The IUM report to the Simons foundation, 2017

Contents

1 Introduction: list of awardees

The Simons foundation supported two programs launched by the IUM:
Simons stipends for students and graduate students;
Simons IUM fellowships.

10 applications were received for the Simons stipends contest. The selection committee consisting of *Yu.Ilyashenko (Chair), G.Dobrushina, G.Kabatyanski, S.Lando, I.Paramonova (Academic Secretary), A.Sossinsky, M.Tsfasman* awarded Simons stipends for 2017 year to the following students and graduate students:

1. Bogachev, Nikolay Vladimirovich
2. Gabdurakhmanov, Ravil Marselevich
3. Ilyin, Alexei Igorevich
4. Ivanov, Alexei Nikolaevich
5. Kalmynin, Alexander Borisovich
6. Kalugin, Alexei Alexeevich
7. Konovalov, Nikolai Sergeevich
8. Loginov, Konstantin Valerevich
9. Solomadin, Grigory Dmitrievich

14 applications were received for the Simons IUM fellowships contest for the first half year of 2017 and 15 applications were received for the second half year. The selection committee consisting of *Yu.Ilyashenko (Chair), G.Dobrushina, B.Feigin, I.Paramonova (Academic Secretary), A.Sossinsky, M.Tsfasman, V.Vassiliev* awarded

Simons IUM-fellowships for the first half year of 2017 to the following researches:

1. Aizenberg, Anton Andreevich
2. Gorodentsev, Alexei Lvovich
3. Kazarian, Maxim Eduardovich
4. Kuznetsov, Alexander Gennadevich
5. Leyenson, Maxim Ilyich

6. Olshansky, Grigory Iosifovich
7. Penskoi, Alexei Victorovich
8. Pushkar, Petr Evgenevich
9. Shabat, George Borisovich
10. Skopenkov, Mikhail Borisovich
11. Smirnov, Evgeni Yurevich
12. Vyugin, Ilya Vladimirovich

Simons IUM-fellowships for the second half year of 2017 to the following researches:

1. Burman, Yurii Mikhailovich
2. Elagin, Alexei Dmitrievich
3. Lashkevich, Mikhail Yurevich
4. Panov, Taras Evgenevich
5. Penskoi, Alexei Victorovich
6. Pushkar, Petr Evgenevich
7. Rybnikov, Leonid Grigorevich
8. Shaposhnikov, Stanislav Valerevich
9. Skopenkov, Arkady Borisovich
10. Skopenkov, Mikhail Borisovich
11. Smirnov, Evgeni Yurevich
12. Talalaev, Dmitry Valerevich
13. Vyugin, Ilya Vladimirovich

The report below is split in two sections corresponding to the two programs above. The first subsection in each section is a report on the research activities. It consists of the titles of the papers published or submitted in the year of 2017, together with the corresponding abstracts. The second subsection of each section is devoted to conference and some most important seminar talks. The last subsection of the second section is devoted to the syllabi of the courses given by the winners of the Simons IUM fellowships. Most of these courses are innovative, as required by the rules of the contest for the Simons IUM fellowships.

The Independent University remains one of the most active centers of Moscow Mathematical life. There is no room here to list its main activities. We only mention that the two invited speakers of the ICM-2018 from Moscow, M. Finkelberg and A. Belavin, are permanent Professors of the IUM.

The support of the Simons foundation have drastically improved the financial situation at the IUM, and the whole atmosphere as well. On behalf of the IUM, I send my deep gratitude and the best New year wishes to Jim Simons, Yuri Tschinkel, and the whole team of the Simons foundation.

Yulij Ilyashenko

President of the Independent University of Moscow

2 Program: Simons stipends for students and graduate students

2.1 Research

2.1.1 Nikolay Bogachev

[1] Classification of (1.2)-reflective anisotropic hyperbolic lattices of rank 4
submitted to Izvestia Mathematics.

A hyperbolic lattice is called (1.2)-*reflective* if the subgroup of its automorphism group generated by all 1- and 2-reflections is of finite index. The main result of this article is a complete classification of (1.2)-reflective anisotropic lattices of rank 4.

[2] With A. Perepechko
Vinberg's Algorithm for Hyperbolic Lattices
submitted to Mathematical Notes.

A hyperbolic lattice (that is a lattice of signature $(n, 1)$) is said to be reflective if the subgroup of its automorphism group generated by all reflections is of finite index. It is well known that reflective hyperbolic lattices exist only for $n < 22$ and there are only finitely many of them. But the classification problem is now completely solved only for $n = 2$ and $n = 4$.

In 1972 Vinberg proposed an algorithm that, given a lattice, enables one to find recursively all faces of the fundamental polyhedron of the corresponding reflection subgroup, determine if there are only finitely many of them and hence, to verify this lattice on reflectivity.

We present in this paper the software implementation of this algorithm, first implemented for hyperbolic lattices of arbitrary form, as well as new reflective hyperbolic lattices, which were found by this program. The program was tested by the large number of hyperbolic lattices.

2.1.2 Ravil Gabdurakhmanov

[1] Spaces of harmonic maps of the projective plane to the four-dimensional sphere
arXiv:1610.05277v2 *submitted to Geometriae Dedicata*

The spaces of harmonic maps of the projective plane to the four-dimensional sphere are investigated in this paper by means of twistor lifts. It is shown that such spaces are empty in case of even harmonic degree. In case of harmonic degree less than 6 it was shown that such spaces are path-connected and an explicit parameterization of the canonical representatives was found. In addition, the last section summarizes known results for harmonic maps of the two-dimensional sphere to the four-dimensional sphere of harmonic degree less than 6.

2.1.3 Alexei Ilyin

[1] Degeneration of Bethe subalgebras in the Yangian of $\mathfrak{gl}_n$,
Ilin, A., Rybnikov L., Letters in Mathematical Physics (2017).

We study degenerations of Bethe subalgebras $B(C)$ in the Yangian $Y(\mathfrak{gl}_n)$, where C is a regular diagonal matrix. We show that closure of the parameter space of the family of Bethe subalgebras, which parametrizes all possible degenerations, is the Deligne-Mumford moduli space of stable rational curves $\overline{M_{0,n+2}}$. All subalgebras corresponding to the points of $\overline{M_{0,n+2}}$ are free and maximal commutative. We describe explicitly the "simplest" degenerations and show that every degeneration is the composition of the simplest ones. The Deligne-Mumford space $\overline{M_{0,n+2}}$ generalizes to other root systems as some De Concini-Procesi resolution of some toric variety. We state a conjecture generalizing our results to Bethe subalgebras in the Yangian of arbitrary simple Lie algebra in terms of this De Concini-Procesi resolution.

2.1.4 Alexei Ivanov

[1] With A. S. Tikhomirov
The moduli component of the space of semistable rank-2 sheaves on $\mathbb{P}^3$ with singularities of mixed dimension

Doklady Mathematics, 2017, Vol. 96, Issue 2, pp. 506-509.

A new irreducible component of the Gieseker-Maruyama moduli scheme $\mathcal{M}(3)$ of semistable coherent sheaves of rank 2 with Chern classes $c_1 = 0, c_2 = 3$, and $c_3 = 0$ on $\mathbb{P}^3$ such that its general point corresponds to a sheaf whose singular locus contains components of dimensions 0 and 1 is described. These sheaves are obtained by elementary transformations of stable reflexive sheaves of rank 2 with Chern classes $c_1 = 0, c_2 = 2$, and $c_3 = 2$ along the projective line. The constructed family of sheaves is the first example of an irreducible component of a Gieseker-Maruyama scheme whose general point corresponds to a sheaf with singularities of mixed dimension.

[2] With A. S. Tikhomirov
Semistable rank 2 sheaves with singularities of mixed dimension on $\mathbb{P}^3$
arXiv:1703:04851, *submitted to Journal of Geometry and Physics.*

We describe new irreducible components of the Gieseker-Maruyama moduli scheme $\mathcal{M}(3)$ of semistable rank 2 coherent sheaves with Chern classes $c_1 = 0, c_2 = 3, c_3 = 0$ on $\mathbb{P}^3$, general points of which correspond to sheaves whose singular loci contain components of dimensions both 0 and 1. These sheaves are produced by elementary transformations of stable reflexive rank 2 sheaves with $c_1 = 0, c_2 = 2$ along a disjoint union of a projective line and a collection of points in $\mathbb{P}^3$. The constructed families of sheaves provide first examples of irreducible components of the Gieseker-Maruyama moduli scheme such that their general sheaves have singularities of mixed dimension.

[3] With V. A. Golubeva
Some applications of Griffiths theorem in theory of Feynman integrals
arXiv:1705.04811 *submitted to Journal of Mathematical Physics*

The present paper provides a method for finding partial differential equations satisfied by the Feynman integrals for diagrams of various types, using the Griffiths theorem on the reduction of poles of rational differential forms. As an application, an algorithm for computing partial differential equations satisfied by Feynman integrals for diagrams of a ladder type is described.

2.1.5 Alexander Kalmynin

[1] Conference to the Memory of Anatoly Alekseevitch Karatsuba on Number Theory and Applications

Number Theory and Applications, 2, Dedicated to the 80th anniversary of professor Anatolii Alekseevich Karatsuba, Sovrem. Probl. Mat., 24, Steklov Math. Institute of RAS, Moscow, 2017, 745;

This paper contains a survey of the contributions of the Conference to the Memory of Anatoly Alekseevitch Karatsuba on Number Theory and Applications, which was held on January 28-30, 2016, at Steklov Mathematical Institute, Russian Academy of Sciences (on January 28 and 29) and at Mechanics and Mathematics Faculty, Lomonosov Moscow State University (on January 30) in Moscow.

[2] On Novák numbers
arXiv:1611.00417, *to appear in Sbornik: Mathematics.*

In this work, we obtain some new lower bounds for the number $\mathcal{N}_B(x)$ of Novák numbers less than or equal to x. We also prove, conditionally on Generalized Riemann Hypothesis, the upper estimates for the number of primes dividing at least one Novák number and give description for the prime factors of Novák numbers N, such that $2N$ is a Novák-Carmichael number.

[3] Omega-theorems for the Riemann zeta function and its derivatives near the line $\operatorname{Re} s = 1$
arXiv:1706.07364 *submitted to Acta Arithmetica*

We introduce a generalization of the method of S. P. Zaitsev. This generalization allows us to prove omega-theorems for the Riemann zeta function and its derivatives in some regions near the line $\operatorname{Re} s = 1$.

[4] Novák-Carmichael numbers and shifted primes without large prime factors
arXiv:1706.07343

We prove some new lower bounds for the counting function $\mathcal{N}_{\mathcal{C}}(x)$ of the set of Novák-Carmichael numbers. Our estimates depend on the bounds for the number of shifted primes without large prime factors. In particular, we prove that $\mathcal{N}_{\mathcal{C}}(x) \gg x^{0.7039-o(1)}$ unconditionally and that $\mathcal{N}_{\mathcal{C}}(x) \gg xe^{-(7+o(1))(\log x)\frac{\log\log\log x}{\log\log x}}$, under some reasonable hypothesis.
[5] Intervals between numbers that are sums of two squares
arXiv:1706.07380

In this paper, we improve the moment estimates for the gaps between numbers that can be represented as a sum of two squares of integers. We consider certain sum of Bessel functions and prove the upper bound for its weighted mean value. This bound provides estimates for the γ-th moments of gaps for all $\gamma \leq 2$.

2.1.6 Alexei Kalugin

[1] On quantization of Lie bialgebras via the Ran space
preprint.

In this paper we develop new approach to Etingof-Kazhdan quantization using the geometry of the Ran space.

[2] Tannakian category of perverse sheaves
preprint.

In this paper we study the category of unipotent perverse sheaves smooth along the stratification defined by hyperplane arrangements.

2.1.7 Nikolai Konovalov

[1] Division Theorems for spaces of sections of equivariant vector bundles
arXiv:1712.02578, *preprint.*

Let G be a semisimple complex Lie group and let X be a smooth complex projective G-variety. Let L be a G-equivariant line bundle over X. Denote by $\Gamma(X, L)$ the vector space of global holomorphic sections of the bundle L. Let $\mathrm{Sing} \subset \Gamma(X, L)$ be the set of all sections whose zero locus is a singular subvariety of X. We study the topology of the complement $\Gamma_{\mathrm{reg}}(X, L) := \Gamma(X, L) \setminus \mathrm{Sing}$.

One can define the natural subring $\mathrm{Lk}(X, L) \subset H^*(\Gamma_{\mathrm{reg}}(X, L), \mathbb{Z})$ only in terms of X. The formula for the orbit map $O^* \colon \mathrm{Lk}(X, L) \to H^*(G, \mathbb{Z})$ is obtained. As a consequence we can generalize previous results on degeneration of the Leray spectral sequence in rational cohomology for the quotient map $\Gamma_{\mathrm{reg}}(X, L) \to \Gamma_{\mathrm{reg}}(X, L)/G$.

2.1.8 Konstantin Loginov

[1] Standard models of degree 1 del Pezzo fibrations. arXiv:1710.02482

We construct a standard birational model (a model that has Gorenstein canonical singularities) for the three-dimensional del Pezzo fibrations $\pi : X \longrightarrow C$ of degree 1 and relative Picard number 1. We also embed the standard model into the relative weighted projective space $\mathbb{P}(1, 1, 2, 3)$. Our construction works in the G-equivariant category where G is a finite group.

2.1.9 Grigory Solomadin

[1] Quasitoric totally normally split representatives in the complex cobordism ring
arXiv:1704.07403, *Sent to Mathematical Notes for reviewing.*

A smooth stably complex manifold is called totally tangentially/normally split if its stable tangential/normal resp. bundle is isomorphic to a sum of complex linear bundles.

We prove that every class of degree higher 2 of the complex cobordism ring contains a quasitoric totally tangentially and normally split manifold.

2.2 Scientific conferences and seminar talks

2.2.1 Nikolay Bogachev

[1] Conference “Gomtrie et Topologie” in the honour of C. Bavard, Bordeaux, France, November, 20 – November, 24

Talk “Reflective hyperbolic lattices and Vinberg’s Algorithm.”

[2] Visit to Prof. Dr. A. Kolpakov, University of Neuchatel, Switzerland, November, 25 – 29.

Talk “Vinberg’s Algorithm for Hyperbolic Lattices”

[3] S.P. Novikov’s Seminar “Geometry, Topology and Mathematical Physics”. Moscow State University, Moscow, Russia, March, 15.

Talk “Arithmetic reflection groups in Lobachevsky spaces”

[4] E.B. Vinberg’s Seminar “Lie groups and invariant theory”, Moscow State University, Moscow, Russia, October, 15.

Talk “Software implementation of Vinberg’s Algorithm for hyperbolic lattices”

[5] Seminar “Geometry and Topology”. Moscow State University, Moscow, Russia, November, 14

Talk “Discrete groups and reflection groups”

2.2.2 Ravil Gabdurakhmanov

[1] The first Summer School on Fontanka: Geometry 2017, PDMI, Fontanka 27, Saint Petersbourg, Russia, July 3-8

[2] The 7th Summer school ”Algebra and geometry”, Yaroslavl, Russia, July 25-31

[3] Research visit to the University of Leeds, Leeds, United Kingdom, November 16-29

2.2.3 Alexei Ilyin

[1] Conference “Algebraic Analysis and Representation Theory”, Kyoto, Japan, June, 26 – June, 30

Poster “Degeneration of Bethe subalgebras in the Yangian“

[2] Visit to Boston, February — May, Talk “Graded algebras“

2.2.4 Alexei Ivanov

[1] Conference "Differential equations and related questions of mathematics", Kolomna, June, 15 – June, 16

Talk "Some applications of Griffiths theorem in theory of Feynman integrals"

2.2.5 Alexander Kalmynin

[1] A.A.Karatsuba's 80th Birthday Conference in Number Theory and Applications , May 22-27, 2017, Moscow

Talk "Omega-theorems for the Riemann zeta function and its derivatives near the line $\mathrm{Re}\, s = 1$."

[2] Workshop: Motives, Periods and L-functions, April 10-12, 2017, Moscow

Talk "Intervals between numbers that are sums of two squares and Jacobi-type forms"

[3] Seminar "Automorphic forms and their applications"', March 28, 2017, Moscow
With V. P Spiridonov

Talk "Elliptic Hypergeometric Functions in Combinatorics, Integrable Systems and Physics. (review of a conference in ESI (Vienna))"

[4] Seminar "Contemporary Problems in Number Theory", December 14, 2017, Moscow

Talk "Large values of character sums"

[5] Seminar "Functional analysis and noncommutative geometry", February 17, 2017, Moscow

Talk "Riemann hypothesis and functional analysis"

[6] Seminar "Functional analysis and noncommutative geometry", October 9, 2017, Moscow, and seminar "Algebras in analysis", October 27, 2017, Moscow.

Talk "Lindenstrauss-Tzafriri theorem and local theory of Banach spaces"

[7] Seminar "Geometric stuctures on manifolds", March 30, 2017, Moscow

Talk "On Erdös-Ulam conjecture"

[8] Seminar "Geometric stuctures on manifolds", June 1, 2017, Moscow

Talk “The multiplication table problem and its generalizations”

[9] Seminar “Contemporary Problems in Number Theory”, April 6, 2017, Moscow

Talk “Novák-Carmichael numbers and shifted primes without large prime factors”

2.2.6 Alexei Kalugin

[1] Visit to Luxembourg, December
Talk “Quantization of Lie bialgebras” at “Quantization and geometry seminar” (University of Luxembourg)
[2] Workshop ”Supergeometry and Applications”, Luxembourg, December 14-15

2.2.7 Alexei Konovalov

[1] Seminar “Geometric structures on manifolds,” Higher School of Economics, Moscow
Talk “Arf-Kervaire invariant”
[2] Seminar “Variety of Varieties,” Higher School of Economics, Moscow
Talk “Introduction to Chern-Simons Theory”.

2.2.8 Konstantin Loginov

[1] Workshop on birational geometry. November 2017, Moscow.
[2] VII Conference on algebraic geometry and complex analysis. August 2017, Koryazhma, Arkhangelsk region.
[3] Inaugural conference for the Laboratory of Mirror Symmetry and Automorphic forms. July 2017, Saint Petersburg.
[4] “Standard model of degree 1 del Pezzo fibrations” at Iskovskikh Seminar (Steklov Mathematical Institute)
[5] “Polynomials with integral coefficients equivalent to a given polynomial” at Iskovskikh Seminar (Steklov Mathematical Institute)
[6] “Sarkisov program in arbitrary dimension” at Iskovskikh Seminar (Steklov Mathematical Institute)
[7] “Birational involutions of the projective plane” at “Geometric structures on varieties” seminar (Higher School of Economics)

2.2.9 Grigory Solomadin

[1] Seminar “Geometry and Topology”, Moscow, MSU, Department of Mathematics and Mechanics, Chair of Higher Geometry and Topology, April.

Talk “Totally normally splitting quasitoric representatives in the unitary cobordism ring”.

3 Program: Simons IUM fellowships

3.1 Research

3.1.1 Anton Aizenberg

[1] Ayzenberg A., Masuda M., Park S., Zeng H. Cohomology of toric origami manifolds with acyclic proper faces, Journal of Symplectic Geometry. 2017. Vol. 15. No. 3. P. 645–685.

A toric origami manifold is a generalization of a symplectic toric manifold (or a toric symplectic manifold). The origami symplectic form is allowed to degenerate in a good controllable way in contrast to the usual symplectic form. It is widely known that symplectic toric manifolds are encoded by Delzant polytopes, and the cohomology and equivariant cohomology rings of a symplectic toric manifold can be described in terms of the corresponding polytope. Recently, Holm and Pires described the cohomology of a toric origami manifold M in terms of the orbit space M/T when M is orientable and the orbit space M/T is contractible. But in general the orbit space of a toric origami manifold need not be contractible. In this paper we study the topology of orientable toric origami manifolds for the wider class of examples: we require that every proper face of the orbit space is acyclic, while the orbit space itself may be arbitrary. Furthermore, we give a general description of the equivariant cohomology ring of torus manifolds with locally standard torus actions in the case when proper faces of the orbit space are acyclic and the free part of the action is a trivial torus bundle.

[2] Locally standard torus actions and sheaves over Buchsbaum posets, Sbornik: Mathematics, Vol. 208 (2017). The version in Russian is published, the English version is in press.

Manifolds with locally standard half-dimensional torus actions represent a large and important class of spaces. Cohomology rings of such manifolds are known in particular cases but in general even Betti numbers are difficult to compute. Our approach to this problem is the following: we consider the orbit type filtration on a manifold with locally standard action and study the induced spectral sequence in homology. It collapses at a second page only in the case when the orbit space is homologically trivial. The cohomology ring in this case was already computed. Nevertheless, we can completely describe the spectral sequence under more general assumptions, namely when all proper faces of the orbit space are acyclic. The theory of sheaves and cosheaves on finite partially ordered sets is used in the computation. The second page of the spectral sequence can be described as the cohomology of a certain sheaf on the dual simplicial poset, whose value on a simplex is the homology of the corresponding toric orbit. We study this and related sheaves and establish the generalizations of the Poincare duality and the Zeeman-McCrory spectral sequence for sheaves of ideals of exterior algebras.

3.1.2 Yurii Burman

[1] With B.Shapiro
On Hurwitz–Severi numbers
to appear in Annali della Scuola Normale Superiore di Pisa, Classe di Scienze, 2017

In this paper we define, for a point $p \in \mathbb{C}P^2$ and a triple (g, d, ℓ) of non-negative integers, a *Hurwitz–Severi number* $\mathfrak{H}_{g,d,\ell}$ as the number of generic irreducible plane curves of genus g and degree $d + \ell$ having an ℓ-fold node at p and at most ordinary nodes as singularities at the other points, such that the projection of the curve from p has a prescribed set of local and remote tangents and lines passing through nodes. In the cases $d + \ell \geq g + 2$ and $d + 2\ell \geq g + 2 > d + \ell$ we express the above Hurwitz–Severi numbers via appropriate ordinary Hurwitz numbers. The remaining case $d + 2\ell < g + 2$ is still widely open.

[2] Higher matrix-tree theorems and Bernardi polynomial
submitted to Journal of Algebraic Combinatorics

The classical matrix-tree theorem discovered by G. Kirchhoff in 1847 relates the principal minor of the $n \times n$ Laplace matrix to a particular sum of monomials of matrix elements indexed by directed trees with n vertices. We prove, for any $k \geq n$, a three-parameter family of identities between degree k polynomials of matrix elements of the Laplace matrix. For $k = n$ and some special values of the parameters the identity turns to be the matrix-tree theorem. The same values of parameters for arbitrary k give analogs of the theorem involving specific polynomials of the matrix elements called higher determinants. We study properties of the higher determinants; in particular, they have an application (due to M. Polyak) in the topology of 3-manifolds.

[3] Abstract matrix-tree theorem and Bernardi polynomial
arXiv:1703.04120

This is an early version of the text “Higher matrix-tree theorems and Bernardi polynomial” submitted to the Journal of Algebraic Combinatorics, see above.

3.1.3 Alexei Elagin

[1] (with J.Xu and S.Zhang) "On exceptional collections of line bundles on weak del Pezzo surfaces", https://arxiv.org/abs/1710.03972 *submitted to International Mathematics Research Notices*

We study full exceptional collections of line bundles on surfaces. We prove that any full strong exceptional collection of line bundles on a weak del Pezzo surface of degree ≥ 3 is an augmentation in the sense of L. Hille and M. Perling, while for some weak del Pezzo surfaces of degree 2 the above is not true. We classify smooth projective surfaces possessing a cyclic strong exceptional collection of line bundles of maximal length: we prove that they are weak del Pezzo surfaces and find all types of weak del Pezzo surfaces admitting such a collection. We find simple criteria of exceptionality/strong exceptionality for collections of line bundles on weak del Pezzo surfaces.

[2] (with V.Lunts) "Regular subcategories in bounded derived categories of affine schemes", https://arxiv.org/abs/1711.01492

Let R be a commutative Noetherian ring such that $X = \mathrm{Spec} R$ is connected. We prove that the category $D^b(cohX)$ contains no proper full triangulated subcategories which are regular. We also bound from below the dimension of a regular category T, if there exists a triangulated functor $T \to D^b(cohX)$ with certain properties. Applications are given to cohomological annihilator of R and to point-like objects in T.

3.1.4 Alexei Gorodentsev

[1] Gorodentsev, Alexey L. Algebra II. Textbook for Students of Mathematics. XV+370 pages with 153 b/w illustrations and 2 illustrations in colour. Springer International Publishing, 2017. eBook ISBN 978-3-319-50853-5, Hardcover ISBN 978-3-319-50852-8, DOI 10.1007/978-3-319-50853-5.
See `http://www.springer.com/gp/book/9783319508528`.

This book is the second part of an intensive "Russian-style" two-year undergraduate course in abstract algebra. It covers multilinear and tensor algebra; symmetric functions and calculus of Young tableaux; introduction to representation theory: semisimple modules, intertwining operators, double commutator theorem; representations of finite groups, the symmetric group S_n, and the Lie algebra $\mathfrak{sl}_2$; introduction to the theory of categories: Yoneda lemma, representable functors, adjoint functors, limits of diagrams; introduction to commutative algebra: integral ring extensions, transcendence generators, structure of a finitely generated algebra over a field, Hibert's Basissatz and Nullstellensatz; introduction to algebraic geometry; the Galois theory; and other topics usually overlooked in standard undergraduate courses. This textbook is based on courses the author has conducted at the Independent University of Moscow and at the Faculty of Mathematics in the Higher

School of Economics. The main content is complemented by a wealth of exercises for class discussion, some of which include comments and hints, as well as problems for independent study.

3.1.5 Maxim Kazarian

[1] Kazaryan M., Zvonkine D., Lando S. Universal Cohomological Expressions for Singularities in Families of Genus 0 Stable Maps // International Mathematical Research Notices. 2017

In this paper we consider families of curve-to-curve maps that have no singularities except those of genus stable maps and that satisfy a versality condition at each singularity. We provide a universal expression for the cohomology class Poincaré dual to the locus of any given singularity. These expressions hold for any family of curve-to-curve maps satisfying the above properties.

[2] M.Kazarian, P.Zograf, Rationality of genus enumeration of maps and hypermaps (in Russian), Algebra i analiz. 2017. V. 29. n. 3. p. 23-33.

It is shown that the generating functions for a fixed genus map and hypermap enumeration become rational after a simple explicit change of variables. Their numerators are polynomials with integer coefficients that obey a differential recursion, and denominators are products of powers of explicit linear functions.

3.1.6 Alexander Kuznetsov

[1] With A. Perry
Derived categories of cyclic covers and their branch divisors
Selecta Math, V. 23 (2017), N. 1, pp. 389–423

Given a variety Y with a rectangular Lefschetz decomposition of its derived category, we consider a degree n cyclic cover $X \to Y$ ramified over a divisor $Z \subset Y$. We construct semiorthogonal decompositions of $\mathbf{D}^b(X)$ and $\mathbf{D}^b(Z)$ with distinguished components $\mathcal{A}_X$ and $\mathcal{A}_Z$, and prove the equivariant category of $\mathcal{A}_X$ (with respect to an action of the n-th roots of unity) admits a semiorthogonal decomposition into $n-1$ copies of $\mathcal{A}_Z$. As examples we consider quartic double solids, Gushel–Mukai varieties, and cyclic cubic hypersurfaces.

[2] Exceptional collections in surface-like categories
Sbornik: Mathematics, V. 208 (2017), N. 9, pp. 1368–1398.

We provide a categorical framework for recent results of Markus Perling on combinatorics of exceptional collections on numerically rational surfaces. Using it we simplify and generalize some of Perling's results as well as Vial's criterion for existence of a numerical exceptional collection.

[3] Derived categories of families of sextic del Pezzo surfaces
preprint math.AG/1708.00522

We construct a natural semiorthogonal decomposition for the derived category of an arbitrary flat family of sextic del Pezzo surfaces with at worst du Val singularities. This decomposition has three components equivalent to twisted derived categories of finite flat schemes of degrees 1, 3, and 2 over the base of the family. We provide a modular interpretation for these schemes and compute them explicitly in a number of standard families. In the Appendix we prove a symmetric version of homological projective duality for $\mathbb{P}^2 \times \mathbb{P}^2$, $\mathrm{Fl}(1, 2; 3)$, and $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$.

[4] With Yu. Prokhorov
Prime Fano threefolds of genus 12 with a $\mathbb{G}_m$-action
preprint math.AG/1711.08504

We give an explicit construction of prime Fano threefolds of genus 12 with a $\mathbb{G}_m$-action, describe their isomorphism classes and automorphism groups.

3.1.7 Mikhail Lashkevich

No papers have been published during this period.

Now I am preparing two papers:

[1] "Braiding and fusing in $GL(1|1)$ WZW model" with A. Babichenko.

A free field representation for the supersymmetric $GL(1|1)$ WZW model used to find the structure constants for typical representations and braiding and fusing matrices. The interest to the model is related to two facts: first, the modules are parameterized by continuous parameters, and, second, as a consequence, it possesses atypical representations at special points.

[2] "Conserved current and irrelevant permutations in complex sinh-Gordon model: form factor approach" (preliminary title) with Ya. Pugai.

We have found a form factor representation for the infinite set of conserved currents in the so called complex sinh-Gordon model. As well form factors for some of their products have been found, which made possible to confirm the conjecture proposed by F. Smirnov and A. Zamolodchikov about the form of the S matrix of the model perturbed by such products.

I am planning to thank the Simons Foundation in both papers.

3.1.8 Maxim Leyenson

"On some correspondences between moduli spaces of vector bundles on algebraic surfaces"
In progress

We define a simple correspondence between moduli spaces of vector bundles on an algebraic surface S and the Hilbert scheme of points on S. We belive that this correspondence plays the role of the classical Abel-Jacobi correspondence for some qustions of the geometry of algebraic surfaces, (though not for the study of rational equvalence).

Assume that we are given a local system f on the surface S. Using the correspondence described above we define a complex b_f on the moduli space of vector bundles on S.

We then define two other correspondences of the moduli space M of vector bundles on S with itself. We believe that both of them play the role of the Hecke-Tyurin transformations on curves. These two correspondences come from two types of submanifolds on a surface: 0-cycles, and curves. (We also give an adelic interpretation of these correspondences).

We conclude by studying the behaviour of the sheaf b_f under these two correspondences, and its "deflection" from being an eigenvector for these two "Hecke operators".

3.1.9 Grigory Olshanski

[1] A. Borodin and G. Olshanski, Representations of the infinite symmetric group. Cambridge University Press, 2017.

Representation theory of big groups is an important and quickly developing part of modern mathematics, giving rise to a variety of important applications in probability and mathematical physics. This book provides the first concise and self-contained introduction to the theory on the simplest yet very nontrivial example of the infinite symmetric group, focusing on its deep connections to probability, mathematical physics, and algebraic combinatorics. Following a discussion of the classical Thoma's theorem which describes the characters of the infinite symmetric group, the authors describe explicit constructions of an important class of representations, including both the irreducible and generalized ones. Complete with detailed proofs, as well as numerous examples and exercises which help to summarize recent developments in the field, this book will enable graduates to enhance their understanding of the topic, while also aiding lecturers and researchers in related areas.

[2] G. Olshanski, Interpolation Macdonald polynomials and Cauchy-type identities, preprint.

Let Sym denote the algebra of symmetric functions and $P_\mu(\,\cdot\,;q,t)$ and $Q_\mu(\,\cdot\,;q,t)$ be the Macdonald symmetric functions (recall that they differ by scalar factors only). The

(q,t)-Cauchy identity

$$\sum_{\mu} P_\mu(x_1, x_2, \dots; q, t) Q_\mu(y_1, y_2, \dots; q, t) = \prod_{i,j=1}^{\infty} \frac{(x_i y_j t; q)_\infty}{(x_i y_j; q)_\infty}$$

expresses the fact that the $P_\mu(\,\cdot\,; q, t)$'s form an orthogonal basis in Sym with respect to a special scalar product $\langle \cdot\,, \cdot \rangle_{q,t}$. The present paper deals with the inhomogeneous *interpolation* Macdonald symmetric functions

$$I_\mu(x_1, x_2, \dots; q, t) = P_\mu(x_1, x_2, \dots; q, t) + \text{lower degree terms}.$$

These functions come from the N-variate interpolation Macdonald polynomials, extensively studied in the 90's by Knop, Okounkov, and Sahi. The goal of the paper is to construct symmetric functions $J_\mu(\,\cdot\,; q, t))$ with the biorthogonality property

$$\langle I_\mu(\,\cdot\,; q, t), J_\nu(\,\cdot\,; q, t) \rangle_{q,t} = \delta_{\mu\nu}.$$

These new functions live in a natural completion of the algebra Sym. As a corollary one obtains a new Cauchy-type identity in which the interpolation Macdonald polynomials are paired with certain multivariate rational symmetric functions. The degeneration of this identity in the Jack limit is also described.

3.1.10 Taras Panov

[1] With V. Buchstaber, N. Erokhovets, M. Masuda, and S. Park.
Cohomological rigidity of manifolds defined by 3-dimensional polytopes.
Russian Math. Surveys, 2017, Vol. 72, No. 2, pp. 199–256; DOI:10.1070/RM9759

A family of closed manifolds is called cohomologically rigid if a cohomology ring isomorphism implies a diffeomorphism for any two manifolds in the family. We establish cohomological rigidity for large families of 3-dimensional and 6-dimensional manifolds defined by 3-dimensional polytopes.

We consider the class $\mathcal{P}$ of 3-dimensional combinatorial simple polytopes P, different from a tetrahedron, whose facets do not form 3- and 4-belts. This class includes mathematical fullerenes, i. e. simple 3-polytopes with only 5-gonal and 6-gonal facets. By a theorem of Pogorelov, any polytope from $\mathcal{P}$ admits a right-angled realisation in Lobachevsky 3-space, which is unique up to isometry.

Our families of smooth manifolds are associated with polytopes from the class $\mathcal{P}$. The first family consists of 3-dimensional small covers of polytopes from $\mathcal{P}$, or hyperbolic 3-manifolds of Löbell type. The second family consists of 6-dimensional quasitoric manifolds over polytopes from $\mathcal{P}$. Our main result is that both families are cohomologically rigid, i. e. two manifolds M and M' from either of the families are diffeomorphic if and only if their cohomology rings are isomorphic. We also prove that if M and M' are diffeomorphic, then their corresponding polytopes P and P' are combinatorially equivalent. These results are intertwined with the classical subjects of geometry and topology, such as combinatorics of 3-polytopes, the Four Colour Theorem, aspherical manifolds, diffeomorphism classification of 6-manifolds and invariance of Pontryagin classes. The proofs use techniques of toric topology.

[2] With Ya. Veryovkin.
On the commutator subgroup of a right-angled Artin group.
arXiv:1702.00446, *to appear in Jounral of Algebra.*

We use polyhedral product models to analyse the structure of the commutator subgroup of a right-angled Artin group. In particular, we provide a minimal set of generators for the commutator subgroup, consisting of special iterated commutators of canonical generators.

[3] With S. Theriault.
The homotopy theory of polyhedral products associated with flag complexes.
arXiv:1709.00388, *submitted to Compositio Mathematica*

If K is a simplicial complex on m vertices the flagification of K is the minimal flag complex K^f on the same vertex set that contains K. Letting L be the set of vertices, there is a sequence of simplicial inclusions $L \to K \to K^f$. This induces a sequence of maps of polyhedral products $(X, A)^L \xrightarrow{g} (X, A)^K \xrightarrow{f} (X, A)^{K^f}$. We show that Ωf and $\Omega f \circ \Omega g$ have right homotopy inverses and draw consequences. We also show that for flag K the polyhedral product of the form $(CY, Y)^K$ is a co-H-space if and only if the 1-skeleton of K is a chordal graph, and deduce that the maps f and $f \circ g$ have right homotopy inverses in this case.

3.1.11 Alexei Penskoi

[1] With M. Karpukhin, N. Nadirashvili and Iosif Polterovich
An isoperimetric inequality for Laplace eigenvalues on the sphere
arXiv:1706.05713 *submitted to Journal of Differential Geometry*

We show that for any positive integer k, the k-th nonzero eigenvalue of the Laplace-Beltrami operator on the two-dimensional sphere endowed with a Riemannian metric of unit area, is maximized in the limit by a sequence of metrics converging to a union of k touching identical round spheres.

This proves a conjecture posed by the Nadirashvili in 2002 and yields a sharp isoperimetric inequality for all nonzero eigenvalues of the Laplacian on a sphere. Earlier, the result was known only for $k = 1$ (J. Hersch, 1970), $k = 2$ (N. Nadirashvili, 2002 and

R. Petrides, 2014) and $k = 3$ (N. Nadirashvili and Y. Sire, 2015). In particular, we argue that for any $k \geq 2$, the supremum of the k-th nonzero eigenvalue on a sphere of unit area is not attained in the class of Riemannin metrics which are smooth outsitde a finite set of conical singularities. The proof uses certain properties of harmonic maps between spheres, the key new ingredient being a bound on the harmonic degree of a harmonic map into a sphere obtained by N. Ejiri.

3.1.12 Petr Pushkar'

[1] With V. Colin and E. Ferrand
International Mathematics Research Notices, Volume 2017, Issue 20, 1 October 2017, Pages 6231–6254

In this paper we give a simple proof that there is no positive loop inside the component of a fiber in the space of Legendrian embeddings in the contact manifold ST^*M, provided that the universal cover of M. We consider some related more general results in the space of one-jet of functions on a compact manifold and we give an application dealing with positive isotopies in homogeneous neighbourhoods of surfaces in a tight contact three-manifold.

3.1.13 Leonid Rybnikov

[1] With Iva Halacheva, Joel Kamnitzer and Alex Weekes
Crystals and monodromy of Bethe vectors
arXiv:1708.05105 [math.RT] *submitted to "Duke Mathematical Journal"*

Fix a semisimple Lie algebra g. Gaudin algebras are commutative algebras acting on tensor product multiplicity spaces for g-representations. These algebras depend on a parameter which is a point in the Deligne-Mumford moduli space of marked stable genus 0 curves. When the parameter is real, then the Gaudin algebra acts with simple spectrum on the tensor product multiplicity space and gives us a basis of eigenvectors. In this paper, we study the monodromy of these eigenvectors as the parameter varies within the real locus; this gives an action of the fundamental group of this moduli space, which is called the cactus group. We prove a conjecture of Etingof which states that the monodromy of eigenvectors for Gaudin algebras agrees with the action of the cactus group on tensor products of g-crystals. In fact, we prove that the coboundary category of normal g-crystals can be reconstructed using the coverings of the moduli spaces. Our main tool is the construction of a crystal structure on the set of eigenvectors for shift of argument algebras, another family of commutative algebras which act on any irreducible g-representation. We also prove that the monodromy of such eigenvectors is given by the internal cactus group action on g-crystals.

[2] With Aleksei Ilin

Degeneration of Bethe subalgebras in the Yangian of $\mathfrak{gl}_n$

arXiv:1703.04147 [math.QA], *published online in "Letters in Mathematical Physics" DOI 10.1007/s11005-017-1031-2 .*

We study degenerations of Bethe subalgebras $B(C)$ in the Yangian $Y(\mathfrak{gl}_n)$, where C is a regular diagonal matrix. We show that closure of the parameter space of the family of Bethe subalgebras, which parametrizes all possible degenerations, is the Deligne-Mumford moduli space of stable rational curves $\overline{M_{0,n+2}}$. All subalgebras corresponding to the points of $\overline{M_{0,n+2}}$ are free and maximal commutative. We describe explicitly the "simplest" degenerations and show that every degeneration is the composition of the simplest ones. The Deligne-Mumford space $\overline{M_{0,n+2}}$ generalizes to other root systems as some De Concini-Procesi resolution of some toric variety. We state a conjecture generalizing our results to Bethe subalgebras in the Yangian of arbitrary simple Lie algebra in terms of this De Concini-Procesi resolution.

3.1.14 Stanislav Shaposhnikov

1. Manita O.A., Romanov M.S., Shaposhnikov S.V. Fokker–Planck–Kolmogorov Equations with a Partially Degenerate Diffusion Matrix, Doklady Mathematics, 2017, Vol. 96, No. 1, pp. 384–388.

The FokkerPlanckKolmogorov equations with a degenerate or partially degenerate diffusion matrix are considered. The distance between probability solutions of these equations with different drift coefficients and different initial conditions is estimated. Sufficient conditions for the existence and uniqueness of probability solutions to nonlinear Fokker-PlanckKolmogorov equations with a partially degenerate diffusion matrix are established.

2. Bogachev V.I., Rockner M., Shaposhnikov S.V. Convergence in variation of solutions of nonlinear Fokker–Planck–Kolmogorov equations to stationary measures, 2017, Preprint Bielefeld University, sfb17027, pp.1–21.

We study convergence in variation of probability solutions of nonlinear Fokker–Planck–Kolmogorov equations to stationary solutions. We obtain sufficient conditions for the exponential convergence of solutions to the stationary solution in case of coefficients that can have an arbitrary growth at infinity and depend on the solutions through convolutions with unbounded discontinuous kernels. In addition, we study a more difficult case where the nonlinear equation has several stationary solutions and convergence to a stationary solution depends on initial data. Finally, we obtain sufficient conditions for solvability of nonlinear Fokker–Planck–Kolmogorov equations.

3.1.15 George Shabat

[1] The Poincare polynomial of the space and $\overline{\mathcal{M}_{0,n}}(\mathbb{C})$ and number of points in the space $\overline{\mathcal{M}_{0,n}}(\mathbb{F}_q)$ (joint with N. Amburg and E. Kreines, in russian). Vestnik Moskovskogo Uni-

versiteta, ser. 1 (mathematics, mechanics), 2017, N. 4, 20-27.

The direct geometric proof of the known relations between the Poincare polynomials is presented.

[2] Generalized Napoleon transformations (joint with P. Makaova and O. Teslya, in russian). Materials of the spring scientific session of the chair of geometry of the mathematical faculty of the Moscow State Pedagogical University and the chair of algebra and geometry of the faculty of natural sciences of the Palatsky University in Olomouts, 35-41. Moscow, MPGU, 2017.

A theorem from elementary geometry is discussed from the viewpoint of linear algebra.

[3] Triangles and cubic curves (in russian). Materials of the spring scientific session of the chair of geometry of the mathematical faculty of the Moscow State Pedagogical University and the chair of algebra and geometry of the faculty of natural sciences of the Palatsky University in Olomouts, 57-64. Moscow, MPGU, 2017.

Some hidden relations between elementary geometry and theory of elliptic curves are discussed.

[4] Formal language of geometry as a semiotic system I. Elementary syntax (joint with G. Kreidlin, in russian). Vestnik RGGU, 2017, 11, 69-87. (Moscow linguistic journal, vol. 19).

A formalization of a language of planimetry is suggested and analyzed.

[5] Counting Belyi pairs over finite fields. To appear in 2016 MATRIX Annals, Springer.

The theory of Belyi pairs over finite fields is outlined and compared with the dessins d'enfants theory.

3.1.16 Arkady Skopenkov

(Abstracts of conference talks are not listed)

[1] A. Skopenkov, How do autodiffeomorphisms act on embeddings, Proceedings A of The Royal Society of Edinburgh, to appear.

`http://arxiv.org/abs/1402.1853`

We work in the smooth category. For an n-manifold N denote by $E^m(N)$ the set of isotopy classes of embeddings $N \to \mathbb{R}^m$. The following problem was suggested by E. Rees in 2002: describe the action of self-diffeomorphisms of $S^p \times S^{n-p}$ on $E^m(S^p \times S^{n-p})$.

Let $g : S^p \times S^{n-p} \to \mathbb{R}^m$ be an embedding such that $g|_{a\times S^{n-p}} : a \times S^{n-p} \to \mathbb{R}^m - g(b \times S^{n-p})$ is null-homotopic for some different points $a, b \in S^p$ and $m \geq n+2+\frac{1}{2}\max\{p, n-p\}$.

Theorem. *For a map $\varphi : S^p \to SO_{n-p}$ define an autodiffeomorphism φ' of $S^p \times D^{n-p}$ by $\overline{\varphi}(a, b) := (a, \varphi(a)b)$. Let φ'' be the S^{n-p-1}-symmetric extension of φ to an autodiffeomorphism of $S^p \times S^{n-p}$. Then for each map $\varphi : S^p \to SO_{n-p}$ embedding $g \circ \varphi''$ is isotopic to embedded connected sum $g\#u$ for some embedding $u : S^n \to S^m$.*

Let N be an oriented n-manifold and $f : N \to \mathbb{R}^m$ an embedding. Denote by $E^m(N)/\#$ the quotient set of $E^m(N)$ by embedded connected sum with embeddings $S^n \to \mathbb{R}^m$. As a corollary we obtain that under certain conditions *for orientation-preserving embeddings $s : S^p \times D^{n-p} \to N$ the class of S^p-parametric embedded connected sum $f\#_s g$ in $E^m(N)/\#$ depends only on f, g and the isotopy (the homotopy or the homology) class of $s|_{S^p\times 0}$.*

[2] A. Skopenkov, A user's guide to the topological Tverberg conjecture. `http://arxiv.org/abs/1605.05141`, Russian Math. Surveys, to appear.

The well-known *topological Tverberg conjecture* was considered a central unsolved problem of topological combinatorics. The conjecture asserts that *for each integers $r, d > 1$ and each continuous map $f\colon \Delta \to \mathbb{R}^d$ of the $(d + 1)(r - 1)$-dimensional simplex Δ there are pairwise disjoint subsimplices $\sigma_1, \dots, \sigma_r \subset \Delta$ such that $f(\sigma_1) \cap \dots \cap f(\sigma_r) \neq \emptyset$.*

A proof for a prime power r was given by I. Bárány, S. Shlosman, A. Szűcs, M. Özaydin and A. Volovikov in 1981-1996. A counterexample for other r was found in a series of papers by M. Özaydin, M. Gromov, P. Blagojević, F. Frick, G. Ziegler, I. Mabillard and U. Wagner, most of them recent. The arguments form a beautiful and fruitful interplay between combinatorics, algebra and topology. In this expository note we present a simplified explanation of easier parts of the arguments, accessible to non-specialists in the area.

[3] A. Skopenkov, On van Kampen-Flores, Conway-Gordon-Sachs and Radon theorems, `http://arxiv.org/abs/1704.00300`, Russian Math. Surveys, to appear as a section of [2].

We exhibit relations between van Kampen-Flores, Conway-Gordon-Sachs and Radon theorems, by presenting direct proofs of some implications between them. The key idea is an interesting relation between the van Kampen and the Conway-Gordon-Sachs numbers for restrictions of a map of $(d+2)$-simplex to R^d to the $(d+1)$-face and to the $[d/2]$-skeleton.

[4] A. Skopenkov, Stability of intersections of graphs in the plane and the van Kampen obstruction. `http://arxiv.org/abs/1609.03727`, to appear in Topology and its Applications

A map $\varphi : K \to \mathbb{R}^2$ of a graph K is *approximable by embeddings*, if for each $\varepsilon > 0$ there is an ε-close to φ embedding $f : K \to \mathbb{R}^2$. Analogous notions were studied in computer science under the names of *cluster planarity* and *weak simplicity*. This short survey is intended not only for specialists in the area, but also for mathematicians from other areas.

We present criteria for approximability by embeddings (P. Minc, 1997, M. Skopenkov, 2003) and their algorithmic corollaries. We introduce *the van Kampen (or Hanani-Tutte) obstruction* for approximability by embeddings and discuss its completeness. We discuss analogous problems of moving graphs in the plane apart (cf. S. Spież and H. Toruńczyk, 1991) and finding closest embeddings (H. Edelsbrunner). We present higher dimensional generalizations, including completeness of the van Kampen obstruction and its algorithmic corollary (D. Repovš and A. Skopenkov, 1998).

[5] S. Avvakumov, I. Mabillard, A. Skopenkov, U. Wagner, Eliminating Higher-Multiplicity Intersections, III. Codimension 2. `http://arxiv.org/abs/1511.03501`, submitted.

We study conditions under which a finite simplicial complex K can be mapped to $\mathbb{R}^d$ without higher-multiplicity intersections. An *almost r-embedding* is a map $f\colon K \to \mathbb{R}^d$ such that the images of any r pairwise disjoint simplices of K do not have a common point. We show that if r is not a prime power and $d \geq 2r + 1$, then there is a counterexample to the topological Tverberg conjecture, i.e., *there is an almost r-embedding of the $(d+1)(r-1)$-simplex in $\mathbb{R}^d$.* This improves on previous constructions of counterexamples (for $d \geq 3r$) based on a series of papers by M. Özaydin, M. Gromov, P. Blagojević, F. Frick, G. Ziegler, and the second and fourth present author.

The counterexamples are obtained by proving the following algebraic criterion in codimension 2: *If $r \geq 3$ and if K is a finite $2(r-1)$-complex then there exists an almost r-embedding $K \to \mathbb{R}^{2r}$ if and only if there exists a general position PL map $f\colon K \to \mathbb{R}^{2r}$ such that the algebraic intersection number of the f-images of any r pairwise disjoint simplices of K is zero.* This result can be restated in terms of cohomological obstructions or equivariant maps, and extends an analogous codimension 3 criterion by the second and fourth author.

It follows from work of M. Freedman, V. Krushkal, and P. Teichner that the analogous criterion for $r = 2$ is false. We prove a beautiful lemma on singular higher-dimensional Borromean rings, yielding an elementary proof of the counterexample. As another application of our methods, we classify *ornaments* $f\colon S^3 \sqcup S^3 \sqcup S^3 \to \mathbb{R}^5$ up to *ornament concordance.*

[6] A. Skopenkov and M. Tancer, Hardness of almost embedding simplicial complexes in $\mathbb{R}^d$, `http://arxiv.org/abs/1703.06305`, submitted.

A map $f : K \to \mathbb{R}^d$ of a simplicial complex is an almost embedding if $f(\sigma) \cap f(\tau) = \emptyset$ whenever σ, τ are disjoint simplices of K.

Theorem. Fix integers $d, k \geq 2$ such that $d = \frac{3k}{2} + 1$.

(a) Assume that $P \neq NP$. Then there exists a finite k-dimensional complex K that does not admit an almost embedding in R^d but for which there exists an equivariant map $K \to S^{d1}$.

(b) The algorithmic problem of recognition almost embeddability of finite k-dimensional complexes in R^d is NP hard.

The proof is based on the technique from the Matoušek-Tancer-Wagner paper (proving an analogous result for embeddings), and on singular versions of the higher-dimensional Borromean rings lemma and a generalized van Kampen–Flores theorem.

[7] A. Skopenkov, High codimension embeddings: classification, submitted to Bull. Man. Atl.

http://www.map.mpim-bonn.mpg.de/High_codimension_embeddings
This page is intended not only for specialists in embeddings, but also for mathematicians from other areas who want to apply or to learn the theory of embeddings.

This article gives a short guide to the Knotting Problem of compact manifolds N in Euclidean spaces and in spheres. After making general remarks we record some of the dimension ranges where no knotting is possible, i.e. where any two embeddings of N are isotopic. We then establish notation and conventions and give references to other pages on the Knotting Problem, to which this page serves as an introduction. We conclude by introducing connected sum and make some comments on codimension 1 and 2 embeddings.

[8] A. Skopenkov, Embeddings just below the stable range: classification, submitted to Bull. Man. Atl. http://www.map.mpim-bonn.mpg.de/Embeddings_just_below_the_stable_range:_classification
This page is intended not only for specialists in embeddings, but also for mathematician from other areas who want to apply or to learn the theory of embeddings.

Recall the Whitney-Wu Unknotting Theorem: if N is a connected manifold of dimension $n > 1$, and $m \geq 2n+1$, then every two embeddings $N \to \mathbb{R}^m$ are isotopic. In this page we summarize the situation for $m = 2n \geq 6$ and some more general situations.

[9] A. Skopenkov, 3-manifolds in 6-space, submitted to Bull. Man. Atl.
http://www.map.mpim-bonn.mpg.de/3-manifolds_in_6-space
This page is intended not only for specialists in embeddings, but also for mathematicians from other areas who want to apply or to learn the theory of embeddings.

The classification of 3-manifolds in 6-space is of course a particular case of the classification of n-manifolds in $2n$-space. In this page we recall the general results as they apply when $n = 3$ and we discuss examples and invariants peculiar to the case $n = 3$.

[10] A. Skopenkov, 4-manifolds in 7-space, submitted to Bull. Man. Atl.
http://www.map.mpim-bonn.mpg.de/4-manifolds_in_7-space
This page is intended not only for specialists in embeddings, but also for mathematician from other areas who want to apply or to learn the theory of embeddings.

Basic results on 4-manifolds in 7-space are particular cases of results on n-manifolds in $(2n-1)$-space for $n = 4$. In this page we concentrate on more advanced results peculiar for $n = 4$.

[11] A. Skopenkov, High codimension links, submitted to Bull. Man. Atl.
http://www.map.mpim-bonn.mpg.de/High_codimension_links
This page is intended not only for specialists in embeddings, but also for mathematician from other areas who want to apply or to learn the theory of embeddings. We describe classification of embeddings $S^{n_1} \sqcup \ldots \sqcup S^{n_s} \to S^m$ for $m - 3 \geq n_i$.

[12] D. Crowley and A. Skopenkov, Embeddings of non-simply-connected 4-manifolds in 7-space, I. Classification modulo knots.
http://arxiv.org/abs/1611.04738, submitted.

We work in the smooth category. Let N be a closed connected orientable 4-manifold with torsion free H_1, where $H_q := H_q(N;\mathbb{Z})$. The main result is *a complete readily calculable classification of embeddings* $N \to \mathbb{R}^7$, up to equivalence which is isotopy and embedded connected sum with embeddings $S^4 \to \mathbb{R}^7$. Such a classification was earlier known only for $H_1 = 0$ by Boéchat-Haefliger-Hudson 1970. Our classification involves Boéchat-Haefliger invariant $\varkappa(f) \in H_2$, Seifert bilinear form $\lambda(f) : H_3 \times H_3 \to \mathbb{Z}$ and β-invariant assuming values in the quotient of H_1 defined by values of $\varkappa(f)$ and $\lambda(f)$.

In particular, for $N = S^1 \times S^3$ we define geometrically a 1–1 correspondence between the set of equivalence classes of embeddings and an explicitly defined quotient of $\mathbb{Z} \oplus \mathbb{Z}$.

[13] D. Crowley and A. Skopenkov, Embeddings of non-simply-connected 4-manifolds in 7-space, II. On the smooth classification. `http://arxiv.org/abs/1612.04776`, submitted.

We work in the smooth category. Let N be a closed connected orientable 4-manifold with torsion free H_1, where $H_q := H_q(N;\mathbb{Z})$. Our main result is *a readily calculable classification of embeddings* $N \to \mathbb{R}^7$ *up to isotopy*, with an indeterminancy. Such a classification was only known before for $H_1 = 0$ by our earlier work from 2008. Our classification is complete when $H_2 = 0$ or when the signature of N is divisible neither by 64 nor by 9.

The group of knots $S^4 \to S^7$ acts on the set of embeddings $N \to \mathbb{R}^7$ up to isotopy by embedded connected sum. In Part I we classified the quotient of this action. The main novelty of this paper is the description of this action for $H_1 \neq 0$, with an indeterminancy.

Besides the invariants of Part I, the classification involves a refinement of the Kreck invariant from our work of 2008 which detects the action of knots.

For $N = S^1 \times S^3$ we give a geometrically defined 1–1 correspondence between the set of isotopy classes of embeddings and a quotient of the set $\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}_{12}$.

[14] A. Skopenkov, Classification of knotted tori, `http://arxiv.org/abs/1502.04470` (the paper is rewritten in 2017, a new version uploaded to arxiv)

We describe the group of (smooth isotopy classes of smooth) embeddings $S^p \times S^q \to R^m$ for $p \le q$ and $m \ge 2p+q+3$. Earlier such a description was known only for $2m \ge 3p+3q+4$. We use a recent exact sequence of M. Skopenkov.

[15] A. Skopenkov, Eliminating higher-multiplicity intersections in the metastable dimension range, `http://arxiv.org/abs/1704.00143`.

The *r-fold* analogues of Whitney trick were 'in the air' since 1960s. However, only in this century they were stated, proved and applied to obtain interesting results, most notably by Mabillard and Wagner. Here we prove and apply a version of the r-fold Whitney trick when general position r-tuple intersections have positive dimension.

Theorem. *Assume that* $D = D_1 \sqcup \ldots \sqcup D_r$ *is disjoint union of* k*-dimensional disks,* $rd \ge (r+1)k+3$, *and* $f : D \to B^d$ *a proper PL (smooth) map such that* $f\partial D_1 \cap \ldots \cap f\partial D_r = \emptyset$. *If the map*

$$f^r : \partial(D_1 \times \ldots \times D_r) \to (B^d)^r - \{(x, x, \ldots, x) \in (B^d)^r \mid x \in B^d\}$$

extends to $D_1 \times \ldots \times D_r$, then there is a proper PL (smooth) map $\overline{f} : D \to B^d$ such that $\overline{f} = f$ on ∂D and $\overline{f}D_1 \cap \ldots \cap \overline{f}D_r = \emptyset$.

Expository publications for university students

[16] Mathematics via problems: from olympiads and math circles to a profession, editors: A. Zaslavsky, A. Skopenkov, and M. Skopenkov. 2017, Moscow, MCCME. `http://www.mccme.ru/circles/oim/sturm.pdf`

In this book we present an approach to 'university' mathematics as sequences of 'high-school' problems.

[17] A. Skopenkov, Embeddings into the plane of graphs with vertices of degree 4, Mat. Prosveschenie, 21 (2017), `http://arxiv.org/abs/1008.4940`

In this expository note we present a proof of the V.A. Vassiliev conjecture on the planarity of graphs with vertices of degree 4 and certain additional structure. Both statement and proof are accessible to high-school students familiar with basic notions of graph theory. The conjecture was first proved by V.O. Manturov (such a proof was one of the main results of his habilitation thesis). In this note the exposition is made clearer and some comments for beginners are added.

[18] A. Volostnov, A. Skopenkov and Yu. Yarovikov, A study on recursive relations, Mat. Prosveschenie, 21 (2017).

In this expository note we present and discuss a short proof of an estimation required for a proof of the Symmetric Local Lovasz Lemma.

[19] A. Remizova and A. Skopenkov, A simple proof of the local Lovasz lemma, Mat. Prosveschenie, to appear.

We present a sequence of elementary problems leading to a short (the original) proof of the Symmetric Local Lovasz Lemma.

[20] A. Skopenkov, How Fermat found extrema, `http://arxiv.org/abs/1610.05968`, submitted.

In this expository note we present a short elementary proof of the well-known criterion for a cubic polynomial to have three real roots. The proof is based on Fermat's approach to calculus for polynomials, and illustrates the idea of a derivative rigorously but without technical ε-δ language. The note is accessible to high-school students.

[21] A. Enne, A. Ryabichev, A. Skopenkov and T. Zaitsev, Invariants of graph drawings in the plane, `http://www.turgor.ru/lktg/2017/6/index.htm`.

In this expository note we present and discuss polynomial algorithm for recognition of graph planarity, as well as Radon and Tverberg Theorems for the plane.

[22] A. Skopenkov, Algebraic Topology From Algorithmic Viewpoint, draft of a book, `http://www.mccme.ru/circles/oim/algor.pdf` (some sections are rewritten in 2017)

In this book we present an 'algorithmic' approach to algebraic topology.

3.1.17 Mikhail Skopenkov

[1] A. Bobenko, M. Skopenkov, Discrete Riemann surfaces: linear discretization and its convergence, J. Reine Angew. Math. 2016:720 (2016) 217-250. This paper was published at the very end of 2016 and did not appear in the report 2016.

We develop linear discretization of complex analysis, originally introduced by R. Isaacs, J. Ferrand, R. Duffin, and C. Mercat. We prove convergence of discrete period matrices and discrete Abelian integrals to their continuous counterparts. We also prove a discrete counterpart of the Riemann–Roch theorem. The proofs use energy estimates inspired by electrical networks.

[2] Pakharev A., Skopenkov M., Surfaces containing two circles through each point and decomposition of quaternionic matrices, Russian Math. Surveys, 72:2 (2017),381–383, http://arxiv.org/abs/1510.06510. This paper does not contain any reference to the support of Simons–IUM fellowship because of the requirement of the other funding organization.

We find all analytic surfaces in space $\mathbb{R}^3$ such that through each point of the surface one can draw two circular arcs fully contained in the surface. This paper announces the result and gives the idea of proof using a new decomposition technique for quaternionic matrices.

[3] Elements of mathematics in problems. Through circles and olympiads to profession. Ed. by A. Skopenkov, M. Skopenkov, and A. Zaslavskiy. Moscow Center for Continuous Mathematical Education, 2018, 592 pp. ISBN 978-5-4439-1239-4 (in Russian).

A problem-based introduction to the subjects of mathematics traditionally studied in circles, with vast coverage of those subjects.

[4] Skopenkov M., Krasauskas R., Surfaces containing two circles through each point, revised version under review, union of http://arxiv.org/abs/1512.09062
and http://arxiv.org/abs/1503.06481.

We find all analytic surfaces in space $\mathbb{R}^3$ such that through each point of the surface one can draw two transversal circular arcs fully contained in the surface. The problem of finding such surfaces traces back to the works of Darboux from XIXth century. We prove that such a surface is an image of a subset of one of the following sets under some composition of inversions:

- the set $\{\, p+q : p \in \alpha, q \in \beta \,\}$, where α, β are two circles in $\mathbb{R}^3$;
- the set $\{\, 2\frac{[p\times q]}{|p+q|^2} : p \in \alpha, q \in \beta, p+q \neq 0 \,\}$, where α, β are two circles in S^2;
- the set $\{\, (x,y,z) : Q(x,y,z,x^2+y^2+z^2) = 0 \,\}$, where $Q \in \mathbb{R}[x,y,z,t]$ has degree 2 or 1.

The proof uses a new factorization technique for quaternionic polynomials.

[5] M. Skopenkov, Discrete field theory: symmetries and conservation laws, https://arxiv.org/abs/1709.0
submitted.

We present a general algorithm constructing a discretization of a classical field theory from a Lagrangian. We prove a discrete Noether theorem relating symmetries to conservation laws and an energy conservation theorem not based on any symmetry. This gives exact conservation laws for several discrete field theories: electrodynamics, gauge theory, Klein-Gordon and Dirac ones. In particular, we construct a conserved discrete energy-momentum tensor, approximating the continuum one at least for free fields. The theory is stated in topological terms, such as coboundary and products of cochains.

In addition to [1]–[5], several talk abstracts have been published in 2017.

3.1.18 Evgeni Smirnov

[1] Singularities of divisors on flag varieties via Hwang's product theorem
Bulletin of the Korean Mathematical Society. 2017. Vol. 54. No. 5. 1773-1778.

We give an alternative proof of a recent result by Pasquier stating that for a generalized flag variety $X = G/P$ and an effective Q-divisor D stable with respect to a Borel subgroup the pair (X, D) is Kawamata log terminal if and only if $\lfloor D \rfloor = 0$.

[2] Multiple flag varieties

35 pages, in Russian. To appear in VINITI, Itogi Nauki i Tekhniki. Proceedings of Algebra and Geometry Seminar of Samara University. English translation: to appear in Journal of Mathematical Sciences.

This is a survey of results on multiple flag varieties, i.e. varieties of the form $G/P_1 \times \cdots \times G/P_k$. We provide a classification of multiple flag varieties of complexity 0 and 1 and results on the combinatorics and geometry of B-orbits and their closures in double cominuscule flag varieties. We also discuss questions of finiteness for the number of G-orbits and existence of an open G-orbits on a multiple flag variety.

3.1.19 Dmitry Talalaev

[1] With G. Sharygin
Deformation quantization of integrable systems
Journal of Noncommutative Geometry, 2017, Vol. 11 , No. 2 , pp. 741-756.

In this paper we address the following question: is it always possible to choose a deformation quantization of a Poisson algebra A so that certain Poisson-commutative subalgebra Cin it remains commutative? We define a series of cohomological obstructions to this, that take values in the Hochschild cohomology of C with coefficients in A. In some particular case of the pair (A,C) we reduce these classes to the classes of the Poisson relative cohomology of the Hochschild cohomology. We show, that in the case, when the algebra C is polynomial, these obstructions coincide with the previously known ones, those which were defined by Garay and van Straten.

[2] Zamolodchikov Tetrahedral Equation and Higher Hamiltonians of 2d Quantum Integrable Systems
SIGMA, 2017, Vol. 13.

The main aim of this work is to develop a method of constructing higher Hamiltonians of quantum integrable systems associated with the solution of the Zamolodchikov tetrahedral equation. As opposed to the result of V.V. Bazhanov and S.M. Sergeev the approach presented here is effective for generic solutions of the tetrahedral equation without spectral parameter. In a sense, this result is a two-dimensional generalization of the method by J.-M. Maillet. The work is a part of the project relating the tetrahedral equation with the quasi-invariants of 2-knots.

[3] With D. Gurevich, P. Saponov
Drinfeld-Sokolov reduction in quantum algebras
arXiv:1710.01806 *submitted to Letters in Mathematical Physics.*

Applying the method of the paper [CT], we perform a quantum version of the Drinfeld-Sokolov reduction in Reflection Equation algebras and braided Yangians, associated with involutive and Hecke symmetries of general forms. This reduction is based on the Cayley-Hamilton identity valid for the generating matrices of these algebras.

3.1.20 Ilya Vyugin

[1] I. V. Vyugin, R. I. Levin, "On the RiemannHilbert problem for difference and qq-difference systems", Order and chaos in dynamical systems, Collected papers. On the occasion of the 125th anniversary of the birth of Academician Dmitry Victorovich Anosov, Proceedings of the Steklov Institute of Mathematics, 2017, V. 297, 297-313

We study an analog of the classical RiemannHilbert problem stated for the classes of difference and q-difference systems. We generalize Birkhoff's existence theorem.

[2] I. V. Vyugin, E. V. Solodkova, I. D. Shkredov, "Additive Energy of Heilbronn Subgroup", Mathematical Notes, 2017, 101:1, 58-70.

A new upper bound for the additive energy of the Heilbronn subgroup is determined and some applications in the case of distribution of Fermat quotients are obtained.

[3] S.V. Konyagin, S.V. Makarychev, I.E. Shparlinski, I.V. Vyugin, "On the new bound for the number of solutions of polynomial equations in subgroups and the structure of graphs of Markoff triples", arXiv:1711.05335, 2017.

We sharpen the bounds of J. Bourgain, A. Gamburd and P. Sarnak (2016) on the possible number of nodes outside the "giant component" and on the size of individual connected components in the suitably defined functional graph of Markoff triples modulo p. This is a step towards the conjecture that there are no such nodes at all. These results are based on some new ingredients and in particular on a new bound of the number of solutions of polynomial equations in cosets of multiplicative subgroups in finite fields, which generalises previous results of P. Corvaja and U. Zannier (2013).

3.2 Scientific conferences and seminar talks

3.2.1 Anton Aizenberg

[1] Workshop "British–Russian Seminar on Toric Topology and Homotopy Theory", March 1415, 2017, Steklov Mathematical Institute, Moscow, Russia.

Talk "Spectral sequences associated with torus actions".

[2] Princeton-Rider Workshop "On the Homotopy Theory of Polyhedral Products", May 29–June 1, Princeton University and Rider University, USA.

Talk "Toric origami structures on quasitoric manifolds and fatness of 3-polytopes".

[3] International Open Chinese-Russian conference "Algebraic topology, geometry and combinatorics of manifolds", December 4–8, Sanya, China.

Talk "Algebras of multi-fans and combinatorics of pseudomanifolds".

[4] Invited talk at Shanghai Jiao Tong University, November, 30.

Talk "Combinatorics and toric topology of spaces of isospectral matrices".

[5] Weekly seminar "Algebraic topology", Moscow State University, October, 24.

Talk: "Manifold of isospectral hermitian arrow-matrices".

[6] Weekly seminar "Differential operators on singular spaces, algebraical integrable systems and quantization", Moscow State University, October, 30.

Talk "Volumes of multi-polytopes and algebras of multi-fans".

[7] Weekly seminar "Geometry, topology and mathematical physics", Moscow State University, November, 15.

Talk "Manifolds of isospectral staircase matrices and Hessenberg varieties".

3.2.2 Yurii Burman

[1] Conference "On crossroads of analysis, algebra, and geometry" (to celebrate Boris Shapiro's 60), Stockholm, May 29–June 2.

Talk "On Hurwitz-Severi numbers"

3.2.3 Alexei Elagin

[1] Workshop "Categorical approach to rationality", Vienna, Austria, May 2-5. Talk "On exceptional collections of line bundles on weak del Pezzo surfaces".

[2] Conference "Mirror Symmetry and Applications", Moscow, December 12-18.

[3] Visit to Indiana University (Bloomington, USA), October 14-31. Joint work with V. Lunts. Talk "Two dimensions of triangulated categories".

3.2.4 Alexei Gorodentsev

I have not participated any conferences during this year.

3.2.5 Maxim Kazarian

[1] Conference "On crossroads of analysis, algebra, and geometry or Boris' 60-th birthday" (May 29-June 2)

Talk "Symplectic Geometry of Topological Recursion"

[2] XXVII Summer School "Contemporary mathematics", 19-30 July, 2017

Minicourse "Continuous frctions"

[3] Vitushkin Seminar on multidimensional complex analisys, 20 Sep 2017.

Talk "Gysin homomorphism for resolutions of Grassmann and Flag degeneracies"

[4] Conference "Contemporary mathematics" dedicated to 80-th Anniversary of V.I.Arnold, Moscow, 18 Dec 2017,

Talk "Thom Polynomials and Nonassociative Hilbert Schemes"

3.2.6 Alexander Kuznetsov

[1] "Derived category and birational geometry", Osaka University, February 20–23, 2017.

Talk "Derived categories of families of sextic del Pezzo surfaces"

[2] "Categorical approach to rationality", ESI, Vienna, May 2–5, 2017.

Talks "Birational geometry and derived category of Gushel-Mukai fourfolds, I, II"

[3] "Closing workshop — future directions", ESI, Vienna, May 15–19, 2017.

Talk "D-equivalence, L-equivalence and families of quadrics"

[4] "Higgs Bundles, K3 Surfaces and Moduli", Humboldt Universität, Berlin, July 10–12, 2017.

Talk "Quadric fibrations in the Grothendieck ring of varieties with an application to Mukai dual K3 surfaces"

[5] "Algebraic Geometry", MFO, Oberwolfach, September 24–30, 2017.

Talk "D-equivalence and L-equivalence"

[6] "Categorical and Analytic Invariants in Algebraic Geometry V", Osaka University, November 27 – December 1, 2017.

Talk "Residual categories for Lefschetz exceptional collections"

3.2.7 Mikhail Lashkevich

No conferences.

3.2.8 Maxim Leyenson

[1] Michael Tsmasman's arithmetic geometry seminar. Independent University of Moscow, Moscow, Russia, March 16.

Talk"On some correspondences between moduli spaces of vector bundles on algebraic surfaces"

3.2.9 Grigory Olshanski

[1] School–conference "Lie algebras, algebraic groups, and invariant theory VI". Moscow, January 30 – February 4, 2017.

Minicourse "Asymptotic theory of characters" (4 talks).

[2] Conference "Representation Theory at the Crossroads of Modern Mathematics", Reims, May 29-June 2, 2017.

Talk "Quantization of harmonic analysis on $U(\infty)$ and applications"

[3] The 39th Conference on Stochastic Processes and their Applications, Moscow July 24-28, 2017.

Talk "Point processes related to q-hypergeometric polynomials"

[4] Conference "Integrable Models in Statistical Mechanics, Limit Shapes and Combinatorics". St.-Petersburg, August 7-11, 2017.

Talk "The infinite-dimensional q-Beta distribution".

[5] Conference "Transformation Groups 2017". Moscow, December 14-18, 2017.

Talk "Combinatorics of characters".

3.2.10 Taras Panov

[1] Conference "Geometry Days in Novosibirsk – 2017", Novosibirsk, Russia, September, 20–23

Plenary talk "Manifolds defined by right-angled 3-dimensional polytopes"

[2] Conference "Princeton-Rider Workshop on the Homotopy Theory of Polyhedral Products", Princeton University, Rider University, USA, May, 29–June, 2

Invited talk "Manifolds defined by right-angled 3-dimensional polytopes"

[3] Conference "Christmas Mathematical Meeting", Independent University of Moscow, Russia, January, 4–6

Invited talk "Polyhedral products, right-angled Coxeter groups, and hyperbolic manifolds"

[4] Visit to China, July

Talk "Calabi–Yau manifolds and SU-bordism" at the Topology Seminar (Fudan University, Shanghai)

Talk "Polyhedral products, right-angled Coxeter groups, and hyperbolic manifolds" at the University Colloquim (Xi'an Jiaotong-Liverpool University, Suzhou)

[5] Visit to UK, September

Talk "Polyhedral products, right-angled Coxeter groups and hyperbolic manifolds" at the Geometry and Topology Seminar (University of Southampton)

Talk "Manifolds defined by right-angled 3-dimensional polytopes" at the Geometry, Topology and Mathematical Physics Seminar (University of Manchester)

3.2.11 Alexei Penskoi

[1] Conference "Christmas meetings", Independent University of Moscow, January 4-6, 2017.

Talk "Isoperimetric inequality for the second non-zero eigenvalue of the Laplace-Beltrami operator on the projective plane"

[2] Conference "Geometric spectral theory", Universite de Neuchâtel, June 19-23, 2017.

Talk "An isoperimetric inequality for Laplace eigenvalues on S^2 and RP^2"

[3] Visit to Institut de Mathématiques de Marseille, UMR 7373 for a scientific collaboration, May-July 2017

[4] Talk "Maximization of the second Laplace-Beltrami eigenvalue on the projective plane" at Geometry, Topology and Mathematical Physics Seminar, Moscow State University & Steklov Mathematical Institute of RAS.

3.2.12 Petr Pushkar'

[1] A participation and organization of seminars "Surgery Theory" and "Spectral sequences" at HSE.

3.2.13 Leonid Rybnikov

[1] Winter school-conference "Lie algebras, Algebraic groups and Theory of Invariants'
January 30 - February 4, 2017 .
Moscow State University, Department of Mechanics and Mathematics, Moscow, Russia.
3 lectures on "Combinatorics and Geometry of Kashiwara Crystals" (in Russian).

[2] Spring School on Representation Theory
March 13–March 17 2017
Graduate School of Mathematical Sciences, University of Tokyo, Komaba Campus.
3 lectures on “Gaudin algebras, Opers and Crystals”.

3.2.14 Stanislav Shaposhnikov

1) The 39th Conference on Stochastic Processes and their Applications (SPA2017) (Moscow, 24.07.2017-28.07.2017),invited speaker, talk: “Estimates of the entropy and Kantorovich distances between stationary distributions of diffusions”

2) International Conference on Elliptic and Parabolic Problems (Gaeta, Italy, 22.05.201726.05.2017), talk: “Integrability and continuity of solutions to double divergence form equations”

3.2.15 George Shabat

[1] International conference dedicated to Prof. G.B.Shabat on the occasion of his 65th birthday. Independent University of Moscow, Moscow, May 25, 2017.

Talk “Half a century in mathematics”.

[2] “ Algebra, Algebraic Geometry, and Number Theory”. Memorial conference for academician Igor Rostislavovich Shafarevich. Steklov Mathematical Institute of RAS, Moscow, June 56, 2017.

Talk “ On the deformations of Belyi pairs”.

[3] B.V. Shabat Centennial Conference “Several Complex Variables”. Krasnoyarsk, September 15, 2017.

Talk “Meromorphic functions on Riemann surfaces with no more than 4 critical values”.

[4] Conference ”Contemporary Mathematics” in honor of 80th birthday of V.I. Arnold. Moscow, Russia, December 21, 2017.

Talk “On the three-term curves”.

[5] The conference in memory of Voevodsky. Moscow, MIAN, Russia, December 28, 2017.

Talk “Drawing curves: decades ago and now.

[6] Topology seminar, EFPL (Ecole Polytechnique Federale de Lausanne, November 14, 2017.

Talk “Dessins d’enfants and the generalized Hurwitz numbers.

3.2.16 Arkady Skopenkov

[1] Conference of Moscow Institute of Physics and Technology, Dolgoprudnyi, November. Talk ‘Hardness of almost embedding simplicial complexes in R^d’

[2] Topology seminar, Institute of mathematics of PAS, Warsaw, September. Talk 'Hardness of embedding and almost embedding simplicial complexes in R^d'.

[3] Faculty of Mathematics and Physics, Charles University, Prague. Talk 'A user's guide to the topological Tverberg conjecture'.

[4] Department of Mathematics and Statistics, Faculty of Science, Masaryk University, Brno. Talk 'Embeddings of 4-manifolds in 7-space'.

[5] Geometry, topology and mathematical physics (S.P. Novikov and V.M. Buchstaber), Moscow State University, Talk 'Hardness of embedding and almost embedding simplicial complexes in R^d'

3.2.17 Mikhail Skopenkov

[1] Skopenkov M., "Surfaces containing two circles through each point", Perspectives in Real Geometry, Luminy (France), 18-22.09.2017, https://conferences.cirm-math.fr/1782.html

[2] Skopenkov M., "Conservation of energy in lattice field theories", Dynamical systems and perturbations, The conference celebrating S.Yu. Pilyugin's 70th birthday, Sankt-Petersburg, 2-4.10.2017, http://pilyugin70.spb.ru/.

[3] Talks at several seminars in Moscow, Sankt-Petersburg, Linz, Vienna.

3.2.18 Evgeni Smirnov

[1] School and conference "Lie algebras, algebraic groups and invariant theory", Moscow, January 30 – February 4, 2017

Talk: "Singularities of divisors on flag varieties"

[2] School on Schubert calculus, Institute of Mathematical Sciences, Chennai, India, October 23 – November 4, 2017

Series of six talks "Singularities of Schubert varieties".

[3] School "Algebra and Number Theory", Kaliningrad, Russia, April 18, 2017.

Series of two talks "Representations of quivers".

[3] Colloquium talk "Horn Conjecture and Littlewood–Richardson Rule", Steklov Mathematical Institute, Moscow, Russia, April 6, 2017

[4] Colloquium talk "Schubert calculus and polyhedra", Institute of Mathematical Sciences, Chennai, India, November 2, 2017

3.2.19 Dmitry Talalaev

[1] Conference on mathematical physics Kezenoi-Am-2017, 8-12 August 2017.

Talk "3-d integrable statistical models and invariants in low dimensional topology"

3.2.20 Ilya Vyugin

[1] Formal and Analytic Solutions of Differential Equations, September 4-8, 2017, Alcala-de-Henares, Spain

Title: “ On the Riemann-Hilbert problem for difference and q-difference systems”.

[2] (joint with Roman Levin) The 8th International Conference on Differential and Functional Differential Equations, Moscow, RUDN University, August 1320, 2017,

Title: “On the Riemann-Hilbert Problem for Difference and q-Difference Systems”.

[3] Conference Diferential Equations and Related Problems, Kolomna–Zaraisk, June 10-11, 2017.

Title: “On the Riemann-Hilbert problem for difference and q-difference systems”.

[4] Talk at the seminar of Dobrushin Mathematics Laboratory (Chair: Mikhail Blank) March 14, 2017.

Title: “On polynomial equations over fields of positive charactreristic”.

[5] Talk at the seminar Dinamical systems and related topics (Chair: Yuliy Ilyashenko) September, 2017.

Title: “ On Birkhoff's theory of difference and q-difference equations”.

3.3 Teaching

3.3.1 Anton Aizenberg

[1] Topology-3. Independent University of Moscow, II year students, February-June 2017, 2 hours (lecture) + 2 hours (seminar) per week.

Program

- Short intro to manifolds. Closed manifolds (topological, smooth, homology). Orientability. Orientable cover. Fundamental cycle. Cap-product. Poincare duality isomorphism. Intersection index of homology cycles. The case of cycles represented by smooth submanifolds. The connection between intersection theory and multiplication in cohomology. Manifolds with boundary. Signature of a closed manifold.
- Principal G-bundles. Universal bundles. Construction of classifying space for general G. Classifying spaces for discrete groups and group cohomology.
- Stiefel and Grassmann manifolds. Vector bundles. Structure group of a vector bundle and associated principal bundle. Tangent bundle of a smooth manifold. The ring of characteristic classes for bundles with the given structure group.
- Cohomology of Grassmann manifolds (both infinite- and finite-dimensional cases). Stiefel–Whitney and Chern classes. Euler class. Pontryagin classes.

- Applications: existence of division algebras, immersions of real projective spaces in euclidean spaces, Borsuk–Ulam theorem. Stiefel–Whitney characteristic numbers.
- The notion of cobordism. Pontryagin theorem. Formulation of Thom theorem. Pontryagin–Thom construction. Geometric cobordism theory.
- (Seminar and home work) Construction of generalized (co)homology theories via spectra. Spectrum of Thom spaces and spectrum of Eilenberg–MacLane spaces.
- Hirzebruch genera. Hirzebruch signature theorem. Multiplicativity of signature on skew products (Chern–Hirzebruch–Serre theorem). Borel–Hirzebruch theorem on multiplicative Hirzebruch genera.
- (Seminar and home work) The notion of K-theory and Chern character.
- Todd genus. Examples of nonexistence of almost complex structures on a manifold.
- (Appendix) Gentle introduction to spectral sequences. General homological technique and basic topological examples: spectral sequence of the filtration, Mayer–Vietoris sequence, Serre sequence.

Conducting seminars at the Faculty of Computer Science, Higher School of Economics, Moscow, Russia:

[2] Calculus-1. January-June 2017 and September-December 2017, 2 groups, each 2 hours per week. Standard I year calculus program: limits, derivatives, Taylor formula, metric spaces, calculus of several variables.

[3] Differential equations. January-June 2017, 2 groups, each 2 hours per week. (Program: general notion of a differential equation, standard methods of exact solution, equations of higher degrees, systems of linear differential equations, matrix exponent, introduction to calculus of variations, introduction to distributions, examples of differential equations in economics and biology)

[4] Calculus-2. September-December 2017, 2 groups, each 2 hours per week. Standard II year calculus program: series, parametric series, Fourier series, basics on Hilbert spaces and orthogonal decompositions, parametric integrals, parametric improper integrals, Fourier transform.

3.3.2 Yurii Burman

[1] Selecta of combinatorics. Independent University of Moscow, 2nd–5th year students, September–December 2017, 2 hours per week.

Program

1. Partially ordered sets

(a) Moebius inversion formula.
(b) Summation over subgraphs: Potts polynomial and random cluster model (partition function by Fortuin-Kastelein)
(c) Special values of Potts polynomial: percollation problem, generating functions of subgraphs and more.
(d) Graph invariants: Tutte polynomial and Bernardi polynomial.

2. Cauchy–Binet formula and its generalizations.
   (a) Generalized Cauchy–Binet formula as a discrete version of the path integral.
   (b) Pluecker relations and discrete path integral in the skew-symmetric case.
   (c) Applications: matrix-tree theorem, its Pfaffian version (Masbaum–Vaintrob identity), its analog for the groups D_n, and more.

3. Matrix-tree theorem in detail.
   (a) Proofs: using Tutte polynomial, using discrete path integration, and others.
   (b) Generalizations: for all minors, for oriented graphs.
   (c) Matrix-tree theorem for higher determinants.

4. Integer points in polyhedra.
   (a) Counting integer points in lattice polyhedra: Ehrhart polynomial.
   (b) Integer points in the interior and on the boundary: Ehrhart–Macdonald duality.
   (c) Summation over the set of integer points: Brion's theorem.

5. Hurwitz numbers.
   (a) Cut-and-join equation.
   (b) Boson-fermion correspondence.
   (c) Center of the group algebra of the permutation group.
   (d) Explicit formula for the generating function.

[2] Basics of differential geometry, Higher School of Economics, 1st year MS students, September–December 2017, 4 hours a week.

Program.

1. Smooth manifolds.

Smooth manifolds, smooth maps, induced topology.

2. Vector bundles.

   Vector bundles. Operations over vector bundles (direct sum, tensor product). Tangent bundles. Derivative of a smooth map.

3. Vector fields.

   Group of diffeomorphisms. Lie algebra of vector fields. Lie derivative.

4. Smooth functions.

   Paracompactness. Partition of unity.

5. Differential forms.

   Superalgebra of differential forms. Exterior derivative, $d^2 = 0$. Cartan's formula.

6. (time permitting) Frobenius' theorem.

   Frobenius' theorem. Almost complex structure. Contact structure.

7. Integration of differential forms.

   Integral of a differential form. Stokes' theorem.

[3] Calculus-2, Higher School of Economics and New Economics School (joint BS program), 2nd year students, September–December 2017, 4 hours a week.
Program

1. Vector-valued functions.

   Geometric sense: curves. Continuity, differentiation, integration. Geometric sense of the derivative: velocity. Arc length.

2. Functions of several variables.

   Continuity. Partial derivative. Directional derivative. Gradient; its geometric meaning. Chain rule: derivative of the composition. Mixed derivatives are equal.

3. Unconditional extrema.

   Fermat's principle. Sufficient conditions of extrema involving 2nd derivatives.

4. Implicit function theorem.

   Contraction mapping principle. Inverse function theorem. Implicit function theorem.

5. Conditional extrema.

   Langrange multipliers. Conditions of 2nd order. Kuhn–Tacker theorem.

6. Multidimensional integration.

   Integrability of continuous functions. The theorem of Fubini: reduction of multidimensional integration to Riemann integration over a segment. Change of variables in a multidimensional integral.

7. Volumes of hypersurfaces.

   Motivation: Santalo's theorem. Volume of embedded hypersurface. Volume of the graph of a function.

[4] Basic notions of mathematics (freshmen's seminar), Higher School of Economics, 1st year students, September–December 2017, 2 hours a week.

Program.

1. Is the principal section of the n-dimensional hypercube a regular polyhedron?

2. How to solve cubic equation and why nobody's doing it.

3. Quadratic roots modulo prime: law of quadratic reciprocity.

4. Doubling of the GDP: Banach–Tarski paradox.

5. Brouwer's theorem, fundamental theorem of algebra and hairy ball theorem: index of a continuous map between circles.

6. Pentagrams: Euler's pentagonal identity.

7. Discontinuous projectivity: maps sending lines to lines and automorphisms of fields.

8. Voting vs dictatorship: ultrafilters, Arrow's impossibility theorem and non-standard analysis.

9. Compass and straightedge: doubling the cube, angle trisection and construction of a regular n-gon.

3.3.3 Alexei Elagin

[1] Algebra - 1. Independent University of Moscow, 1st year students, Fall 2017.
Program:

1. **Polynomials.** Polynomials. Degree, Euclidean division. Bezout's theorem. g.c.d., Euclidean algorithm for polynomials. Irreducible polynomials. Unique factorization into prime factors for polynomials. Gauss lemma. Unique factorization for integer coefficients and for several variables. Eisenstein's criterion. Fundamental theorem of algebra. Irreducible polynomials over $\mathbb{R}$ and $\mathbb{C}$.

2. **Algebraic structures.** Rings, fields, groups. Zero divisors. Ideals, their sum and intersection. Principal ideals. Quotient ring. Ring homomorphisms. Isomorphism theorems. Chinese remainder theorem for ideals. Its special cases for integers and for polynomials. Interpolation. Modules. Free and finitely generated modules. Direct sum of modules. Sum and intersection of submodules. Homomorphisms. Cyclic modules. Submodules of free modules over Principal ideal domains. Structure theorem for finitely generated modules over Euclidean domains.

3. **Arithmetics.** Euclidean algorithm and Fundamental theorem of arithmetic. Greatest common divisor. Domains. Primes and units. Fundamental theorem of arithmetic in Euclidean domains. Principal Ideal Domains and Unique Factorization Domains. Modular arithmetic. Group of units of a ring. Multiplicative group of a finite field is cyclic. Gaussian numbers. Gaussian primes and integer primes. Decomposition of integers into a sum of two squares. Quadratic residues. Legendre symbol. Some modular equations in integers.

4. **Group theory.** Symmetric group. Cycles, transpositions, parity. Inversions. Order of a permutation. Conjugation. Decomposition into a product of cycles. Groups, subgroups. Cosets, index of a subgroup. Lagrange's theorem. Normal subgroups. Quotient group. Group action on a set. Orbits, stabilizers. Transformation groups. Order of an element. Cyclic groups, their subgroups and generators. finitely generated abelian groups, structure theorems. Free abelian groups.

5. **Linear algebra.** Systems of linear equations, elementary operations with rows and columns. Gaussian elimination. Row echelon form. Space of solutions. Matrices, algebraic operations with matrices, elementary operations. Vector spaces. Bases, dimension. Subspaces, their sum, intersection. Linear morphisms, matrices. Kernel, image, rank of a linear morphism. Invertible, injective, surjective maps. Determinant of a matrix and of an operator. Explicit formula and axiomatic definition. Minors, decomposition formulas. Resultant and discriminant. Inverse matrix, its computation.

See *http://ium.mccme.ru/f17/f17-algebra1.html* for the exercise sheets.

3.3.4 Alexei Gorodentsev

[1] Geometry. Independent University of Moscow, 1st year, semester 2, February–May 2017, 4 hours per week.
See http://gorod.bogomolov-lab.ru/ps/stud/geom_ru/1617/list.html

Program.

1. Projective spaces, homogeneous coordinates, affine charts. $\mathbb{RP}_1 = S^1$, $\mathbb{CP}_1 = S^2$, $\mathbb{RP}_2$, $\mathbb{RP}_3 = \mathrm{SO}_3(\mathbb{R})$ as topological spaces. Geometry of $\mathbb{P}_1$: linear fractional maps, cross-ratio, involutions, harmonicity. Pencils of lines on $\mathbb{P}_2$, Dezargus theorems, constructions by ruler. Geometry of conics, Pascal theorem, Ponselet porism. Complementary subspaces and projections, projective duality. The Group PGL, affine transformation group vs projective transformation group. Veronese curve. Spaces of hypersurfaces.

2. Examples of quadrics: Veronese, Segre, Plücker. Tangent lanes and tangent space of a quadric, smoothness, singular subspace. Linear subspaces laying on a smooth quadric. Space of quadrics, tangent space to a hypersurface of smooth quadrics. Pencils of quadrics. Affine an projective classification of quadrics over $\mathbb{C}$ and $\mathbb{R}$. Euclidean geometry of degree-2 curves and surfaces in $\mathbb{R}^2$ and $\mathbb{R}^3$.

3. Spinor decomposition of euclidean 4-space $\mathbb{R}^4 \otimes \mathbb{C} \simeq \mathrm{End}(\mathbb{C}^2)$ such that $\mathbb{C}$-bilinear extension of the euclidean form from $\mathbb{R}^4$ turns to the polarization of det. Ytrmitean conjugation on the space of complex 2×2-matrices. Quaternions: generators and relations, norm, division, pure imaginary quaternions of norm 1. The universal coverings $\mathrm{SU}_2 \to \mathrm{SO}_3(\mathbb{R})$ and $\mathrm{SU}_2 \times \mathrm{SU}_2 \to \mathrm{SO}_4(\mathbb{R})$ (the *geometric* approach). Hopf's bundle $\mathrm{SU}_2 = S^3 \to S^2$.

4. Linear models of elliptic and hyperbolic geometries in real projective space $\mathbb{P}(V)$: geodesics are projective lines, length and angles are computed in terms of inner product in V, the absolute consists of isotropic vectors. Conformal models of hyperbolic geometry, Lobachevski plane as the unit disk and as the upper half-plane in $\mathbb{C}$.

5. Isometry groups in Elliptic and hyperbolic geometries, isomertry group is spanned by reflections. Discrete subgroups of the isometry group of the Lobachevski plane, fundamental polygon, Poincare theorem. The modular group.

[2] Geometric introduction to algebraic geometry. Math in Moscow program of IUM joint with the Faculty of Mathematics, HSE. 1 semester course for 2-4 year students. September–December 2017, 4 hours per week.
See http://gorod.bogomolov-lab.ru/ps/stud/projgeom/1718/list.html

Program.

1. Projective spaces and projective geometry: projective groups, projections, cross-ratio, the Veronese curve. Projective quadrics: singular points and tangent spaces, linear subspaces on a quadric, polar mapping, pencils of quadrics.

2. The Veronese, Segre, and Grassmann varieties, and their projective embeddings. $\mathrm{Gr}(2,4) \subset \mathbb{P}_5$ in details. Polarizations of polynomials, linear support of a tensor, Plücker relations. Tangent spaces and polars of affine and projective hypersurfaces.

3. Commutative algebra draught: Noetherian rings, integral extensions, the Nullstellensatz, transcendence degree, resultants.

4. The anti-equivalence between the categories of affine algebraic varieties over algebraically closed field $\Bbbk$ and finitely generated reduced $\Bbbk$-algebras. Maximal spectrum and Zariski topology. Geometric properties of algebra homomorphisms.

5. The structure sheaf of affine algebraic variety and its sections over the principal open sets. Definition of an algebraic manifold. Separability. Morphisms from projective to separable varieties are closed. Blow up of a point, projection of projective variety from an external point is finite. Normalization theorems, the normalization in an algebraic family.

6. The dimension of an algebraic variety. Dimensions of fibers, semi-continuity and constructibility theorems. Generic plane section of a projective variety, geometric definition of the dimension of a projective variety. 27 lines on a smooth cubic surface.

7. Locally trivial vector bundles and locally free $\mathcal{O}_X$-modules. Line bundles and Picard group, $\mathrm{Pic}(\mathrm{Spec}\,A) = 0$ for factorial A, $\mathrm{Pic}(\mathrm{Gr}(k,n)) = \mathbb{Z}$. Linear systems and projective embeddings via line bundles. Splitting of locally trivial vector bundles on the projective line.

3.3.5 Maxim Kazarian

[1] Differential Geometry. Independent University of Moscow, II year students, January-May 2017, 4 hours per week.

Program.

Vector fields and differential forms. *Lie bracket, exterior differential and wedge product, distributions, Frobenius criterium of integrability.*

Plane and space curves. *Length, curvature, focal set of a plane curve, normal and geodesic curvature of a space curve on a surface.*

Surface geometry. *Riemann structure, IInd quadratic form, principal curvatures, Gaussian curvature.*

Gauss' Theorema Egregium. *Connection and curvature forms of a metric on a surface, Euclidean coordinates of a flat metric.*

Topological connection. *Fiber bundles, trivializations, parallel transport, curvatur as an infinitesimal holonomy.*

Covariant derivative. *Vector bundles, sections, connection matrix, structure Cartan equation, curvature tensor.*

Riemann manifolds. *Levi-Civita connection, Riemann tensor, geodesics.*

[2] Symplectic and Differential Geometry, National Reserch University Higher School of Economics, III year students, September-December 2017, 2 hours per week.

Program.

Linear symplectic algebra

Symplectic structure Darboux coordinates. Symplectic structure on the cotangent bundle space.

Hamiltonians. Symplectomorphisms. Hamilton equations. Liouville theorem. Caracteristics. Hamiltonian reduction. Poisson bracket.

Lagrangian manifolds. Generating function. Generating families.

Caustics. Caustics of system of rays. Cusps and swallowtails. Caustics as discrininants of generating families.

Elements of singularity theory. Degenerate and non-degenerate critical points. Morse lemma. Corank and Milnor number. Normal form of functions. Versal deformation. ADE-classification of simple singularities. Caustics and critical point singularities.

Contact manifolds. Contact structure. Legendrian manifolds and their fronts. Generating families of hypersurfaces.

Maslov class. Geometry of Lagrangian Grassmannian: charts and sell partition. Geometry of trails.

Lagrangian and Legendrian characteristic classes. Universal cohomological complex of singularities. Topological restrictions on the coexistence of singularities.

3.3.6 Alexander Kuznetsov

[1] "Higher-dimensional projective geometry", Spring 2017, IUM/Science-educational center MIAN.

Program.

- Lecture 1 (14.02.2017). Determinatal loci. Veronese and Segre varieties.
- Lecture 2 (21.02.2017). (by Anton Fonarev) Kähler differentials.
- Lecture 3 (28.02.2017). Grassmannians.
- Lecture 4 (07.03.2017). Relative constructions.

- Lecture 5 (14.03.2017). Blow up.
- Lecture 6 (21.03.2017). Locally complete intersections.
- Lecture 7 (28.03.2017). Examples of blowups.
- Lecture 8 (04.04.2017). Chow scheme.
- Lecture 9 (11.04.2017). Moduli spaces.
- Lecture 10 (18.04.2017). Examples of Hilbert schemes.

[2] "Fano varieties" (minicourse, 4 lectures), "Fontanka Summer School", POMI, Saint-Petersburg, July 3–8, 2017.

Abstract: I will talk about Fano varieties — one of the most important class of algebraic varieties. By definition a Fano variety is a smooth projective variety with ample anticanonical class. In a contrast with varieties of general type (varieties with ample canonical class) there is only a finite number of deformation classes of Fano varieties in each dimension, however a complete classification is only available in dimensions up to 3. Fano varieties are close to rational varieties, in particular every Fano variety is rationally connected. However, many Fano varieties are non-rational, and in general the question of rationality of Fano varieties is difficult and interesting. On the other hand, Fano varieties are extremely interesting from the derived category perspective. In particular, the derived category of coherent sheaves of a Fnao variety always admits an interesting semiorthogonal decomposition, and the structure of its component is tightly related to the geometric properties of the variety. I will discuss classification of Fano varieties in dimensions up to 3, and connections between their rationality and the structure of their derived categories.

3.3.7 Mikhail Lashkevich

General relativity theory. September – December 2017. 3 hours per week.

Program

1. Geometry and physics of special relativity theory:
   - nondegenerate symmetric form in a linear space and signature;
   - metric in an affine space, index and indexless notation;
   - principle of least action for particles and fields.

   Seminar: Lorentz transformation, coordinate systems, metric.
2. Geometry of pseudo-Riemannian space-time:

- affine connection and covariant derivatives;
- Levi–Civita connection;
- Riemann curvature tensor

Seminar: Connection, metric, curvature: examples

3. Particles and fields in a curved space-time:

   - a particle in a curved space-time, geodesics, external electromagnetic field;
   - Hamilton–Jacobi equation;
   - fields in a curved space-time, canonical energy-momentum tensor.

   Seminar: Physical interpretation of metric: time and space intervals, synchronization of clocks.

4. Equations of gravitation field:

   - transformation of fields, Lie derivative;
   - metric energy-momentum tensor, its connection to the canonical one;
   - Hilbert–Einstein action and Einstein equations, their structure;

   Seminar: Energy-momentum tensor for various physical systems

5. Conservation laws in general relativity:

   - Einstein and Landau–Lifshitz energy-momentum pseudotensor;
   - superpotential;
   - energy, momentum and angular momentum as surface integrals.

   Seminar: Symmetries and Killing vector fields.

6. Weak gravitational field:

   - linearized Einstein equations, gauge fixing;
   - static solutions to linearized Einstein equations, residual gauge freedom;
   - energy and angular momentum via asymptotics of gravitational field.

   Seminar: Problems on gravitational field in the linear approximation.

7. Gravitational waves:

   - free solutions of homogeneous linearized Einstein equations;

- plane waves, monochromatic waves, gauge fixing, polarizations;
- energy-momentum pseudotensor of a plane gravitational wave.

Seminar: Other cases of gravitational wave in vacuum: gravitational waves on a curved background and strong gravitational waves.

8. Emission of gravitation waves:

   - retarded solution to the wave equation, simplification in dimension 4;
   - non-relativistic source: restrictions by the energy-momentum conservation and quadrupole emission;
   - angular distribution and calculation of the total emitted energy.

   Seminar: Interaction of gravitational waves with matter: examples.

9. Schwartzschild solution:

   - spherically symmetric Einstein equation, its direct solution;
   - geodesics in the Schwartzschild metric, incompleteness of the Schwartzschild coordinates;
   - Eddington–Finkelstein coordinates, event horizon and singularity;
   - Kruskal coordinates and Penrose diagram, maximally extended Kruskal manifold.

   Seminar: Gravitational field of static spherically symmetric body, static equilibrium condition.

10. Motion of a particle in the Schwartzschild metric:

    - solution of the Hamilton–Jacobi equation;
    - four types of motion in the Schwartzschild metric, conditions for their realization.

    Seminar: Hamilton–Jacobi equation: variable separation method, integrability, Liouville theorem, examples.

11. Motion in a rather weak gravitational field and experimental checks of general relativity:

    - nearly-Newtonian field and perihelion precession;
    - deviation of a light beam in a weak gravitational field.

    Seminar: Fall of a layer of dustlike matter on a black hole.

12. Charged and rotating black holes:
    - Reissner–Norstrm solution of Maxwell and Einstein equations;
    - singularity, event horizon, Cauchy horizon, Penrose diagram;
    - Kerr–Newman solution, ergosphere, ring singularity, Penrose diagram.

    Seminar: last seminar continued.

13. Cosmological solutions. Friedmann models:
    - homogeneous and isotropic Universe, constant curvature spaces;
    - Friedmann equations, their solutions for dustlike matter and ultrarelativistic gas;
    - Cosmological constant and accelerating expansion, modern model of expanding Universe, dark matter and dark energy.

    Seminar: Properties of isotropic hypersurfaces, images invariance, focusing, optical scalars.

14. De Sitter spaces and inflation models:
    - problems of models with initially decelerating expansion: flatness problem, homogeneity and isotropy problem, horizons problem;
    - primordial accelerating expansion: inflation;
    - de Sitter spaces of 1st and 2nd (AdS) order;
    - possible physical origins of inflation, equation for a homogeneous scalar field in an expanding Universe.

3.3.8 Maxim Leyenson

Introduction into algebraic surfaces – 1. Independent University of Moscow, 2nd-5th year students, September-December 2017, 2 hours per week.

Program

- Examples of algebraic surfaces, and some basic numerical invariants.
- Smooth surfaces of degree d in $\mathbb{P}^3$, and their Hodge numbers. Quadric surfaces are rational.

- Cubic surfaces in $\mathbb{P}^3$
  - ⋆ Linear system of plane cubic curves through 6 points on $\mathbb{P}^2$, and corresponding cubic surfaces. E_6
- Riemann-Roch theorem on a surface – easy variant, (without Noether equality), with a proof.
- Pluricanonical linear systems. Kodaira dimension.
- Notion of a rational surface. Hirzebruch surfaces F_n, and vector bundles of rank two on a line.
- Examples of birational isomorphisms. Cremona transformations of the plane.
- Ruled surfaces, briefly.
- Rational equivalence and Chern classes.
  - ⋆ Rational equivalence of 0-cycles on a surface
  - ⋆ Chern classes of a line bundle on a surface (with values in Chow groups)
  - ⋆ Chern classes of a split vector bundle
  - ⋆ Vector bundles with a filtration, and splitting principle.
  - ⋆ Porteus definition of Chern classes.
- Max Noether's equality, and the stromg form of the Riemann-Roch theorem on a surface (with idea of the classical proof.)
- Elliptic surfaces: examples. Degenerate fibers.
- Abelian surfaces: examples.
- K3 surfaces.
  - ⋆ Space quartics, and the notion of K3-surfaces.
  - ⋆ Classical definition of K3 surfaces.
  - ⋆ K3 surfaces of genus g.
  - ⋆ Examples of K3 surfaces of genus 2, 3, 4 and 5.
- Notion of surfaces of general type.
- Classification theorem of algebraic surfaces, with sketch of a proof.
- Surfaces of general type – 2.

- ⋆ Surface geography.
- ⋆ Some surfaces of general type.
- ⋆ Bogomolov's inequality (statement only).

3.3.9 Grigory Olshanski

[1] Representations and Probability. Joint seminar. National Research University Higher School of Economics and Independent University of Moscow.

January–April 2016, 2 hours per week.

Program:

Lectures at the research seminar on combinatorial, algebraic, and probabilistic aspects of representation theory. Main topics: positive definite kernels; conditionally negative definite kernels; isometric embeddings of metric spaces into a real Hilbert space; Chentsov's geometric construction; connections with unitary representations; Bochner's theorem; Gaussian measures; random processes; symmetric functions; applications to characters of the infinite-dimensional unitary group.

3.3.10 Taras Panov

[1] Geometry. Independent University of Moscow, I year students, September-December 2017, 2 hours per week.

Program:

1. Linear spaces.
2. Affine geometry.
3. Euclidean geometry.
4. Groups of transformations.
5. Convex geometry.

http://higeom.math.msu.su/people/taras/teaching/panov-geometry.pdf

[2] Linear algebra and geometry, Department of Mathematics and Mechanics, Moscow State University, I year students, February–May 2017, 4 hours per week.

Program:

1. Vector spaces.
2. Linear operators.
3. Geometry of Euclidean and Hermitian spaces.
4. Operators in Euclidean and Hermitian spaces.
5. Bilinear functions.
6. Tensors.

http://higeom.math.msu.su/people/taras/teaching/panov-linalg.pdf

[3] Algebraic topology (advanced course), Department of Mathematics and Mechanics, Moscow State University, II–VI year students, September–December 2017, 2 hours per week

Program:
1. Simplicial homology.
2. Singular homology.
3. Cellular homology.
4. Homotopy groups and homology groups.
5. Cohomology and multiplication.

http://higeom.math.msu.su/people/taras/teaching/panov-topology2.pdf

3.3.11 Alexei Penskoi

[1] Differential geometry, National Research University — Higher School of Economics, 2-4 year students, January-May 2017, 4 hours per week (lecture 2 hours + exersise class 2 hours).

Program.

1. Curves and surfaces in the plane and the three-dimensional space. Curvature, torsion, Frenet frame. First and second fundamental forms. Principal curvatures, mean curvature and Gauß curvature. Mean curvature normal vector. Euler formula for the normal section curvature.

2. Surfaces in n-dimensional space. First and second fundamental forms. Connections in the tangent and normals bundles on a surface. Second fundamental form and Weingarten operator. Gauß-Weingarten derivational equations. Gauß-Bonnet theorem for surfaces.

3. Basic theory of Lie groups ans algebras.

4. Vector bundles and gluing cocycles. Structure group. Euclidean and hermitian bundles. Natural operations with bundles. Orientable bundles.

5. Connections in vector bundles. Connection local form, Christoffel symbols. Connections in euclidean and hermitian bundles. Connections compatible with metrics and their curvature.

6. Riemannian manifolds. Curvature, torsion. Levi-Civita connection. Symmetries of curvature tensor. Ricci tensor. Scalar curvature.

7. Riemannian manifolds II. Geodesics. Geodesic coordinates. Lagrangian approach to geodesics. Second variation.

8. Submanifolds of Riemannian manifolds. First and second fundamental forms.
9. Laplace-Beltrami operator and minimal submanifolds, Takahashi theorem.
10. Characteristic classes. Chern-Weil construction of characteristic classes. Chern, Pontryagin and Euler classes and their properties.

[2] Differential geometry, Moscow State University, 3 year students, September-December 2017, 8 hours per week (lecture 2 hours + exersise class 6 hours).

Program.

1. Reminiscences from Calculus: implicit function theorem, inverse function theorem, rank theorem. Surfaces in affine spaces and different ways of their definition.
2. Smooth manifolds. Partition of unity. Maps of manifolds.
3. Tangent vectors and differential of a map. Tangent and cotangent spaces.
4. Immersions, submanifolds, submersions.
5. Vector fields. Commutator of vector fields. Integral curves of a vector filed. One-parametric group generated by a vector field.
6. Tensor fields, differential forms. Riemann metric, volume form. Exterior differential.
7. Relation between d and grad, rot and div.
8. Orientation of a manifold. Integration of forms over manifolds.
9. Manifolds with boundary. Stokes theorem for manifolds with boundary. Relation to Green, Stokes and Gauß-Ostrogradsky formulas in calculus.
10. De Rham cohomologies. Poincaré lemma. Mayer-Vietoris long exact sequence.
11. Properties of de Rham cohomologies (finite dimension, Künneth formula etc).
12. Vector bundles.
13. Connections in vector bundles.
14. Levi-Civita connection.
15. Curvature operator, curvature tensor.
16. Parallel transport.
17. Geodesics, exponential map.

[3] Riemannian geometry, Moscow State University, 4-6 year students, February-May 2017, 2 hours per week.

Program.

1. Riemannian manifolds
2. Riemannian curvature
3. Riemannian coverings
4. Riemannian geometry of surfaces
5. Isoperimetric inequalities
6. Comparison theorems

[4] Complex analytic manifolds and holomorphic vector bundles-I, Independent University of Moscow, 3-6 year students, September-December 2017, 2 hours per week.

Program.

1. Complex analytic manifolds and holomorphic vector bundles
2. Complex analysis of several variables
3. Sheafs
4. Connections and its curvature
5. Characteristic classes

[5] Harmonic maps, Independent University of Moscow, 3-6 year students, February-May 2017, 2 hours per week.

Program.

1. Energy functional and Euler-Lagrange equation for energy. Harmonic maps.
2. Harmonic and minimal isometric immersions.
3. Maps to Riemannian manifolds, case of S^2.
4. Maps of surfaces to surfaces, case of maps $S^2 \longrightarrow S^2$.
5. Existence and regularity results.
6. Harmonic maps $S^2 \longrightarrow S^n$, Calabi and Barbosa theorems.
7. Harmonic maps $S^2 \longrightarrow S^4$, $S^2 \longrightarrow S^6$, Bryant theorem.

[6] Calculus on manifolds. "Math in Moscow" program at the Independent University of Moscow for undergraduate students from the U.S. and Canada, February-May 2017, 4 hours per week (lecture 2 hours + exersise class 2 hours).

Program

1. Definition and examples of smooth manifolds.
2. Orientability and orientation.
3. Tangent vectors and tangent space to a manifold at a point. Tangent bundles. Vector fields.
4. Skew-symmetric forms on linear spaces. Wedge product.
5. Differential forms on manifolds. Exterior differential.
6. Smooth maps of manifolds. Diffeomorphisms. The transformation rule under coordinate change for functions, vector fields and differential forms.
7. Integration. Coordinate change in the integral. Integration of differential forms. Stokes theorem. Green's formula, Gauss-Ostrogradskii divergence theorem, Stokes formula for a surface in $\mathbb{R}^3$.
8. Closed and exact forms. The Poincare lemma. De Rham cohomology.

[7] Calculus on manifolds. "Math in Moscow" program at the Independent University of Moscow for undergraduate students from the U.S. and Canada, September-December 2017, 4 hours per week (lecture 2 hours + exersise class 2 hours).

Program as in [6].

[8] Exercise classes at Moscow State University: Classical Differential Geometry, February-May 2017, 2 hours per week.

[9] Exercise classes for various courses at National Research University — Higher School of Economics: Calculus-I, September-December 2017, 4 hours per week, Calculus on Manifolds, September-December 2017, 2 hours per week, Topology-I, January-March 2017, 2 hours per week. Topology-I, December 2017, 3 hours per week.

3.3.12 Petr Pushkar'

[1] Complex Analysis. Independent University of Moscow, 2 year students, January-May 2017, 4 hours per week.

Program

1. Complex-valued functions. Holomorphic functions. CauchyRiemann equations.
2. Holomorphic forms. Cauchy Theorem. Expansion as a convergent power series.

3. Meromorphic functions. Loran series.
4. Maximum principle, Cauchy's argument principle, open mappings.
5. Residues.
6. Isolated singlar points. The CasoratiWeierstrass theorem
7. Schvarz lemma. Automorphisms.
8. Uniformization theorem.
9. Holomorphic and harmonic functions.
10. Riemmanian surfaces. Elements of elliptic functions theory. Abel theorem.

[2] Ordinary Differential Equations, 2 year students, September-December 2017, 4 hours per week.

Program

1. Tangent vectors, Vector fields
2. Main theorems
3. Linear systems and equations, exponential function, Quasi-polynomials
4. Linearization and Lyapunov stability
5. Obstruction to integrability of distributions
6. First-order differential equations
7. Examples.

3.3.13 Leonid Rybnikov

[1] Representations of Lie Algebras (joint with Boris Feigin). Independent University of Moscow (joint with PhysTech), 3 year students (mostly from PhysTech), September-December 2017, 3 hours per week.

Program.

1. Basic notions of Representation Theory. Schur Lemma.
2. Finite dimensional representations of a 1-dimensional Lie algebra.
3. Heisenberg group and Heisenberg Lie algebra. Finite dimensional and infinite dimensional representations.
4. Algebra of differential operators and its representations. Global differential operators on $\mathbb{CP}^1$. Some representations of $\mathfrak{sl}_2$.
5. Clifford algebra (*).
6. Universal enveloping algebra and Casimir elements.

7. Finite dimensional representations of $\mathfrak{sl}_2$ and their characters.

8. Verma modules for $\mathfrak{sl}_2$. Casimir eigenvalues on highest weight representations. Application to Harmonic Analysis on S^2.

9. Tensor product of representations of $\mathfrak{sl}_2$.

10. Generalization to $\mathfrak{sl}_3$. Weight diagrams and characters of finite dimensional representations.

11. Verma modules and parabolically induced modules for $\mathfrak{sl}_3$.

[2] Algebra-2, NRU HSE, Department of Mathematics, I year students, January–June 2017, 3 hours per week.

Program.

1. Finite groups. Actions of finite groups on sets. Burnside's formula.

2. Sylow's theorems.

3. classification of finitely generated Abelian groups.

4. Finitely generated modules over Euclidean rings.

5. Jordan normal form.

6. Normal form of self-adjoint and orthogonal operators.

7. Polynomial rings. Gauss Lemma.

8. Noetherian rings. Hilbert's Theorem.

9. Symmetric polynomials.

10. Tensor products. Convolutions and natural isomorphisms.

11. Symmetric and exterior algebras.

3.3.14 Stanislav Shaposhnikov

1) Mathematical calculus. Independent University of Moscow, 1 year students, September – December 2017, 4 hours per week.

Program:

1. Sets. Functions. Equivalence relations. Partially or linearly ordered sets. Mathematical induction.
2. Real numbers: an axiomatic approach. Complex numbers.
3. Sequences and series. The Cauchy sequences.
4. Real numbers are the completion of the rational numbers. p-adic numbers.
5. Topology of the real line. Compact sets. The Cantor set.
6. Continuous functions.
7. Pointwise and uniform convergence.
8. Differentiable functions. The Weierstrass function.
9. Taylor series.

3.3.15 George Shabat

[1] Algebraic curves. Independent University of Moscow, 3-5 year students, February-May 2017, 2 hours per week.

The program

0. Brief historic introduction

0.0. Algebraic curves in the antiquity
0.1. Fermat's descent
0.2. Newton and cubic curves
0.3. Euler and elliptic integrals
0.4. Gauss and agm
0.5. The Abel-Jacobi addition theorems
0.6. Riemann and moduli spaces
0.7. Italian algebraic geometry
0.8. Abstract algebraic manifolds (A. Weil)
0.9. Deligne-Mumford moduli spaces
0.10. Non-unirationality of moduli spaces (Harris-Mumford)

1. Algebro-geometric basics

1.0. "Ground" field and its extensions; transcendency degree
1.1. Grothendieck (pre)schemes
1.2. Projective varieties: irreducibility, dimension

4.0. Moduli spaces
4.1. Effective Mordell
4.2. Periods and Schottky's problem
4.3. Counting curves over number fields
4.4. The theory of Witten–Kontsevich–...

[2] Mordell conjecture – Faltings theorem. Independent University of Moscow, 3-5 year students, September-December 2017, 2 hours per week.

Program

1. Introduction.
2. Individual equations and families of equations
3. Parshin's trick
4. Generalized jacobians
5. Shafarevich implies Mordell
6. Weak Mordell
7. Galois cohomology and applications
8. Heights
9. Problems of effectiveness
10. Overview of the alternative proofs

3.3.16 Arkady Skopenkov

A list of university courses taught by A. Skopenkov in 2017

[1] Discrete structures and algorithms in topology, III year students, September-December 2017, 4 hours per week. Moscow Institute of Physics and Technology (DIHT)

Program. It is shown how in the course of solution of interesting geometric problems (close to discrete mathematics and computer science) naturally appear main notions of algebraic topology (homology groups, obstructions and invariants). Thus main ideas of algebraic topology are presented with minimal technicalities.

Detailed information in Russian: `http://www.mccme.ru/circles/oim/home/combtop13.htm#combtop14`

[2] Algorithms for recognition of realizability of graphs and hypergraphs, Independent University of Moscow, February-May 2016, 2 hours per week.

Program. It is shown how in the course of solution of interesting geometric problems (close to discrete mathematics and combinatorial geometry) naturally appear main notions of algebraic topology (homology groups, obstructions and invariants). Thus main ideas of algebraic topology are presented with minimal technicalities.

Detailed information in Russian: `http://www.mccme.ru/circles/oim/home/combtop13.htm#nmuspr15`

[3] Classification of links, Independent University of Moscow, September-December 2016, 2 hours per week.

Program. Some examples, invariants and classification results for links are presented. We study both the classical case of 1-links in $\mathbb{R}^3$ and higher-dimensional generalization.

Detailed information in Russian: http://www.mccme.ru/circles/oim/home/combtop13.htm#link

[4] Basic topology, II year students, September-December 2017, 2 hours per week. Moscow Institute of Physics and Technology (DIHT)

Program. It is shown how in the course of solution of interesting geometric problems (close to discrete mathematics) naturally appear main notions of algebraic topology (2-dimensional manifolds, continuous maps, obstructions and invariants). Thus main ideas of algebraic topology are presented with minimal technicalities.

Detailed information in Russian: http://www.mccme.ru/circles/oim/home/combtop13.htm#fivt

[5] Introduction to topology, IV-V year students, September-December 2017, 2 hours per week. Moscow Institute of Physics and Technology (DIHT)

Program. It is shown how in the course of solution of interesting geometric problems (close to physics) naturally appear main notions of algebraic topology (homology groups, obstructions and invariants). Thus main ideas of algebraic topology are presented with minimal technicalities.

Detailed information in Russian: http://www.mccme.ru/circles/oim/home/combtop13.htm#fopf15

[6] Discrete analysis (exercises), II year students, February-December 2017, 2 hours per week. Moscow Institute of Physics and Technology (DIHT)

Program. We study certain topics in combinatorics and graph theory (including random graphs).

Detailed information in Russian: http://www.mccme.ru/circles/oim/home/discran1314.htm

Other educational activities by A. Skopenkov in 2017

[7] International Summer Conference of Tournament of Towns, Jury member, June-August, Moscow-Adyghea. Detailed information:

http://www.turgor.ru/en/lktg/index.php

[8] Moscow Mathematical Conference of High-school Students, Programme Committee member, September-December, Moscow. Detailed information in Russian: http://www.mccme.ru/mmks/index.htm

[9] Seminar on advanced mathematical education (A.D. Blinkov), Talk ‘Elements of topology in high-school teaching’.

[10] A course on ‘special’ mathematics for high-school students, high-school ‘Intellectual’, January-December. Detailed information in Russian:

http://www.mccme.ru/circles/oim/index.htm#il

[11] Math circle ‘Olympiads and Mathematics’ for high-school students, MCCME, January-December. Detailed information in Russian:

http://www.mccme.ru/circles/oim/index.htm#oim

[12] Minicources on mathematics for high-school students, Moscow ‘olympic’ schools, April, June and November, Moscow region.

3.3.17 Mikhail Skopenkov

[1] Quarks game. Independent University Moscow, I-III year students, Fall 2017, 2 hours per week. The course is the one supported by the fellowship.

This is a play-based introduction to basic ideas of field theory describing elementary particles. It is going to give better intuition in both physics and mathematical branches such as differential geometry and complex analysis. For each particular theory and each new notion, we try to show how they appear naturally in a solution of a practical problem and which are their further applications. This makes most of the objects visual and simple.

The material is studied via problem solving, with detailed hints and discussion in classes. No prerequisites in physics are assumed; knowledge of school-level mathematics is sufficient. The course is accessible for 1st year students.

Program.

1. Toy model of lattice gauge theory: exchange of goods between cities. Relation to magnetic field. Quantization: random exchange rates. Exact solution of 1- and 2-dimensional lattice gauge theory. Numerical analysis in dimension 3 and 4. An example of non-Abelian gauge theory. Confinement of quarks. The essence of the problem in Yang-Mills theory (one of the “Millenium problems”). Higgs mechanism*. Strong- and weak-coupling expansions*.

2. Mathematical model of an electrical network - the simplest lattice field theory. Existence and uniqueness of the potential in an electrical network. Maximum principle. Energy conservation. Variational principle. Magnetic field. Relation to toy gauge theory. Discrete harmonic and discrete analytic functions. Electromagnetic field*. Discrete Maxwell’s equations*.

3. Feynman’s checkerboard: the simplest model of an electron. Spin. Discrete Dirac’s equation*. Convergence of Feynman’s checkerboard to Dirac’s theory*.

[2] Discrete mathematics. Independent University Moscow, I-III year students, Spring 2017, 2 hours per week. The course is also supported by the fellowship.

Program (short version).

1. Intro to combinatorics: counting, the Pascal triangle, linear recurrences, generating functions.

2. Intro to probability: classical probability, independence, random variables, the Bernoulli process, law of large numbers.

[3] Introduction to discrete mathematics. Higher School of Economics, I year students, Spring 2017, 4 hours per week.

Program (short version).

1. Intro to combinatorics: counting, the Pascal triangle, linear recurrences, generating functions.

2. Intro to probability: classical probability, independence, random variables, the Bernoulli process, law of large numbers.

3. Intro to graphs: number of edges, paths and cycles, connected graphs, trees.

[4] Random walks and electric networks, Higher School of Economics, I year students, Spring 2017, 2 hours per week.

Program.

1. Definition of an electric network.

2. The existence and uniqueness of a potential in an electric network. Conductance.

3. Physical interpretation of dissections of a rectangle into squares. The Dehn theorem on tiling of a rectangle.

4. Definition of a random walk. Physical interpretation of the hitting probability. The 1-dimensional random walk is recurrent.

5. Conductance and its probabilistic interpretation. Energy conservation. The variational principle. The short-cut principle. Conductance between the center and the boundary of a square lattice $n \times n$. The 2-dimensional random walk is recurrent.

7. Conductances of trees. The 3-dimensional random walk is not recurrent.

In addition to [1]–[4]: assistance teaching in basic courses at Higher School of Economics (I-III year students, 4 hours per week throughout 2017), supervision of 9 students, supervision of a pupil for Moscow Mathematical Conference for School Pupils, popular-science lectures for pupils in 6 schools in Moscow, Sankt-Petersburg, Saratov.

3.3.18 Evgeni Smirnov

[1] Linear algebra, 2nd year, 1st semester, Higher School of Economics, Department of Physics, September–December 2017, 2 hours of lectures and 2 hours of exercise sessions per week

Course outline:

1. Systems of linear equations. Gaussian elimination, row-echelon form.
2. Vector spaces, subspaces. Linear dependence and independence.
3. Linear maps, matrices, multiplication of matrices
4. Determinants.
5. Eigenvectors, eigenvalues, diagonalization of linear operators.
6. Bilinear forms, Jacobi's theorem, Sylvester's criterion.
7. Euclidean and Hermitian spaces.

8. Tensors, tensor product of vector spaces.

[2] Algebra, 1st year, 2nd semester, Independent University of Moscow, February–May 2017, 2 hours of lectures and 2 hours of exercises per week

Course outline:

1. Bilinear and quadratic forms.
2. Gram–Schmidt orthogonalization.
3. Hermitian, skew-Hermitian, unitary operators.
4. Quadratic residues, Gaussian reciprocity. Legendre–Jacobi symbol.
5. p-adic numbers, Hänsel's lemma.
6. Legendre's theorem. Hilbert's symbol.
7. Minkowski–Hasse theorem.
8. Tensor product of vector spaces.
9. Tensor product of modules over a ring.
10. Representations of finite groups. Characters.
11. Representations of quivers.

[3] Algebra, 2nd year, 3rd semester, Independent University of Moscow, September–December 2017, 2 hours of lectures and 2 hours of exercises per week

Course outline:

1. Field extensions. Finite and algebraic extensions.
2. Transcendence of e and π.
3. Decomposition field of a polynomial. Separability
4. Galois extension. Main theorem of Galois theory.
5. Cyclotomic and abelian extensions.
6. Solvability and insolvability in radicals.
7. Representation theory of associative algebras: main notions.
8. Jacobson's double centralizer theorem.
9. Symmetric group algebra, Hecke algebras.
10. Representations of symmetric groups: the approach of Vershik and Okounkov.

3.3.19 Dmitry Talalaev

[1] Integrable models of statistical physics and some problems of low-dimensional topology. Independent University of Moscow, Special curse, September-December 2017, 2 hours per week.

1. Statement of the problem of constructing invariants of knots, 2-knots, 4-dimensional manifolds

2. Description of the classical methods of investigating the invariants of knots a. The fundamental group of the complement b. Alexander polynomial c. Jones polynomial

3. Quandles, their cohomology and applications to the problem of 1-knot invariants

4. A Brief Introduction to Quantum Groups a. Bialgebras, Hopf algebras b. The Yang-Baxter equation c. Yangians d. Some facts from the representation theory

5. A Brief Introduction to Statistical Physics a. The Gibbs paradigm b. Physical phenomena of integrable statistical models c. The Bethe ansatz method d. Thermodynamic limit

6. Invariants of the Turaev-Reshetikhin type

7. The Jones polynomial and the representation of the Temperley-Lieb algebra

8. Quandles and 2-knot invariants

9. The tetrahedral equation of Zamolodchikov a. Physical interpretation b. Examples of solutions c. Higher braids d. Cluster implementation

10. Three-dimensional integrable statistical models

11. Quasi-invariants of 2-knots

12. Combinatorial 4-dimensional topological field theories

[2] Analytic geometry, Moscow State University, I year students, September-December 2017, 4 hours per week.

1. Coordinate systems on the plane and in space
2. Lines in the plane
3. Lines and planes in the space
4. Affine and orthogonal coordinate changes
5. Curves of the second order
6. Surfaces of the second order
7. Projective classification of second-order curves

3.3.20 Ilya Vyugin

[1] Calculus II (lectures and seminars). Independent University of Moscow, second year students, January-June, 2017, 2+2 hours per week.

Program

1. Numerical and functional series.
2. Lebesgue measure.
3. Measurable functions.
4. The Lebesgue integral.
5. Fubini's theorem and the Radon-Nikodym theorem.
6. The space L_2.
7. Orthogonal systems of functions. Fourier series.

[2] Calculus on manifolds (lectures and seminars). Independent University of Moscow, I year students, September-December, 2017, 2+2 hours per week.

Program

1. Manifolds. Tangent vectors.
2. Submanifolds.
3. Differential forms on manifolds.
4. Commutators. Frobenius theorem.
5. Integral of the differential form. Stokes theorem.
6. De Rham cohomology. Poincaré theorem.
7. Sard's theorem.Thom's transversality theorem.

[3] Variation Calculus and Optimal Control (lectures) Course in HSE, January-June, 2017.

Program

1. Necessary condition for an extremum. Euler–Lagrange equation.
2. Lagrange multiplier method
3. Sufficient condition for an extremum of linear functional. Theory of the second variation.
4. Fundamentals of the theory of optimal control. Pontryagin's maximum principle.
5. Elements of convex analysis.
6. Applications.

[4] Complex Differential Equations (joint with V. Poberezhny) Special cours in HSE, 2017.

Program

Distributions. Integrability. Symmetries.