geometric Correspondence of Gauge Theories

The Abdus Salam International Centre for Theoretical Physics (ICTP, Trieste, Italy) The Workshop co-sponsored by SISSA and INFN. 12-16 September 2016.

Talk. Topological CFT and Frobenius manifolds.

[2] The second French-Russian Scientific Conference Random Geometry and Physics. Paris, Institut Henri Poincare, 17-21 2016

Talk. Superstrings and Frobenius manifolds

3.2.3 Alexei Elagin

[1] International conference "Categorical and analytic invariants in Algebraic geometry 3", Moscow, September 12-16, talk "On Serre dimension of finite dimensional algebras".

[2] Visit to Indiana University (Bloomington, USA), October 12-26. Joint work with V. Lunts and S. Zhang.

[3] Sergey Galkin's seminar at IUM, March 19, talk "Exceptional collections of line bundles on del Pezzo surfaces".

3.2.4 Alexei Gorodentsev

I have not participated any conferences during this year.

3.2.5 Maxim Kazarian

[1] International Conference on Combinatorics of Moduli Spaces, Hurwitz Numbers, and Cohomological Fields Theories, June 6 - 11, 2016, Moscow, Russia, Laboratoire J.-V. Poncelet, Steklov Mathematical Institute, Higher School of Economics and Skolkovo Institute of Science and Technology. The talk 'Universal cohomological expressions for strata in genus zero Hurwitz spaces,

3.2.6 Anton Khoroshkin

Talk "On categorification of Macdonald polynomials"

at *Seminaire on mathematical physics*, NIU HSE Moscow;

Talk "Compactified Moduli spaces of curves as a Homotopy quotients of operads"

at *Characteristic classes and intersection theory* seminar, NIU HSE Moscow;

3.2.7 Iosif Krasilshchik

[1] International Conference "Algebraic Geometry and Mathematical Physics", Tromsø (Norway), June, 30 – July, 1

Talk "Algebraic models of integrable systems"

[2] International Conference "Differential Geometry and its Applications", Brno (Czech Republic), July, 11 – 15

Talk "Nonlocal geometry of PDEs and integrability"

3.2.8 Alexander Kuznetsov

[1] "Triangulated Categories in Algebra, Geometry and Topology", University of Stuttgart, March 14–18, 2016.

Talk "Rectangular Lefschetz decompositions and fractional Calabi–Yau categories"

[2] "Homological Algebraic Geometry", University of Sheffield, June 9–10, 2016.

Talk "Categorical joins"

[3] "New methods in birational geometry", University of Toulouse, June 22–July 1, 2016.

Talk "Derived categories and birational properties of Gushel–Mukai varieties"

[4] "Categorical and analytic invariants in Algebraic geometry 3", HSE, Moscow, September 12–16, 2016.

Talk "On double covers of quadratic and Lagrangian degeneracy loci"

[5] "Non-commutative, derived and homotopical methods in geometry", University of Antwerp, September 19–24, 2016.

Talk "Categorical joins and homological projective duality"

[6] "Categorical and Analytic invariants IV", IPMU, Tokyo, November 14–18, 2016.

Talk "Exceptional collections in surface-like categories"

3.2.9 Maxim Leyenson

[1] Examples and projective models of Hilbert modular surfaces

Six talks at the Ossip Schwarzman and Andrey Levin seminar, HSE, Moscow, November-December 2016.

Program:

- ⋆ Resolution of cusp singularities of Hilbert modular surfaces;
- ⋆ Hilbert modular surfaces $X(5)$ and $X(5,2)$ for the field $\mathbb{Q}(\sqrt{5})$ of levels 1 and 2;
- ⋆ Configuration of diagonals on $X(5,2)$, and the proof of rationality;
- ⋆ Study of curves on $X(5,2)$: it is a blowup of a cubic surface.
- ⋆ Sylvester form for cubic surfaces; cubic surfaces with Eckardt points. Clebsch cubic surface as a unique surface with 10 Eckardt points.
- ⋆ Klein model for the Clebsch cubic surface.
- ⋆ Eisenstein series for the Hilbert modular group and its congruence subgroups;
- ⋆ Explicit map of $X(5,2)$ to the Klein surface given by the Eisenstein series;
- ⋆ $X(5)$ as a double cover of $\mathbb{P}^2$, and study of the ramification curve;
- ⋆ Ring of even Hilbert modular forms on $X(5)$, and $X(5,2)$;
- ⋆ Decomposition of the discriminant for $X(5)$ into a product of two Hilbert modular forms of odd weight.

3.2.10 Grigory Olshanski

[1] Conference "Mathematics, Theoretical Physics and Data Science 2016 dedicated to anniversaries of Yakov Sinai and Grigory Margulis".

Moscow, Russia, July 5–7, 2016

Talk "New results in asymptotic representation theory"

3.2.11 Alexei Penskoi

[1] Conference "Doppler Institute-CRM Workshop on the occasion of 80th birthdays of Jiři Patera and Pavel Winternitz", Prague, Czech Republic, May 30 - June 3, 2016.

Talk "Spectral geometry and symmetry reduction"

[2] Visit to Weizmann Institute of Science, Rehovot, Israel, March 2016

Talk "Recent advances in geometric optimization of eigenvalues of the Laplace-Beltrami operator on closed surfaces" at the Joint session of Seminar in Geometric functional analysis and probability & Geometry and Topology seminar, March 3, 2016.

[3] Session of Moscow Mathematical Society, April 19, 2016.

Talk "Spectral Geometry: hear the Shape, see the Sound"

3.2.12 Petr Pushkar'

[1] Seminar on Symplectic Geometry, HSE, Moscow
Series of talks and lectures. "From reduction to symplectic fixed points and generating families"

3.2.13 Sergey Rybakov

[1] Higher Dimensional Algebraic Geometry and Characteristic p.
September 12 - 16, 2016, Marseille, France
Title: On zeta functions of cubic surfaces over finite fields
http://scientific-events.weebly.com/1376.html

3.2.14 George Shabat

[1] Workshop "Grothendieck-Teichmuller Theories", Chern Institute of Mathematics, Nankai University, Tianjin, China, July 24 – July 31. Talk "Belyi pairs in the critical filtrations of Hurwitz spaces".

[2] Conference "Interactions between Topological Recursion, Modularity, Quantum Invariants and Low-Dimensional Topology", Melbourne, Nobember 28 – December 23. Talk "Counting Belyi pairs over finite fields".

3.2.15 Arkady Skopenkov

[1] Mathematics of Jiri Matiusek, Prague, July, poster 'Eliminating Higher-Multiplicity Intersections, Codimension 2'

[2] Conference of Moscow Institute of Physics and Technology, Dolgoprudnyi, November, Talk "Stability of intersections of graphs in the plane and the van Kampen obstruction"

[3] Discrete Geometry Seminar, Institute of Science and Technology, Austria,
Talk "Stability of intersections of graphs in the plane and the van Kampen obstruction"

[4] Topology seminar of PDMI, St Petersburg,
Talk "A user's guide to topological Tverberg conjecture"

[5] Joint seminar of Faculty of Computer Science, Higher School of Economics, and Faculty of Innovations and High Technology, Moscow Institute of Physics and Technology,
Talk "Stability of intersections of graphs in the plane and the van Kampen obstruction"

[6] Seminar on Lie groups, Independent University of Moscow,
Talk "A user's guide to topological Tverberg conjecture"

[7] Postnikov memorial seminar, Moscow State University,
Talk "Classification of knotted tori"

3.2.16 Mikhail Skopenkov

[1] Skopenkov M., Surfaces containing two circles through each point, 'Mathematics, Theoretical Physics and Data Science 2016 dedicated to anniversaries of Yakov Sinai and Grigory Margulis', 5-7 July 2016, IUM, http://mpd2016.iitp.ru/schedule.

[2] Skopenkov M., Tiling by rectangles and alternating current, 'Transversal aspects of tilings' 11.06.2016 21.06.2016, https://oleron.sciencesconf.org/.

[3] Talks at several seminars in Moscow and Kirov.

3.2.17 Evgeni Smirnov

[1] International Conference on Algebraic Geometry, Complex Analysis, and Computer Algebra, Koryazhma, Arkhangelsk region, Russia, August 3–9, 2016
Talk: "Multiple flag varieties"

[2] Laboratory of Algebraic Geometry and its Applications seminar, HSE, Moscow, Russia, March 18, 2016
Talk: "Chord diagrams"

[3] Seminar "Combinatorics of Vassiliev invariants", HSE, Moscow, Russia, January 2016
Series of two talks: "Ribbon graphs and Lagrangian subspaces"

3.2.18 Mikhail Verbitsky

1. Simons Symposium on Geometry Over Nonclosed Fields,
   20.04.2016, "Constructing automorphisms of hyperkähler manifolds".

2. 1-st International Conference on Differential Geometry,
   14.04.2016, Fez, Morocco, plenary talk "Degenerate twistor deformations"

3. Twenty-third Gökova Topology/Geometry conference,
   30.05.2016-4.06.2016, Gökova, Turkey, a minicourse of 4 lectures, "Unobstructed symplectic packing" and "Teichmüller spaces and moduli of geometric structures".

4. Conference: Hitchin 70: A celebration of Nigel Hitchin's 70th birthday in honour of his contributions to mathematics (QGM, Aarhus University), 05.09.2016, "Ergodic complex structures".

5. Derived categories and Chow groups of hyperkaehler and Calabi-Yau varieties, Simons Center for Geometry and Physics, 20.09.2016, " Perverse coherent sheaves on hyperkähler manifolds and Weil conjectures".

3.2.19 Ilya Vyugin

[1] Combinatorial and Additive Number Theory 2016, January 4-8, 2016 Graz, Austria.

Title: "Solutions of polynomial equation over $\mathbb{F}_p$ and new bounds of energy of multiplicative subgroups".

[2] Information Technology and Systems 2016, September, 25-30, Repino, St. Petersburg, Russia

Title: "New bounds of polynomial energy" (joint with Sergey Makarychev).

[3] Information Technology and Systems 2016, September, 25-30, Repino, St. Petersburg, Russia

Title: "Polynomial equations over $\mathbb{F}_p$ in subgroups of $\mathbb{F}_p^*$" (joint with Sergey Makarychev).

[4] Information Technology and Systems 2016, September, 25-30, Repino, St. Petersburg, Russia

Title: "On the Riemann-Hilbert Problem for Linear q-Difference Equations" (joint with Roman Levin).

[5] Conference Diferential Equations and Related Problems, Kolomna–Konstantinovo, June 10-11, 2016.

Title: "On Birkhoff's Theory of Difference Equations".

[6] Talk at the seminar Contemporary Problems in Number Theory (Chairs: Sergei Konyagin and Ilya Shkredov) December 8, 2016.

Title: "The upper bound of the number of solutions of a set of polynomial equations".

[7] Talk at the seminar Contemporary Problems in Number Theory (Chairs: Sergei Konyagin and Ilya Shkredov) May 5, 2016.

Title: "Polynomial Energy of Subgroup and Application".

3.3 Teaching

3.3.1 Anton Aizenberg

[1] Topology-3. Independent University of Moscow, II year students, February-May 2016, 2 hours (lecture) + 2 hours (seminar) per week.

Program

- ⋆ Closed manifolds (topological, smooth, homology). Orientability. Orientable cover. Fundamental cycle, degree of a map. Cap-product. Poincare duality isomorphism.

- ⋆ Intersection index of homology cycles. The case of cycles represented by smooth submanifolds. The connection between intersection theory and the product in cohomology. Manifolds with boundary. Signature of a closed manifold.
- ⋆ Principal G-bundles. Universal bundles. Construction of classifying space for general G. Classifying spaces for discrete groups and group cohomology.
- ⋆ Stiefel and Grassmann manifolds. Vector bundles. Structure group of a vector bundle and associated principal bundle. Tangent bundle of a smooth manifold. The ring of characteristic classes for bundles with the given structure group.
- ⋆ Cohomology of Grassmann manifolds (both infinite- and finite-dimensional cases). Stiefel–Whitney and Chern classes. Euler class. Pontryagin classes.
- ⋆ Applications: existence of division algebras, immersions of real projective spaces in euclidean spaces, Borsuk–Ulam theorem. Stiefel–Whitney characteristic numbers.
- ⋆ The notion of cobordism. Pontryagin theorem. Formulation of Thom theorem. Construction of invariants of smooth manifolds using formal power series: Hirzebruch genera. Hirzebruch signature theorem. The notion of K-theory and Chern character.

[2] Combinatorics, topology and algebra of simplicial complexes. Steklov Mathematical Institute, February-June 2016, 3 hours per week.

Program

- ⋆ Simplicial complexes, basic operations. Reminder on simplicial (co)homology. Euler characteristic.
- ⋆ Important classes of simplicial complexes: convex, simplicial and homology spheres, simplicial and homology manifolds. Question of algorithmic recognizability.
- ⋆ h-vector. Dehn–Sommerville relations for homology spheres and homology manifolds.
- ⋆ Nonnegativity of h-vector and the upper bound theorem. Stanley–Reisner algebra.
- ⋆ Review of necessary notions in commutative algebra: projective and injective modules, resolutions, Tor and Ext functors, Cohen–Macaulay rings and modules, local cohomology, Gorenstein algebras and Poincare duality algebras.
- ⋆ Cohen–Macaulay complexes. Reisner's theorem.
- ⋆ g-theorem and g-conjecture for spheres.
- ⋆ Buchsbaum complexes. Schenzel theorem and Novik–Swartz theorems. h'- and h''-vectors. Generalized g-conjecture for homology manifolds.

- ⋆ Regular cell complexes and their barycentric subdivisions. Balanced complexes. Flag f- and h-numbers, cd-index. Nonnegativity of the cd-index of convex spheres.
- ⋆ h-vectors of balanced convex spheres: Juhnke-Kubitzke–Murai theorem.
- ⋆ Tor-algebra of a simplicial complex. Hochster and Baskakov–Buchstaber–Panov theorems. Topological proof of Reisner's theorem. Topological characterization of the depth of Stanley–Reisner algebra.
- ⋆ Topological models arising in toric topology.

[3] Toric topology, Independent University of Moscow, III-V year students, September-December 2016, 2 hours per week.

Program

- ⋆ Group action on a topological space. Borel construction. Equivariant cohomology.
- ⋆ Short intro in spectral sequences: spectral sequence of a filtration, Serre spectral sequence, Mayer–Vietoris sequence. Equivariantly formal spaces.
- ⋆ Two Leray spectral sequences associated with the action. Localization theorem for equivariant cohomology.
- ⋆ Statement of Chang–Skjelbred and Atiyah–Bredon theorems. GKM-method to study torus actions. GKM-manifolds and GKM-graphs.
- ⋆ Examples: Grassmann manifolds, flag manifolds. Quasitoric manifolds. Locally standard torus actions.
- ⋆ Reconstructing quasitoric manifold from its characteristic data. Smooth structure and (equivariant) cohomology of quasitoric manifolds. Moment-angle manifolds.
- ⋆ Short intro in symplectic geometry: de Rham complex, Cartan's magical formula, hamiltonian action, moment map, symplectic reduction. Maximal actions: symplectic toric manifolds.
- ⋆ Statements of Atiyah–Guillemin–Sternberg convexity theorem and Delzant's classification theorem. Reconstruction of a symplectic toric manifold from the standard action using symplectic reduction. Statement of Duistermaat–Heckmann theorem.
- ⋆ Equivariant versions of Gysin homomorphism and characteristic classes. Atiyah–Bott localization formula. Analytical interpretation: integration of Cartan forms. Equivariant symplectic form.
- ⋆ Computation of symplectic volumes via localization. Proof of Duistermaat–Heckmann theorem via localization. Fourier transform of a simple polytope.

- ⋆ Comments: stationary phase method, Lawrence formula for the volume of polytope, exact integration of polynomials over polytopes. Harish-Chandra–Itzykson–Zuber integral. Torus manifolds and multi-polytopes.
- ⋆ Moment-angle complexes and coordinate subspace arrangements. Davis–Januszkiewicz spaces. Cohomology of moment-angle complexes: Buchstaber–Panov theorem and Hochster formula. Eilenberg–Moore spectral sequence.
- ⋆ Goresky–MacPherson formula and Mayer–Vietoris spectral sequence.

[4] Calculus, Faculty of Computer Science, Higher School of Economics, I year students, September-December 2016, 2 hours x 2 groups per week.

Program

Standard program in basic calculus: real numbers, sequences, limits, functions, derivatives.

3.3.2 Alexander Belavin

[1] Introduction to Supersring Theory. Independent University of Moscow, IV-VI year students, September-December 2016, 4 hours per week.

Program

I. Quantization of bosonic string.

1.Introductio. Why strings? 2. Relativistic particle. 3. Relativistic string. Nambu-Goto action. 4. Polyakov approach. Symmetries. 5.Conformal gauge. 6.Conformal theory of massless boson field . 7. The quantization, modes , commutation relations. 8. Virasoro Algebra. Physical states. 9. Physical state at levels 0,1 and 2. Positivity of physical states with a = 1 and d = 26. 10. Kac theorem and the number of physical statess at an arbitrary level.

II. Quantization of fermionic string Neveu-Schwarz-Ramond.

1. Action in superconformal gauge. 2. NS - and R - fields. Modes . The commutation relations. 3. Constrains . N = 1 Super Virasoro. 4. The physical state at the lower levels. 5. The positive rate for a = 1/2 (5/8) in NS (R) -sector and d = 10. 6. Feigin-Fuks theorem.Numbers of the physical states at an arbitrary level. 7. GSO projection and Space-Time supersymmetry in NRS strings.

III. BRST quantization of string theory.

1. BRST symmetry. 2.Faddeev-Popov ghosts d = 26 string. 3.The vertex for the massless vector particle. 4.F-P ghosts and BRST operator in the NSR string. 5.Bosonization. 6. Bosonization of ghostss in NSR strings.(Pictures). 7. Vertex for the massless spin field in Ramond sector.

IV. N = 2 Super Virasoro on the world-sheet and Supersymmetry in space-time in the NSR string theory.

1.N = 2 Super Virasoro generators, commutation relations, U (1) -current, spectral flux. 2.N = 2 Super Virasoro in the NSR string. 3.Spectral flux action on in the space of states. 4. Using the spektral flow for construction of the odd generator of Super Poincare.

3.3.3 Alexei Elagin

[1] Introduction to homological algebra. Independent University of Moscow, from II year, February – May 2016, 2 hours per week.

Program:

1. Complexes and cohomology.
2. Categories and functors.
3. Classical derived functors.
4. Derived functors: examples and applications.
5. Abelian categories.
6. The derived category.
7. Triangulated categories.
8. Morphisms in derived category.
9. The derived functor.
10. Spectral sequences.

See *http://ium.mccme.ru/s16/s16-elagin.html* for the lecture notes.

3.3.4 Alexei Gorodentsev

[1] Geometry. Independent University of Moscow, 1st year, 1st term. September–December 2016, 4 hours per week.
See `http://gorod.bogomolov-lab.ru/ps/stud/geom_ru/1617/list.html`

Program.

1. The spaces $\mathbb{R}^n$ for small $n = 1, 2, 3$. Points and vectors. Center of masses, barycentric coordinates. Areas and volumes, Cramer rules. Lines and planes. Scalar product, the CauchySchwarz and triangle inequalities. Euclidean transformation groups, compositions of shifts, reflections, and rotations. Stereographic projections and inversions. Möbius group of the plane.

2. Higher dimensional linear and affine spaces. Bases, repers, and coordinates. Dimensions of sums and intersections. $\mathrm{Af}(V) = V \rtimes \mathrm{GL}(V)$. The semiaffine group (the bijective maps sending the lines to the lines are the affine maps twisted by automorphism of the ground field over $\mathbb{Q}$).

3. Volumes of parallelepipeds and simplexes. Multilinear sign alternating forms and Grassmannian polynomials. Generalized vector products and Cramer rules.

4. Euclidean spaces. Orthogonal complements and orthogonal projections, orthogonalization. Euclidean volume, Gramians. Computation of lengths and angles. Examples of polyhedrons: the cube, cocube, simplex etc in dimension n. Euclidean and orthogonal groups, canonical form of a linear orthogonal transformation, decomposition of a Euclidean transformations into a composition of reflections in hyperplanes.

5. Finite reflection groups. Weyl chambers, root systems, and Coxter graphs. Classification of connected Coxter graphs and regular polyhedrons in Euclidean spaces.

6. Convex geometry of $\mathbb{R}^n$. Geometric description and topological equivalence of norms on $\mathbb{R}^n$. Supporting hyperplanes, faces, vertexes, and extreme points of closed convex sets. Krein-Milman and Minkowski–Weyl theorems, Farkas lemma. Enumeration of faces for convex polyhedrons. Linear optimization and the Gale duality. The simplex-algorithm.

[2] Geometric introduction to algebraic geometry. Faculty of Mathematics at the Higher School of Economics, 1 term course for 2-4 year students. September–December 2016, 4 hours per week.
See `http://gorod.bogomolov-lab.ru/ps/stud/giag_ru/1617/list.html`

Program.

1. Projective spaces and projective geometry: projective groups, projections, cross-ratio, the Veronese curve. Projective quadrics: singular points and tangent spaces, linear subspaces on a quadric, polar mapping, pencils of quadrics.

2. The Veronese, Segre, and Grassmann varieties, and their projective embeddings. $\mathrm{Gr}(2,4) \subset \mathbb{P}_5$ in details. Polarizations of polynomials, linear support of a tensor, Plücker relations. Tangent spaces and polars of affine and projective hypersurfaces.

3. Commutative algebra draught: Noetherian rings, integral extensions, the Nullstellensatz, transcendence degree, resultants.

4. The anti-equivalence between the categories of affine algebraic varieties over algebraically closed field $\Bbbk$ and finitely generated reduced $\Bbbk$-algebras. Maximal spectrum and Zariski topology. Geometric properties of algebra homomorphisms.

5. The structure sheaf of affine algebraic variety and its sections over the principal open sets. Definition of an algebraic manifold. Separability. Morphisms from projective to separable varieties are closed. Blow up of a point, projection of projective variety from an external point is finite. Normalization theorems, the normalization in an algebraic family.

6. The dimension of an algebraic variety. Dimensions of fibers, semi-continuity and constructibility theorems. Generic plane section of a projective variety, geometric definition of the dimension of a projective variety. 27 lines on a smooth cubic surface.

7. Locally trivial vector bundles and locally free $\mathcal{O}_X$-modules. Line bundles and the Picard group, $\mathrm{Pic}(\mathrm{Spec}\, A) = 0$ for factorial A, $\mathrm{Pic}(\mathrm{Gr}(k, n)) = \mathbb{Z}$. Linear systems an projective embeddings via line bundles. Splitting of locally trivial vector bundles on a line.

8. Geometric schemes, the base change. Cones and projective cones over an algebraic variety. Blow up and the normal cone of a subvariety in an an algebraic variety. Kähler differentials, derivations, cotangent sheaf and tangent bundle. Computation of tangent spaces and tangent cones. The local properties of smooth points.

9. The degree of a projective variety. Hilbert's polynomials and Poincare series. The Bézout's theorem.

3.3.5 Maxim Kazarian

[1] Calculus. Independent University of Moscow, Ist year students 2nd term), February-May 2016, 4 hours per week.

Program

Representation of curves given parametrically and implicitely. Singular and other special points, behaviour near the special points and at infinity, asymptotes.

Differentiation of functions in several variables. Directional derivative, examples of non-differentiable functions possessing partial derivatives, equality of mixed partial derivatives, chain rule for the derivative of a composition.

Taylor theorem for functions in several variables.

Critical points. Maxima, minima, and saddle points of functions in two variables, second differential, Hessian, Morse index, Morse lemma.

Inversion function theorem. Invertibility of a mapping and non-degeneracy of the Jacobian, derivative of the inverse function, curvilinear coordinates.

Contraction mapping principle. The proof of the inversion mapping theorem.

Implicit function theorem. The derivative of an implicit function, smooth multidimensional surfaces in Euclidean spaces, local coordinates.

Conditional extremum. The derivative of the restriction function, Lagrange multipliers, Morse index of the restriction function.

Typical singularities of maps of surfaces. Foldings and pleats, developable of a family of curves, evolute of a plane curves and rigidity of its singular points.

[2] Calculus on manifolds. Independent University of Moscow, IInd year students 3d term), September-December 2016, 4 hours per week.

Program

Curves in $\mathbf{R}^n$. Path integral. Change of parametrization. Behavior under a change of a path.

Manifolds. Submanifolds in $\mathbf{R}^n$ and abstract manifold. Local coordinates. Orientability and orientation.

Tangent vector. Tangent vector as a velocity of a motion along a curve. Directional derivative of a function. Derivation in the ring of functions. Tangent space to a manifold at a point. Tangent bundles. Derivative of a mapping. Chain rule.

Vector fields. Phase curves and phase flows. Rectification of vector fields. Commutator. Frobenius theorem on integrable dif=stributions.

Differential forms on manifolds. Differential of a function. Wedge product. Exterior differential. Volume form, area form, and Gelfand-Leray form. The transformation rule under coordinate change for functions, vector fields and differential forms.

Integration of differential forms. Manifold with boundary. Induced orientation of the boundary. Stokes theorem.

Lie derivative. Cartan identity.

Poincare lemma. Closed and exact forms. De Rham cohomology.

Differential forms in classical vector calculus and mathematical physics. Green's formula, Gauss-Ostrogradskii divergence theorem, Stokes formula for a surface in $\mathbf{R^3}$. Maxwell equation.

Harmonic functions. Mean value theorem. Maximum principle.

3.3.6 Anton Khoroshkin

[1] Quantum Groups II. Independent University of Moscow, III year and higher level students, February-May 2015, 4 hours per week (2 hours lecture + 2 hours seminar).

This is the second part of the course which wants to clarify the notions and ideas invented by V.Drinfeld developed in the series of papers on quantum groups.

program:

- Coalgebras, Hopf algebras and tensor categories;
- Quantization and classical limit, Poisson algebras;
- Poisson-Lie groups, Lie bialgebras;
- Coboundary, triangular and quasitriangular Lie bialgebras;
- Drinfeld Double and universal R-matrix;

- Operad theory and Tamarkin's quantization;
- Little discs operad and Deligne's conjecture;
- Braided tensor categories;
- KZ-equations and Drinfeld category;

[2] Algebraic theory of D-modules. Higher School of Economics, Fall 2016, faculty of mathematics, III year and higher level students, 3 hours per week (2 hours lecture + 1 hours seminar)

program:

Analitic continuation and Bernstein-Sato polynomials;
Gelfand-Kirillov dimension;
Holonomic modules;
Singular support, singular cycles and Habbers theorems;
Homological properties of functional dimension;
Sheaf of differential operators and D-modules on general algebrac;
Kashiwara's theorem;
Functors on D-modules;
D-affine property of projective spaces;
Perverse sheaves and Riemann-Hilbert correspondanc;
Kazhdan-Lustig polynomials.

[3] Algebra-I (Fall 2016) Higher School of Economics, faculty of mathematics, I year undergraduate students (4 hours lectures + 2 hours seminar per week)

[4] Galois theory (Spring 2016) Higher School of Economics, faculty of mathematics, II year undergraduate students (3 hours seminars per week)

[5] Geometry (Fall 2016) Higher School of Economics, faculty of mathematics, I year undergraduate students (2 hours seminars per week)

3.3.7 Iosif Krasilshchik

[1] Geometry of infinitely prolonged differential equations. Independent University of Moscow, II–V year and PhD students, February–May 2016, 4 hours per week.

Program

1. Spaces and bundles of infinite jets. The Cartan distribution on the space $J^\infty(\pi)$.
2. Symmetries of the Cartan distribution on $J^\infty(\pi)$. Evolutionary derivations.
3. Linearizations of nonlinear differential operators and lifts of linear ones. Cartan connection. $\mathcal{C}$-differential operators. Horizontal de Rham complex.
4. Adjoint operator and the Green formula.
5. Infinitely prolonged differential equations. Higher symmetries and their generating functions. Theorem about the structure of symmetries.

6. Examples of computing higher symmetries. Recursion operators.

7. Homological theory of recursion operators. Variational Nijenhuis bracket.

8. Nonlocal geometry of infinitely prolonged equations. Differential coverings. Nonlocal symmetries and shadows.

9. Examples of computations. Bäcklund transformations.

10. Conservation laws and Abelian coverings. Generating functions of conservation laws. Geometric theory of recursion operators.

11. The tangent covering. Variational symplectic structures.

[2] Cohomological invariants of nonlinear differential equations. Independent University of Moscow, II–V year and PhD students, September–December 2016, 4 hours per week.
Program

1. Reminder: infinite jets, infinitely prolonged differential equations and geometric structures on them.

2. The Vinogradov $\mathcal{C}$-spectral sequence (variational bicomplex) on $J^\infty(\pi)$. One-line theorem.

3. Cohomological framework of the Lagrangian formalism The $\mathcal{C}$-spectral sequence of an infinitely prolonged equation.

4. Compatibility complex of a $\mathcal{C}$-differential operator and the p-lines theorem.

5. Reminder: nonlocal geometry of equations and differential coverings.

6. Variational symplectic structures.

7. Two-line equations. Cotangent covering.

8. Variational Schouten bracket.

9. Hamiltonian equations. Variational Poisson bracket. Compatible Poisson structures. Bi-Hamiltonian equations. Magri theorem.

[3] Calculus I, Russian State University for the Humanities, I year students, September–December 2016, 4 hours per week.
Program.

1. Sets and maps.

2. Real numbers, Euclidean plane, Cartesian coordinates polar coordinates.
3. Limits and their properties. The "remarkable limits" $\lim_{x\to 0}\frac{\sin x}{x}$ and $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$.
4. Functions in one real variable. Continuity. Basic properties of continuous functions. Continuity of elementary functions.
5. Geometrical and dynamical meaning of the first derivative. The rigorous definition. First differential.
6. Basic properties of first derivative.
7. Derivatives of of the composite and inverse functions.
8. Derivatives of the elementary functions.
9. Higher derivatives. The Taylor formula.
10. L'Hôpital's rules.
11. The Newton method of solving equations $f(x) = 0$.
12. Derivatives and extrema.
13. Drawing graphs of functions using differential calculus.

[4] Ordinary differential equations, Russian State University for the Humanities, II year students, October–December 2016, 4 hours per week.

Program.

1. Problems in geometry and mechanics leading to ODEs.
2. General definition of ODEs and their solutions. Types of solutions.
3. The phase portrait and isolines.
4. Initial data and Cauchy problem. The uniqueness and existence theorem.
5. First-order equations.
   (a) Separation of variables.
   (b) Homogeneous equations.
   (c) Inhomogeneous equations.
   (d) Equations in total differentials. Integrating factor.

6. Second-order equations.
   (a) Equations $f(x, y', y'') = 0$.
   (b) Equations $f(y, y', y'') = 0$.
7. Linear equations of arbitrary order.
   (a) Homogeneous equations. Fundamental set of solutions. Wronskian.
   (b) Solving inhomogeneous equations by the Lagrange method.
   (c) Equations with constant coefficients. The characteristic polynomial.
      i. The case of simple real roots.
      ii. The case of multiple real roots.
      iii. The case of simple complex roots.
      iv. The case of multiple complex roots.
8. Solving ODEs by means of power series.

[5] Calculus II, Russian State University for the Humanities, I year students, February–June 2016, 4 hours per week.

Program.

1. Integral calculus
   (a) Problems leading to integration. Indefinite integral and its properties. Change of variables. Integration by parts.
   (b) Integration of some classes of elementary functions.
   (c) The definite integral. Riemann and Darboux sums. Basic properties. The Newton-Leibniz Theorem. Change of variables.
   (d) Areas of flat figures. Length of arcs. Volumes of axially symmetrical bodies.
   (e) Improper integrals.
2. The theory of series
   (a) Convergence tests: positive series
   (b) Convergence tests: all series
   (c) The Taylor series
   (d) Power series

3.3.8 Alexander Kuznetsov

[1] “Categorical resolutions of singularities” (minicourse, 4 lectures), summer school “New methods in birational geometry”, University of Toulouse, June 22–July 1, 2016.

Abstract: I will discuss an approach to resolution of singularities of schemes via derived categories of coherent sheaves. I will define what a categorical resolution of singularities is, and will describe a construction of a categorical resolution for any separable scheme of finite type over a field of characteristic zero (thus showing that categorically any such singularity behaves as a rational singularity). Then I will discuss the question of constructing small categorical resolutions, concentrating on the notions of crepancy and explicit examples.

[2] “Root systems and Dynkin diagrams” (minicourse, 4 lectures), summer school “Modern mathematics”, Dubna, July 19–30, 2016.

Abstract: A root system is a finite collection of vectors in a Euclidean space such that for each of these vectors v the reflection s_v with respect to the hyperplane H_v orthogonal to v preserves the system, and moreover, for each vector v' from the system the difference $s_v(v') - v'$ is an integral multiple of v. In a two-dimensional space there are only three (reduced and irreducible) root systems. It turns out that one can completely classify: there are several series and several “exceptional” systems. We will discuss root systems in spaces of arbitrary dimension, their classification, and Dynkin diagrams, naturally appearing in the classification problem. Besides, we will discuss an important generalization of root systems — affine systems, and describe some fields pf mathematics, where root systems appear.

[3] “Derived categories and moduli spaces” (minicourse, 5 lectures), summer school “Advanced School and Workshop on Moduli Spaces, Mirror Symmetry and Enumerative Geometry”, ICTP, Trieste, August 1–19, 2016.

Abstract: I will discuss the point of view of derived categories on various aspects of theory of moduli spaces. We will start with a short introduction into derived categories of coherent sheaves on algebraic varieties and the techniques of exceptional collections and semiorthogonal decompositions. After that we will discuss various applications of these techniques. Among these are monadic descriptions of moduli spaces of vector bundles on projective spaces and other varieties admitting a full exceptional collection. Also we will discuss symplectic forms on moduli spaces of sheaves on cubic fourfolds and other varieties admitting a K3 admissible subcategory.

3.3.9 Maxim Leyenson

[1] Introduction into algebraic geometry – 2

Program

The main idea of this course is to get basic results of the classical algebraic geometry while introducing only as much (or as little!) modern techniques as needed.

When do sheaf cohomology simplify things, and when do they obfuscate them? What is the meaning of the Serre duality without the cohomology groups? And so on.

The base field is always algebraically closed, unless specified otherwise. Characteristic of the base field is any, but sometimes we will assume that it is large enough (most often 5 is enough), or zero.

Projective varieties - 2

- ⋆ Vector bundles. Tangent bundle of a smooth variety. Dictionary (vector bundles) ↔ (locally free sheaves)
- ⋆ Differentials on a smooth variety. The Canonical class.

Algebraic curves - 2

- ⋆ Smooth plane cubics are not rational.
- ⋆ Intersection index of curves in a surface. Bezout theorem on a plane.
- ⋆ Geometric genus of a curve.
- ⋆ Picard group of a curve. Elliptic curve: detailed study of linear systems.
- ⋆ Riemann-Roch theorem: sheaf-theoretic and geometric (classical) proofs.
- ⋆ Canonical class. Riemann-Hurwitz formula. Residues on a curve and Serre duality: sketch of the [algebraic] proof.
- ⋆ Picard variety: existence (statement only). Morphism from a symmetric power of a curve to the Picard variety.
- ⋆ Canonical embedding of a curve.
- ⋆ Hyperelliptic curves.
- ⋆ Curves of genus 2 are hyperelliptic.
- ⋆ Curves of genus 3 and 4: canonical embedding, and classification.

⋆ Curves of genus 5, and trisecant lines.

⋆ Coarse moduli space of curves: definition.

Algebraic surfaces - 1

⋆ Adjunction formula.

⋆ Intersection index of properly intersecting curves.

⋆ Self-intersection of a curve on a surface.

⋆ First Chern class of a line bundle.

⋆ Néron-Severi group of a surface. Examples: $\mathbb{P}^2$, quadric $\mathbb{P}^1 \times \mathbb{P}^1$.

3.3.10 Grigory Olshanski

[1] Representations and Probability. Joint seminar. National Research University Higher School of Economics and Independent University of Moscow.

January–April 2016, 2 hours per week.

Program

Lectures at the research seminar on combinatorial, algebraic, and probabilistic aspects of representation theory. Main topics: The classical de Finetti theorem; its q-analogue (based on my joint work with Alexander Gnedin); the classical symmetrization map $S(\mathfrak{g}) \to U(\mathfrak{g})$; the special symmetrization map in the case of $\mathfrak{g} = \mathfrak{gl}(N, \mathbb{C})$.

[2] Lie groups, Lie algebras, and their representations –1. One-semester course, National Research University Higher School of Economics and Independent University of Moscow.

September-December 2016, 2 hours per week.

Program

Definition and basic properties of real Lie groups (a closed subgroup of a Lie group is a Lie subgroup; exponential map; a continuous homomorphism of Lie groups is a smooth map; a homomorphism of connected Lie groups is uniquely determined by its differential at the unit element, etc). Vector fields; definition and basic properties of real Lie algebras; elements of the Lie theory. Definition of universal enveloping algebra; the Poincaré-Birkhoff-Witt theorem. The center of $U(\mathfrak{gl}(N, \mathbb{C}))$. Descriptions of finite-dimensional $\mathfrak{sl}(2, \mathbb{C})$-modules. Haar measure on a Lie group. Classification of irreducible representations of $U(N)$. Irreducible characters of $U(N)$; Schur polynomials; elements of combinatorics of symmetric functions.

3.3.11 Alexei Penskoi

[1] Differential Geometry. Independent University of Moscow, II year students, February-May 2016, 4 hours per week (lecture 2 hours + exersise class 2 hours). Program

1. Curves and surfaces in the plane and the three-dimensional space. Curvature, torsion, Frenet frame. First and second fundamental forms. Principal curvatures, mean curvature and Gauß curvature. Mean curvature normal vector. Euler formula for the normal section curvature.

2. Surfaces in n-dimensional space. First and second fundamental forms. Connections in the tangent and normals bundles on a surface. Second fundamental form and Weingarten operator. Gauß-Weingarten derivational equations. Gauß-Bonnet theorem for surfaces.

3. Basic theory of Lie groups ans algebras.

4. Vector bundles and gluing cocycles. Structure group. Euclidean and hermitian bundles. Natural operations with bundles. Orientable bundles.

5. Connections in vector bundles. Connection local form, Christoffel symbols. Connections in euclidean and hermitian bundles. Connections compatible with metrics and their curvature.

6. Riemannian manifolds. Curvature, torsion. Levi-Civita connection. Symmetries of curvature tensor. Ricci tensor. Scalar curvature.

7. Riemannian manifolds II. Geodesics. Geodesic coordinates. Lagrangian approach to geodesics. Second variation.

8. Submanifolds of Riemannian manifolds. First and second fundamental forms.

9. Laplace-Beltrami operator and minimal submanifolds, Takahashi theorem.

10. Characteristic classes. Chern-Weil construction of characteristic classes. Chern, Pontryagin and Euler classes and their properties.

[2] Spectral Geometry. Independent University of Moscow, II year students, February-May 2016, 2 hours per week.

Program

1. Laplace-Beltrami operator on Riemannian manifolds

2. Eigenvalues of Laplace-Beltrami operator (Dirichlet problem, Neumann problem, problem on manifolds without boundary)

3. Variational desription of eigenvalues, Rayleigh quotient
4. Weyl function and its asymptotics
5. Inequalities for eigenvalues, Dirichlet-Neumann bracketing
6. Nodal domains, Courant nodal domain theorem
7. Isoperimetric inequalities, symmetrization
8. Cheeger isoperimetric constant, Cheeger inequality
9. Conformal volume, Yang-Yau inequality
10. Extremal metrics and minimal submanifolds of spheres

[3] Riemannian Geometry, A joint course of National Research University — Higher School of Economics and "Math in Moscow" program at the Independent University of Moscow for undergraduate students from the U.S. and Canada, Independent University of Moscow, II-IV year students, February-May 2016, 4 hours per week (lecture 2 hours + exersise class 2 hours).

Program

1. Riemannian manifolds
2. Riemannian curvature
3. Riemannian coverings
4. Riemannian geometry of surfaces
5. Isoperimetric inequalities
6. Comparison theorems

[4] Topology-I. "Math in Moscow" program at the Independent University of Moscow for undergraduate students from the U.S. and Canada, February-May 2016, 4 hours per week (lecture 2 hours + exersise class 2 hours).

Program

1. The language of topology. Continuity, homeomorphism, compactness for subsets of $\mathbb{R}^n$ (from the epsilon-delta language to the language of neighborhoods and coverings).
2. The objects of topology: topological and metric spaces, cell spaces, manifolds. Topological constructions (product, disjoint union, wedge, cone, suspension, quotient spaces, cell spaces, examples of fiber bundles).

3. Examples of surfaces (2-manifolds), orientability, Euler characteristic. Classification of surfaces (geometric proof for triangulated surfaces).

4. Homotopy and homotopy equivalence, fundamental groups and their elementary properties.

5. Fundamental group and covering spaces. Algebraic classification of covering spaces (via subgroups of the fundamental group of the base). Branched coverings, Riemann-Hurwitz theorem.

6. Knots and links in 3-space. Reidemeister moves. Polynomial invariants.

[5] Topology-I. "Math in Moscow" program at the Independent University of Moscow for undergraduate students from the U.S. and Canada, September-December 2016, 4 hours per week (lecture 2 hours + exersise class 2 hours).

Program is exactly as in the Spring Semester

[6] Exercise classes for various courses at Moscow State University: Classical Differential Geometry, February-May 2016, 2 hours per week; Analytic Geometry, September-December 2016, 4 hours per week;

[7] Exercise classes for Calculus-I, National Research University — Higher School of Economics, September-December 2016, 2 hours per week.

3.3.12 Petr Pushkar'

[1] Complex Analysis. Independent University of Moscow, 2 year students, January-May 2016, 4 hours per week.

Program

1. Complex-valued functions. Holomorphic functions. CauchyRiemann equations.
2. Holomorphic forms. Cauchy Theorem. Expansion as a convergent power series.
3. Meromorphic functions. Loran series.
4. Maximum principle, Cauchy's argument principle, open mappings.
5. Residues.
6. Isolated singlar points. The CasoratiWeierstrass theorem
7. Schvarz lemma. Automorphisms.
8. Uniformization theorem.
9. Holomorphic and harmonic functions.
10. Riemmanian surfaces. Elements of elliptic functions theory. Abel theorem.

[2] Morse Theory, Independent University of Moscow, 2 year students and higher, special course, September-December 2016, 2 hours per week.

Program.

1. Morse functions, Morse lemma
2. Morse inequalities

3 Arnold's problem on estimation of a number of critical points on a compact manifold with boundary.

4. Pairs of complexies, attempts of classifications.
5. Combinatorics.
6. Robin Forman Morse theory.
7. Cerf diagramms and application for contact topology.

3.3.13 Sergey Rybakov

[1] Introduction to p-adic cohomology. Independent University of Moscow, September-December 2016, 4 year students, 2 hours per week.

There are several (equivalent) definitions of the groups of p–adic cohomology of a smooth projective algebraic variety over a field of positive characteristic. For example, one can use crystalline theory or cohomology of the de Rham–Witt complex. If the variety has a lift to characteristic zero, then p–adic cohomology is the de Rham cohomology of the lift.

In this course we focus on applications of general theory to abelian varieties and K3 surfaces. It is assumed that students are familiar with basic algebraic geometry and algebraic de Rham cohomology of smooth algebraic varieteies over a field of characteristic zero.

Program:

1. Cartier operator.
2. Lifting algebraic varieties to the rings of Witt vectors. Hodge to de Rham degeneration.
3. Diedonne modules of finite group schemes. Oda's theorem on the group H^1_{dR} of an abelian variety.
4. Barsotti–Tate groups and Diedonne–Manin classification. Crystalline cohomology of abelian varieties.
5. Manin's theorem: computing the Zeta function of an abelian variety modulo p using Cartier operator.
6. The Nygaard and Ogus theorem on superspecial abelian varieties.
7. Formal groups and K3 surfaces.

3.3.14 George Shabat

[1] Algebra-2. Independent University of Moscow, I year students, September-December 2016, 2 hours per week.

Program

0. Fragments of theory of categories and fumctors

0.0. Initial-terminal objects

0.1. Categories $\mathcal{SET}^\bullet$,$\mathcal{TOP}^\bullet$; $\mathcal{MON}, \mathcal{GRP}, \mathcal{AB}$

0.2. Functor $\pi_1 : \mathcal{TOP}^\bullet \to\to \mathcal{GRP}$

0.3. On the functors $\pi_{>1} : \mathcal{TOP}^\bullet \to\to \mathcal{AB}$; Whitehead algebra

0.4. Monomorphisms and epimorphisms

0.5. Equalizers

0.6. Cartesian closed categories

0.7. $Z^{Y\times X} \cong \left(Z^Y\right)^X$, $Z^{Y+X} \cong Z^Y \times Z^X$, $(Z \times Y)^X \cong Z^X \times Y^X$

1. Abelian categories

1.0. Coincidence of sums and products

1.1. First examples: $G\_\mathcal{MOD}\ \ R - \mathcal{MOD}$

1.2. Kernels and cokernels

1.3. Categories of complexes

1.4. Exact sequences

1.5. Additive structure on the sets of morphisms

1.6. Injective and projective objects

1.7. Resolvents

1.8. Categories of presheaves on ringed spaces

1.9. Freyd-Mitchell embedding theorem (formulation)

2. Groups– 2

2.0. Listing small groups

2.1. Sylow subgroups of finite groups

2.2. Compositional series; Jordan-Holder theorem

2.3. Solvable and nilpotent groups

2.4. Nilpotence of p-groups

2.5. Solvability of (p, q)-groups (formulation of Burnside's theorem)

2.6. Abelian normal factors in the groups of order ≤ 23

2.7. Extension of groups with abelian kernels

2.8. Cohomology of groups $\mathrm{H}^2(G; M)$

3. Derived functors

3.0. Semi-exact functors

3.1. Definition of derived functors

3.2. The existence and uniqueness of derived functors

3.3. Derivatives of the functor invar :: $G_{\mathcal{MOD}} \longrightarrow\longrightarrow \mathcal{AB}$

3.4. Standard resolvent

3.5. Cohomology of groups $\mathrm{H}^1(G; M)$ and $\mathrm{H}^3(G; M)$
3.6. Group H^1 and crossed morphisms
3.7. Group H^1 and functional equations
3.8. Exterior actions $G \to \mathrm{Out}(N)$
3.9. Groups H^3 and obstacles to the extension of groups
3.10. Classification of general extensions of groups

4. Elements of finite group representation theory

4.0. Categories $G_{\mathbb{C}}\mathcal{MOD}$
4.1. Contragradient cofunctor $(V, \rho) \mapsto\mapsto (V^*, \rho^*)$
4.2. Operations $\oplus$, $\pitchfork$ and $\otimes$ in $G_{\mathbb{C}}\mathcal{MOD}$
4.3. Irreducible representations
4.4. Maschke theorem of complete reducibility
4.5. Characters and their behavior with respect to the operations
4.6. Orthogonality of characters
4.7. Regular representation and its decomposition
4.8. Burnside formula
4.9. Proof of Burnside's theorem on the solubility of (p, q)-groups
4.10. Finite groups of order ≤ 60

5. Rings − 2

5.0. On the dimension of rings
5.1. Cohomological dimension
5.2. Krull dimension
5.3. Nilradical and Jacobson radical
5.4. Artinian rings
5.5. Noether local rings
5.6. Extensions of local rings; ramification

[1] Algebra-3. Independent University of Moscow, II year students, September-December 2016, 2 hours per week.

Program

1. Adjoint functors

1.0. Definition
1.1. Simplest examples
1.2. The adjoint functors to forgetting ones
1.3. Cartesian closed categories-2
1.4. Galois correspondences

2. Commutative rings

2.0. Sets of all, maximal and prime ideals of a ring
2.1. Rings of continuous functions on compacts
2.2. Structures on the spectrum of a ring

2.3. Spectrum as a functor from category $\mathcal{ANN}$
2.4. Extensions of rings and morphisms of spectra
2.5. Regular local rings
2.6. Residue fields
2.7. Operations over rings and their spectra
2.8. Fibered products

3. Fields – 2
3.0. Transcendence degree of field extensions
3.1. Separable and purely inseparable extensions
3.2. Algebraic closure of a field
3.3. Field of algebraic numbers
3.4. Field of Puiseux series
3.5. Profinite groups
3.6. Groups of automorphisms of algebraic closures of fields
3.7. Galois theory of algebraic extensions of fields
3.8. Inverse Galois problem
3.9. Abelian extensions of $\mathbb{Q}$

4. Finitely-generated algebras over fields
4.0. Factor-rings of rings of polynomials over a field
4.1. Extensions of ground field
4.2. Affine algebraic manifolds
4.3. Systems of polynomial equations as functors of points
4.4. Primary decomposition of ideals in the rings of polynomials
4.5. Hilbert Nullstellensatz
4.6. Fields of rational functions on irreducible manifolds
4.7. Normalization
4.8. Finite extensions of functional fields

5. Fraction fields of Dedekind rings
5.0. Valuations
5.1. Dedekind rings and discrete valuation rings
5.2. Metric and topology defined by a valuation
5.3. Extensions of a ground field of transcendence degree 1
5.4*. Algebraic curves and their moduli spaces
5.5. Fields of formal Laurent series
5.6. Fields of p-adic numbers
5.7. Locally compact fields

6. General overview of the course of algebra

6.0. Language of algebra
6.1. Categorical approach to algebra
6.2. Axiomatic theories
6.3. Classification problems of algebra
6.4. Open problems

3.3.15 Arkady Skopenkov

A list of university courses taught by A. Skopenkov in 2016

[1] Discrete structures and algorithms in topology, III year students, September-December 2016, 4 hours per week. Moscow Institute of Physics and Technology (DIHT)

Program. It is shown how in the course of solution of interesting geometric problems (close to discrete mathematics and computer science) naturally appear main notions of algebraic topology (homology groups, obstructions and invariants). Thus main ideas of algebraic topology are presented with minimal technicalities.

Detailed information in Russian: `http://www.mccme.ru/circles/oim/home/combtop13.htm#combtop14`

[2] Topological theory of vector fields on manifolds, Independent University of Moscow, February-May 2014, 2 hours per week.

Program. It is shown how in the course of solution of interesting geometric problems (close to dynamical systems and physics) naturally appear main notions of algebraic topology (homology groups, obstructions and invariants). Thus main ideas of algebraic topology are presented with minimal technicalities.

Detailed information in Russian: `http://www.mccme.ru/circles/oim/home/combtop13.htm#vefi`

[3] Topological Tverberg conjecture: combinatorics, algebra and topology, Independent University of Moscow, September-December 2016, 2 hours per week.

Program. The well-known topological Tverberg conjecture was considered a central unsolved problem of topological combinatorics. A proof for a prime power r was given by I. Bárány, S. Shlosman, A. Szűcs, M. Özaydin and A. Volovikov in 1981-1996. A counterexample for other r was found in a series of papers by M. Özaydin, M. Gromov, P. Blagojević, F. Frick, G. Ziegler, I. Mabillard and U. Wagner, most of them recent. The arguments form a beautiful and fruitful interplay between combinatorics, algebra and topology. We present a simplified explanation of easier parts of the arguments, accessible to non-specialists in the area.

Detailed information in Russian: `http://www.mccme.ru/circles/oim/home/combtop13.htm#tver`

[4] Discrete analysis (exercises), II year students, February-December 2016, 2 hours per week. Moscow Institute of Physics and Technology (DIHT)

Program. We study certain topics in combinatorics and graph theory (including random graphs).

Detailed information in Russian: http://www.mccme.ru/circles/oim/home/discran1314.htm

Other educational activities by A. Skopenkov in 2016

[5] International Summer Conference of Tournament of Towns, Jury member, June-August, Pereslavl Detailed information:

http://www.turgor.ru/en/lktg/index.php

[6] Moscow Mathematical Conference of High-school Students, Programme Committee member, September-December, Moscow Detailed information in Russian: http://www.mccme.ru/mmks/index.htm

[7] Conference on Advanced Education, October, invited speaker, Kirov Detailed information in Russian: http://cdoosh.ru/conf/conf.html

[8] A course on 'special' mathematics for high-school students, high-school 'Intellectual', January-December. Detailed information in Russian:

http://www.mccme.ru/circles/oim/index.htm#il

[9] Math circle 'Olympiads and Mathematics' for high-school students, MCCME, January-December. Detailed information in Russian:

http://www.mccme.ru/circles/oim/index.htm#oim

[10] Minicources on mathematics for high-school students, Kirov region summer school, July, Kirov region.

[11] Minicources on mathematics for high-school students, Moscow 'olympic' schools, April, June and November, Moscow region.

[12] Chebyshev lab lecture for high-school students, December, invited lecturer, St Petersburg.

3.3.16 Mikhail Skopenkov

[1] Visual Geometry. Independent University Moscow, I-III year students, Spring 2016, 2 hours per week. The course is the one supported by the fellowship.

Program (short version).

1. Affine and Projective geometry.
2. Spherical geometry.
3. Lobachevsky geometry
4. Intro to topology: graphs on surfaces.

[1] Visual Geometry. Independent University Moscow and Higher School of Economics, I-III year students, September-December-2016, 2 hours per week.

Program (short version).
1. Intro to topology: graphs on surfaces.
2. Affine and Projective geometry.
3. Spherical geometry.
4. Lobachevsky geometry
5. Minkowsky geometry.

[3] Distant courses for mathematical olympiads winners (http://school.dist-math.ru/moodle), 2007–Spring 2016. Program in Russian available at the website http://school.dist-math.ru/moodle

In addition to [1]–[3]: assistance teaching in basic courses at Higher School of Economics (I-III year students, 5 hours per week throughout 2016), supervision of 6 students, consulting a pupil for Moscow Mathematical Conference for School Pupils, coordination of online problem solution submission system for IUM, writing a scenario for a short popular-science movie.

3.3.17 Evgeni Smirnov

[1] Algebra, 1st year, 2nd semester, Higher School of Economics, January–June 2016, 3 hours of lectures and 3 hours of exercise sessions per week
Course outline:
1. Finite Abelian groups
2. Eigenvectors and eigenvalies
3. Jordan and Frobenius normal forms
4. Classification of finitely generated modules over Euclidean rings
5. Bilinear and quadratic forms over R and C. Symmetric/Hermitian, orthogonal/unitary operators
6. Tensors. Tensor algebra. Symmetric and exterior algebras.

[2] Algebra, 2nd year, 1st semester, Higher School of Economics, September–December 2016, 2 hours of lectures and 2 hours of exercise sessions per week
Course outline:
1. Groups. Derived subgroup. Solvable and nilpotent groups.
2. Sylow theorems. Semisimple products. Finite groups of small order.
3. Introduction to representation theory of finite groups: Maschke's theorem, Schur's lemma
4. Characters of representations of finite groups. Orthogonality relations. Burnside's formula.
5. Group algebra, its semisimplicity, the center of a group algebra.
6. Representations of compact groups. Complete reducibility.
7. Representations of SU(2).

[3] Reflection groups and Coxeter groups, Independent University of Moscow, February–April 2016, 2 hours of lectures per week

Course outline:

1. Finite Reflection Groups
2. Root systems
3. Generators and Relations
4. Classification of finite reflection groups
5. Crystallographic reflection groups
6. Classification of regular polytopes
7. Polynomial invariants
8. Chevalley–Shepard–Todd theorem

[4] Algebra, 1st year, 1st semester, Independent University of Moscow, September–December 2016, 2 hours of lectures and 2 hours of exercises per week

Course outline:

1. Rings

1.1. Rings, fields, ring homomorphisms. Modular arithmetic. Fermat's little theorem. Chinese remainder theorem for remainder rings.

1.2. Ideals, quotient rings, the homomorphism theorem. Eucledian domains, principal ideal domains. GCD and LCD, associated elements. Chinese remainder theorem for principal ideal domains.

1.3. Unique factorization domains. Prime and irreducible elements. Integral principal ideal domains are unique factorization domains. Polynomials. Gauss's Lemma.

1.4 The polynomial ring over a UFD is a UFD. Main theorem on symmetric polynomials.

2. Vector spaces

2.1. Vector spaces. Examples. The n-dimensional arithmetic vector space. Linear dependence and independence. Basis, dimension. Dual space.

2.2. Linear maps, isomorphism. Basis as an isomorphism with the arithmetic vector space. Dual space. The canonical isomorphism between a vector space and its double dual. Annihilator of a vector subspace. Direct sums of vector spaces. Quotient space.

2.3. Matrix of a linear map. Product of matrices. Linear operators. Grassmann (skew) polynomials. Determinant. $\det AB = \det A \det B$.

2.4. The symmetric group. The sign of a permutation. Exterior powers of an operator. Minors as matrix entries of an exterior power. Computation of the determinant: an explicit formula, row/column decomposition.

2.5. Skew-symmetric multilinear forms. Determinant as a skew-symmetric multilinear form. Eigenvectors, diagonalizable operators, Cayley–Hamilton Theorem.

3. Modules

3.1. Modules over rings. Examples. Vector space with an operator is a $k[t]$-module. Homomorphisms. System of generators, basis. Free modules, rank. Submodule of a free module is free.

3.2. Description of finitely generated modules over principal ideal domains. Applications: classification of finitely generated abelian groups, Jordan normal form of an operator. Jordan normal form theorem.

3.3.18 Mikhail Verbitsky

1. "Metric geometry", IUM, 2-3 year students, February-May 2016. Program:
   (a) Metric spaces, path metrics, geodesics, Hopf-Rinow theorem.
   (b) CAT-inequalities, CAT(0)-spaces, Cartan-Hadamard theorem.
   (c) Gromov hyperbolic groups, quasiisometries of metric spaces, main examples of hyperbolic and non-hyperbolic groups.
   (d) Isoperimetric inequality and solvability of word problem in hyperbolic groups.
2. "Kazhdan property T", summer school "Algebra and geometry" Yaroslavl, July 2016. Program:
   (a) Fell topology.
   (b) Expander graphs.
   (c) Gelfand-Naimark-Segal construction.

3.3.19 Ilya Vyugin

[1] Calculuas I (lectures and seminars). Independent University of Moscow, I year students, September-December 2016, 2+2 hours per week.

Program

1. Introduction to the Set Theory.
2. Sequences and metric spaces.
3. Limit of function.
4. Derivatives and Taylor's formula.
5. Riemann Integral
6. Functions of several variables.

[2] Variation Calculus and Optimal Control (lectures) Course in HSE.

Program

1. Necessary condition for an extremum. Euler–Lagrange equation.
2. Lagrange multiplier method
3. Sufficient condition for an extremum of linear functional. Theory of the second variation.

4. Fundamentals of the theory of optimal control. Pontryagin's maximum principle.
5. Elements of convex analysis.
6. Applications.

[3] Complex Differential Equations (joint with V. Poberezhny) Special cours in HSE.
Program
1. Fuchs conditions.
2. Riccati equation.
3. Linear differential equations: monodromy, singular points. Levelt decomposition.
4. Hypergeometric equation and hypergeometric functions.
5. Riemann–Hilbert problem and isomonodromic deformations.
6. Symplectic and Poisson structures of isomonodromic deformations.