The IUM report to the Simons foundation, 2019

Contents

1 Introduction: list of awardees

The Simons foundation supported two programs launched by the IUM:
Simons stipends for students and graduate students;
Simons IUM fellowships.

13 applications were received for the Simons stipends contest. The selection committee consisting of *Yu.Ilyashenko (Chair), G.Dobrushina, G.Kabatyanski, S.Lando, I.Paramonova (Academic Secretary), A.Sossinsky, M.Tsfasman* awarded Simons stipends for 2019 year to the following students and graduate students:

1. Abramyan Semyon Arturovich
2. Guseva, Lyalya Andreevna
3. Kalmynin, Alexander Borisovich
4. Krutovskii, Roman Vladimirovich
5. Loginov, Konstantin Valerevich
6. Matushko, Maria Georgievna
7. Prikhodko, Artem Nikolaevich
8. Sagdeev, Arsenii Alexeevich

19 applications were received for the Simons IUM fellowships contest for the first half year of 2019 and 21 applications were received for the second half year. The selection committee consisting of *Yu.Ilyashenko (Chair), G.Dobrushina, B.Feigin, I.Paramonova (Academic Secretary), A.Sossinsky, M.Tsfasman, V.Vassiliev* awarded

Simons IUM-fellowships for the first half year of 2019 to the following researches:

1. Adler, Vsevolod Eduardovich
2. Burman, Yurii Mikhailovich
3. Fedorovsky, Konstantin Yuryevich
4. Krasilshchik, Iosif Semenovich
5. Litvinov, Alexei Victorovich
6. Panov, Taras Evgenyevich

7. Penskoi, Alexei Victorovich
8. Poberezhny, Vladimir Andreevich
9. Pushkar, Petr Evgenyevich
10. Shabat, George Borisovich
11. Sharygin, George Igorevich
12. Skopenkov, Arkadii Borisovich

Simons IUM-fellowships for the second half year of 2019 to the following researches:

1. Belavin, Alexander Abramovich
2. Burman, Yurii Mikhailovich
3. Gorchinsky, Sergei Olegovich
4. Lashkevich, Mikhail Yuryevich
5. Leenson, Maxim Ilyich
6. Penskoi, Alexei Victorovich
7. Pugai, Yaroslav Petrovich
8. Pushkar, Petr Evgenyevich
9. Shabat, George Borisovich
10. Shaposhnikov, Stanislav Valeryevich
11. Smirnov, Evgeni Yurevich
12. Sossinsky, Alexei Bronislavovich
13. Khoroshkin, Anton Sergeevich

The report below is split in two sections corresponding to the two programs above. The first subsection in each section is a report on the research activities. It consists of the titles of the papers published or submitted in the year of 2019, together with the corresponding abstracts. The second subsection of each section is devoted to conference and some most important seminar talks. The last subsection of the second section is devoted to the syllabi of the courses given by the winners of the Simons IUM fellowships. Most of these courses are innovative, as required by the rules of the contest for the Simons IUM fellowships.

The Independent University remains one of the most active centers of Moscow Mathematical life. There is no room here to list its main activities. We only mention that, out of the 17 last winners of the very prestigious Moscow Mathematical Society Prize for young mathematicians, 10 are the alumni of the IUM. Amidst them is Natalya Goncharuk, graduated 2012, awarded 2018, the first women ever to receive this Prize.

The support of the Simons foundation have drastically improved the financial situation at the IUM, and the whole atmosphere as well. On behalf of the IUM, I send my deep gratitude and the best New year wishes to Jim Simons, Yuri Tschinkel, and the whole team of the Simons foundation.

Yulij Ilyashenko

President of the Independent University of Moscow

2 Program: Simons stipends for students and graduate students

2.1 Research

2.1.1 Semyon Abramyan

[1] With T. Panov

Whitehead products in moment-angle complexes and substitution of simplicial complexes. Tr. Mat. Inst. Steklova, 305, Steklov Math. Inst. RAS, Moscow, 2019, 7–28 (in Russian; Proc. Steklov Inst. Math., 305 (2019), 1–21.

We describe the simplicial complex $\partial\Delta_w$ which realises a given general iterated higher Whitehead bracket $w \in \pi(\mathcal{Z}_{\mathcal{K}})$. For a particular form of brackets inside w, we prove that $\partial\Delta_w$ is the smallest complex that realises w. We describe the canonical cellular chain which represents the Hurewicz image of a general iterated higher Whitehead product w. Using coalgebraic Taylor resolution we describe another canonical representative for the Hurewicz image of a general iterated higher Whitehead product w in terms of missing faces of simplicial complex $\mathcal{K}$.

2.1.2 Lyalya Guseva

[1] On the derived category of $\mathrm{IGr}(3, 8)$

arXiv:1810.07777 *to appear in Sbornik: Mathematics.*

In this paper we construct a full exceptional collection of vector bundles in the bounded derived category of coherent sheaves on the Grassmannian $\mathrm{IGr}(3, 8)$ of isotropic 3-dimensional subspaces in an 8-dimensional symplectic vector space.

2.1.3 Alexander Kalmynin

[1] Large values of short character sums
Journal of Number Theory, 2019, Vol. 198, p. 200-210, https://doi.org/10.1016/j.jnt.2018.09.027

In this paper, we prove that for any $A > 0$ there exist infinitely many primes p for which sums of the Legendre symbols modulo p over an interval of length $(\ln p)^A$ can take large values.

[2] Intervals between consecutive numbers which are sums of two squares
Mathematika, 2019, Vol. 65, No. 4, p. 1018-1032, https://doi.org/10.1112/S0025579319000299

In this paper, we improve the moment estimates for the gaps between numbers that can be represented as a sum of two squares of integers. We consider certain sum of Bessel functions and prove the upper bound for its weighted mean value. This bound provides estimates for the γ-th moments of gaps for all $\gamma \leq 2$.

[3] (with S. V. Konyagin) Large gaps between sums of two squares, arXiv:1906.09100

We study the properties of a sequence c_n defined by the recursive relation

$$\frac{c_0}{n+1} + \frac{c_1}{n+2} + \ldots + \frac{c_n}{2n+1} = 0$$

for $n \geq 1$ and $c_0 = 1$. This sequence also has an alternative definition in terms of certain norm minimization in the space $L^2([0,1])$. We prove estimates on the growth order of c_n and the sequence of its partial sums, infinite series identities, connecting c_n with the harmonic numbers H_n and also formulate some conjectures based on numerical computations.

[4] On symmetry graph of prime numbers, submitted to journal "Integers".

A pair $\{p, q\}$ of odd primes is called symmetric if $|p - q| = (p - 1, q - 1)$. Symmetry graph of primes is the graph whose vertices are primes and there is an edge between p and q if and only if $\{p, q\}$ is a symmetric pair. In this paper we partially answer some questions regarding properties of symmetry graph of primes stated in a recent article by W. Banks, P. Pollack and C. Pomerance. In particular, we provide an explicit example of connected component of this graph that has two vertices and prove that Dickson's conjecture implies existence of connected components isomorphic to a given finite connected graph.

2.1.4 Roman Krutovskii

[1] With T. Panov
Dolbeault cohomology of complex manifolds with torus action
arxiv: 1908.06356

In this paper we describe the basic Dolbealut cohomology algebra of the canonical foliation on a class of complex manifolds with a torus symmetry group. This class includes complex moment-angle manifolds, LVM- and LVMB-manifolds and, in most generality, complex manifolds with a maximal holomorphic torus action. We also provide a dga model for the ordinary Dolbeault cohomology algebra. The Hodge decomposition for the basic Dolbeault cohomology is proved by reducing to the transversely Khler (equivalently, polytopal) case using a foliated analogue of toric blow-up.

2.1.5 Konstantin Loginov

[1] On non-rational fibers of del Pezzo fibrations.

Mathematical Notes 106 (6), 2019.

We consider threefold del Pezzo fibrations over a curve germ whose central fiber is non-rational. Under the additional assumption that the singularities of the total space are at worst ordinary double points, we apply a suitable base change and show that there is a 1-to-1 correpspondence between such fibrations and certain non-singular del Pezzo fibrations equipped with a cyclic group action.

[2] On semistable degenerations of Fano varieties.

arXiv e-print, 1909.08319, *submitted to European Journal of Mathematics.*

Consider a family of Fano varieties $\pi : X \longrightarrow B \ni o$ over a curve germ with a smooth total space X. Assume that the generic fiber is smooth and the special fiber $F = \pi^{-1}(o)$ has simple normal crossings. Then F is called a semistable degeneration of Fano varieties. We show that the dual complex of F is a simplex of dimension $\leq \dim F$. Simplices of any admissible dimension can be realized for any dimension of the fiber. Using this result and the Minimal Model Program in dimension 3 we reproduce the classification of the semistable degenerations of del Pezzo surfaces obtained by Fujita. We also show that the maximal degeneration is unique and has trivial monodromy in dimension ≤ 3.

2.1.6 Maria Matushko

[1] With S. Khoroshkin
Fermionic limit of the Calogero-Sutherland system
Journal of Mathematical Physics, 2019, Vol. 60, No. 7, 071706.

We present a construction of an integrable model as a projective type limit of Calogero-Sutherland models of N fermionic particles, when N tends to infinity. Explicit formulas for limits of Dunkl operators and of commuting Hamiltonians by means of vertex operators are given.

[2] With S. Khoroshkin
Matrix elements of vertex operators and fermionic limit of spin Calogero-Sutherland system
arXiv:1910.08966 *Submitted to Journal of Physics A: Mathematical and Theoretical*

We present a construction of an integrable model as a projective type limit of spin Calogero-Sutherland model with N fermionic particles, where N tends to infinity. It is implemented in the multicomponent fermionic Fock space. Explicit formulas for limits of Dunkl operators and the Yangian generators are presented by means of fermionic fields.

[3] Calogero-Sutherland system at free fermion point
Submitted to Theoretical and Mathematical Physics

We present two ways to obtain precise expressions of the commuting Hamiltonians of the integrable system regarded as a fermionic limit of the quantum Calogero-Sutherland system with coupling constant $\beta = 0$. The construction is realized in the Fock space. The classical limit of the system is examined.

2.1.7 Artem Prikhodko

[1] Equivariant Grothendieck-Riemann-Roch theorem via formal deformation theory. With G. Kondyrev
arXiv:1906.00172 *submitted to the Journal of Algebraic Geometry*

We use the formalism of traces in higher categories to prove a common generalization of the holomorphic Atiyah-Bott fixed point formula and the Grothendieck-Riemann-Roch theorem. The proof is quite different from the original one proposed by Grothendieck et al.: it relies on the interplay between self dualities of quasi- and ind- coherent sheaves on X and formal deformation theory of Gaitsgory-Rozenblyum. In particular, we give a description of the Todd class in terms of the difference of two formal group structures on the derived loop scheme $\mathcal{L}X$. The equivariant case is reduced to the non-equivariant one by a variant of the Atiyah-Bott localization theorem.

[2] Hodge-to-de Rham degeneration for stacks. With D. Kubrak
arXiv:1910.12665 *submitted to International Mathematics Research Notices*

We introduce a notion of a Hodge-proper stack and extend the method of Deligne-Illusie to prove the Hodge-to-de Rham degeneration in this setting. In order to reduce the statement in characteristic 0 to characteristic p, we need to find a good integral model of a stack (a so-called spreading), which, unlike in the case of schemes, need not to exist in general. To address this problem we investigate the property of spreadability in more detail by generalizing standard spreading out results for schemes to higher Artin stacks and showing that all proper and some global quotient stacks are Hodge-properly spreadable. As a corollary we deduce a (non-canonical) Hodge decomposition of the equivariant cohomology for certain classes of varieties with an algebraic group action.

2.1.8 Arsenii Sagdeev

[1] On the partition of an odd number into three primes in a prescribed proportion
Mathematical Notes, 2019, Vol. 106, No. 1, pp. 98–107

We prove that, for any partition $1 = a + b + c$ of unity into three positive summands, each odd number n can be subdivided into three primes $n = p_a(n) + p_b(n) + p_c(n)$ so that the fraction of the first summand will approach a, that of the second, b, and that of the third, c as $n \to \infty$

[2] With A. Raigorodskii
On a Frankl–Wilson theorem and its geometric corollaries
Acta Math. Univ. Comenianae, 2019, Vol. 88, No. 3, pp. 1029–1033

We find a new analogue of the Frankl–Wilson theorem on the independence number of distance graphs of some special type. We apply this new result to two combinatorial geometry problems.

First, we improve a previously known value c such that $\chi(\mathbb{R}^n; S_2) \geq (c + o(1))n$, where $\chi(\mathbb{R}^n; S_2)$ is the minimum number of colors needed to color all points of R n so that there is no monochromatic set of vertices of a unit equilateral triangle S_2.

Second, given $m \geq 3$ we improve the value ξ_m such that for any $n \in \mathbb{N}$ there is a distance graph in $\mathbb{R}^n$ with the girth greater than m and the chromatic number at least $(\xi_m + o(1))^n$

[3] On a Frankl–Wilson theorem
Submitted to Problems of Information Transmission

We obtain a new theorem on the independence number of Johnson type distance graphs. This result can be considered as an analogue of the classic Frankl–Wilson theorem. We apply our result to two Euclidean Ramsey Theory problems.

2.2 Scientific conferences and seminar talks

2.2.1 Semeyon Abramyan

[1] Course "Advanced Topology", Independent University of Moscow, Moscow, Russia, Spring 2019, Lecturer
[2] Course "Advanced Topology: Cobordism Theory", Independent University of Moscow, Moscow, Russia, Fall 2019, Lecturer
[3] Conference "Topology, Geometry, and Dynamics: Rokhlin 100" Saint-Petersburg, August, 1923
Talk "Higher Whitehead products in moment-angle complexes and substitution of simplicial complexes"
[4] Talk "Grothendieck-Riemann-Roch theorem" at Topology seminar (Skoltech)
[5] Conference "One day seminar in Toric Topology", Osaka, November, 15
Talk "Pontryagin Algebras for Moment-Angle Complexes"
[6] Conference "Toric Topology 2019 in Okayama", Okayama, November, 18–22
Talk "Higher Whitehead products in Moment-Angle Complexes"
[7] Talk "Unitary bordism and Landweber Exactness Theorem" at "Smooth Structures on Manifolds" seminar (Higher School of Economics)

2.2.2 Lyalya Guseva

[1] Conference "Hodge ideals and mixed Hodge modules", Angers, April 1 – 5.
[2] Workshop on birational geometry, Moscow, March 25 – 29.
[3] Conference "Birational Geometry and Degeneration of Kahler-Einstein metrics", Moscow, April 8–13.
[4] Conference "Who is Who in Mirror Symmetry", Moscow, May 27 – June 3.
[5] Conference "Hyperkhler Event", Moscow, June 19 – 23.
[6] Conference "Birational Geometry and Fano varieties", Moscow, June 24 – 28.

2.2.3 Alexander Kalmynin

[1] Seminar "Functional analysis and noncommutative geometry", Moscow, Russia, February 1, 2019

Talk ”Van der Corput sets and Banach limits”

[2] Number Theory Seminar, TU Graz, Graz, Austria, March 6, 2019

Talk ”On the distribution of gaps between consecutive sums of two squares”

[3] Memorial conference dedicated to the memory of Ivan Matveevich Vinogradov, March 28, 2019

Talk ”Orthorecursive expansion of unity”

[4] Seminar ”Automorphic forms and their applications”, Moscow, April 9, 2019

Talk ”Cohen-Kuznetsov construction and arithmetical functions in short intervals”

[5] Seminar ”Functional analysis and noncommutative geometry”, Moscow, April 5, 2019

Talk ”Groupoid C*-algebras associated with one-dimensional tilings”

[6] International conference ”Baikal Number Theory”, Olkhon island, Russia, August 26-30, 2019

Talk ”Positivity of character sums”

[7] XVII International Conference ”Algebra, Number Theory and Discrete Geometry: modern problems, applications and problems of history”, Tula, September 23-28, 2019

Talk ”Nonnegativity of long character sums”

[8] Seminar ”Functional analysis and noncommutative geometry”, Moscow, October 10, 2019

Talk ”Gowers norms”

[9] Seminar ”Automorphic forms and their applications”, Moscow, November 12, 2019

Talk ”Sphere packings and automorphic forms”

[10] Seminar ”Approximation theory”, Moscow, November 28, 2019

Talk ”Orthorecursive expansions of unity”

[11] Young Mathematicians — 2020, Laboratory of Mirror Symmetry and Automorphic Forms, Moscow, December 6, 2019

Talk ”Nonnegativity of long character sums”

[12] Seminar ”Contemporary problems in Number Theory”, Moscow, December 12, 2019

Talk ”Positivity of long character sums”

2.2.4 Roman Krutovskii

[1] Conference “Toric Topology in Okayama 2019”, Okayama, November, 18 – 22

Talk “Basic Dolbeault Cohomology of manifolds with torus action”

[2] Conference “Topology, Geometry, Dynamics: Rokhlin – 100”, Saint-Petersburg, August, 19 – 23

Talk “Basic cohomology of moment-angle manifolds”

[3] One-day seminar on Toric Topology, Osaka, November, 14

Talk “Basic de Rham Cohomology of canonical folations on moment-angle manifolds”

[4] Japanese-Russian seminar, Moscow, August, 27 – 28
Talk “Basic Dolbeault cohomology of moment-angle manifolds”

2.2.5 Konstantin Loginov

[1] University of Loughborough, October 2019,
Talk: “Semistable degenerations of Fano varieties”.
[2] University of Bristol, October 2019,
Talk: “Semistable degenerations of Fano varieties”.
[3] Edinburgh Geometry seminar (EDGE), October 2019,
Talk: “Semistable degenerations of Fano varieties”.
[4] Laboratory of Algebraic Geometry and Homological Algebra, Moscow Institute of Physics and Technology, Dolgoprudny, Moscow region, September 2019,
Talk: “Semistable degenerations of Fano varieties”.
[5] Iskovskikh seminar (Steklov Mathematical Institute), September 2019,
Talk: “Upper bounds of Manin type”.
[6] HSE Laboratory of Algebraic Geometry seminar, September 2019,
Talk: “On semistable degenerations of Fano varieties”.
[7] Iskovskikh seminar (Steklov Mathematical Institute), April 2019,
Talk: “Snc degenerations of Fano varieties”.
[8] Student Birational Geometry Seminar (Steklov Mathematical Institute), February 2019,
Talk: “On the classification of three-dimensional Fano varieties with Picard rank 1”.
[9] Iskovskikh seminar (Steklov Mathematical Institute), January 2019.
Talk: “On the rationality problem for del Pezzo fibrations of degree 4”

2.2.6 Maria Matushko

[1] The XXVIth International Colloquium on Integrable Systems (ISQS-26), Prague, Czech Republic, July 7 – 12
Talk “Fermionic limit of the Calogero-Sutherland model”
[2] Workshop on Classical and Quantum Integrable Systems (CQIS-2019), Euler International Mathematical Institute, St. Petersburg, Russia, July 22 – 26
Talk “Matrix elements of vertex operators and fermionic limit of Spin Calogero-Sutherland system”
[3] Joint Seminar on Mathematical Physics of National Research University HSE and Skoltech Center for Advanced Studies, Moscow, Russia, November 13
Talk “Calogero-Sutherland system at infinity”
[4] Seminar of the Department of Theoretical Physics, Steklov Mathematical Institute of RAS, Moscow, Russia, March 13

Talk “Fermionic limit of the Calogero-Sutherland system”

2.2.7 Artem Prikhodko

[1] Seminar “Arithmetic geometry seminar of HSE”, Moscow, October, 28 and November, 11

Talk “Hodge-to-de Rham degeneration for stacks”

2.2.8 Arsenii Sagdeev

[1] International conference “3rd Hungarian-Russian Combinatorics workshop”, Moscow, May 20 - 25

Talk “On the number of edges in distance graphs of some special type”

[2] International conference “Transcendence and Diophantine Problems”, Moscow, Russia, June 10 - 14

Talk “On asymmetric Diophantine approximation”

[3] International conference “European Conference on Combinatorics, Graph Theory and Applications EUROCOMB2019”, Bratislava, Slovakia, August 26 - 30

Talk “On a Frankl–Wilson theorem and its geometric corollaries”

[4] International conference “Autumn Mathematical Readings in Adygea”, Maykop, Russia, October 15 - 20

Talk “On a Frankl–Wilson theorem and its applications to Euclidean Ramsey theory”

[5] Russian conference “62nd scientific conference MIPT”, Moscow, Russia, November 18 - 23

Talk “On asymmetric Diophantine approximation”

[6] Seminar “Moscow seminar on Number Theory”, MSU, Moscow, Russia, November

Talk “On asymmetric analogues of Hurwitz’s theorem on Diophantine approximation”

3 Program: Simons IUM fellowships

3.1 Research

3.1.1 Vsevolod Adler

[1] Nonautonomous symmetries of the KdV equation and step-like solutions

arXiv:1911.04770, *to appear in Journal of Nonlinear Mathematical Physics.*

We study solutions of the KdV equation governed by a stationary equation for symmetries from the non-commutative subalgebra, namely, for a linear combination of the master-symmetry and the scaling symmetry. The constraint under study is equivalent to a sixth order nonautonomous ODE possessing two first integrals. Its generic solutions have a singularity on the line $t = 0$. The regularity condition selects a 3-parameter family of solutions which describe oscillations near $u = 1$ and satisfy, for $t = 0$, an equation equivalent to degenerate P_5 equation. Numerical experiments show that in this family one can distinguish a two-parameter subfamily of separatrix step-like solutions with power-law approach to different constants for $x \to \pm\infty$. This gives an example of exact solution for the Gurevich–Pitaevskii problem on decay of the initial discontinuity.

3.1.2 Alexander Belavin

[1] With K. Aleshkin and A. Litvinov,
JKLMR conjecture and Batyrev construction,
J. Stat. Mech., 1903, 034003 (2019)

In this paper we study a mirror interpretation of the relation between the exact partition functions of N=(2,2) gauged linear sigma-models (GLSM) on the 2d sphere and Kahler potentials on the moduli spaces of the CY manifolds proposed by Jockers et al. We use the Batyrev mirror construction for establishing the explicit relation between GLSM and the corresponding mirror family of the Calabi-Yau manifolds, defined as hypersurfaces in weighted projective spaces. We demonstrate how to do this by the explicit calculation in the case of the quintic threefold and its mirror.

[2] Special Geometry on Calabi-Yau Moduli Spaces and Q-Invariant Milnor Rings,
Proc. International Congress of Mathematicians (ICM2018), World Scientific, Vol. 2, p.2567?2580 (2019);

The moduli spaces of Calabi–Yau (CY) manifolds are the special K?hler manifolds. The special K?hler geometry determines the low-energy effective theory which arises in Superstring theory after the compactification on a CY manifold. For the cases, where the CY manifold is given as a hypersurface in the weighted projective space, a new procedure for computing the K?hler potential of the moduli space has been proposed in our works earlier. The method is based on the fact that the moduli space of CY manifolds is a marginal subspace of the Frobenius manifold which arises on the deformation space of the corresponding Landau–Ginzburg superpotential. I review this approach and demonstrate its efficiency by computing the Special geometry of the 101-dimensional moduli space of the quintic threefold around the orbifold point

3.1.3 Yurii Burman

[1] Higher matrix-tree theorems and Bernardi polynomial

Journal of Algebraic Combinatorics, vol. 50(2019), no. 4, pp. 427–446.

The classical matrix-tree theorem discovered by G. Kirchhoff in 1847 expresses the principal minor of the $n \times n$ Laplace matrix as a sum of monomials of matrix elements indexed by directed trees with n vertices. We prove, for any $k \geq n$, a three-parameter family of identities between degree k polynomials of matrix elements of the Laplace matrix. For $k = n$ and special values of the parameters the identity turns to be the matrix-tree theorem.

For the same values of parameters and arbitrary $k \geq n$ the left-hand side of the identity becomes a specific polynomial of the matrix elements called higher determinant of the matrix. We study properties of the higher determinants; in particular, they have an application (due to M. Polyak) in the topology of 3-manifolds.

[2] With R. Fröberg and B. Shapiro
Algebraic relations between harmonic and anti-harmonic moments of plane polygons
arXiv:1908.07621v1, *to appear in International Mathematics Research Notices*

In this paper we describe the algebraic relations satisfied by the harmonic and anti-harmonic moments of simply connected, but not necessarily convex planar polygons with a given number of vertices.

3.1.4 Konstantin Fedorovsky

[1] With P. Paramonov
On Lip^m*-reflection of harmonic functions over boundaries of simple Carathéodory domains*
Anal. Math. Phys., **9** (2019), no. 3, 1031-1042.

In this paper we obtain several new sharp necessary and sufficient Lip^m-continuity conditions for operators of harmonic reflection of functions over boundaries of simple Carathéodory domains in $\mathbb{R}^N$. These results are based on our Lip^m-continuity criterion for the Poisson operator in the aforementioned domains.

[2] With Yu. Belov and A. Borichev
Nevanlinna domains with large boundaries
J. Funct. Anal., **266** (2019), no. 8, 2617-2643.

Nevanlinna domains are an important class of bounded simply connected domains in the complex plane; they are images of the unit disc under mappings by univalent functions belonging to model spaces (i.e. the subspaces of the Hardy space H^2 invariant with respect to the backward shift operator). Nevanlinna domains play a crucial role in recent progress in problems of uniform approximation of functions on compact sets in $\mathbb{C}$ by polyanalytic polynomials and polyanalytic rational functions. We give a complete solution to the following problem posed in the early 2000-s: how large (in the sense of dimension) can be the boundaries of Nevanlinna domains? We establish the existence of Nevanlinna domains with large boundaries. In particular, these domains can have boundaries of positive planar measure. The sets of accessible points can be of any Hausdorff dimension between 1 and 2. As a quantitative counterpart of these results, we construct rational functions univalent in the unit disc with extremely long boundaries for a given amount of poles.

[3] with E. Borovik
On the Lip(ω)*-continuity of the operator of harmonic reflection over boundaries of simple Carathéodory domains*
Zap. nauchn. sem. POMI, **480**, 2019, 62–72; Engl. transl.: J. Math. Sci. (N.Y.) (to appear)

We study the continuity conditions for the operator of harmonic reflection of functions over boundaries of simple Caratheodory domains. This operator is considered as one acting from from a space of functions of Lipschitz-Golder type, defined by a general modulus of continuity, into another space of such kind. The obtained results are based on the continuity criterion for the Poisson operator (acting in the same spaces of functions) in the aforementioned domains, which also have been obtained in the paper; they generalize and refine the results of the recent work by the second author and P. Paramonov (Analysis and Mathematical Physics, 2019).

3.1.5 Sergei Gorchinsky

[1] S. O. Gorchinskiy, Vik. S. Kulikov, A. N. Parshin, V. L. Popov, "Igor Rostislavovich Shafarevich and his scientific heritage", *Proc. Steklov Inst. Math.*, **307** (2019).

In this paper, we give a survey of almost all scientific results of Igor Rostislavovich Shafarevich, from his youth to the very end of his life. Explanation of results is detailed and popular enough to make it clear for a broad audience of mathematicians. Further developments towards problems posed by Shafarevich are described. We show the beauty and depth of mathematical discoveries made by Shafarevich.

[2] S. O. Gorchinskiy, "Orthogonal to principle ideles", accepted to publication in *Mathematical Notes*.

We describe the orthogonal ${K^*}^{\perp}$ to the group of principal ideles K^* with respect to the global tame symbol pairing on the group of ideles $\mathbb{A}_X^*$ of a smooth projective algebraic curve X over a field k. More precisely, we describe the quotient ${K^*}^{\perp}/(K^* \cdot U)$, where U is the kernel of the global tame symbol pairing. When k is algebraically closed, this quotient maps surjectively to $\mathrm{Pic}^0(X)$ with the kernel being $\mathrm{Hom}\big(\mathrm{Pic}^0(X), k^*\big)$. When k is finite, the quotient is trivial.

3.1.6 Anton Khoroshkin

[1] "Quadratic Algebras arising from Hopf operads generated by a single element"
`math.arXiv`:1907.05573 submitted to Letters in Mathematical Physics

The operads of Poisson and Gerstenhaber algebras are generated by a single binary element if we consider them as Hopf operads (i.e. as operads in the category of cocommutative coalgebras). In this note we discuss in details the Hopf operads generated by a single element of arbitrary arity. We explain why the dual space to the space of n-ary operations in this operads are quadratic and Koszul algebras. We give the detailed description of generators, relations and a certain monomial basis in these algebras.

[2] with Thomas Willwacher
`math.arXiv`:1905.04499 submitted to Journal of EMS

We give a description of the operad formed by the real locus of the moduli space of stable genus zero curves with marked points $\overline{\mathcal{M}_{0,n+1}}(\mathbb{R})$ in terms of a homotopy quotient of an operad of associative algebras. We use this model to find different Hopf models of the algebraic operad of Chains and homologies of $\overline{\mathcal{M}_{0,n+1}}(\mathbb{R})$. In particular, we show that the operad $\overline{\mathcal{M}_{0,n+1}}(\mathbb{R})$ is not formal. The manifolds $\overline{\mathcal{M}_{0,n+1}}(\mathbb{R})$ are known to be Eilenberg-MacLane spaces for the so called pure Cacti groups. As an application of the operadic constructions we prove that for each n the cohomology ring $H^{\bullet}(\overline{\mathcal{M}_{0,n+1}}(\mathbb{R}),\mathbb{Q})$ is a Koszul algebra and that the manifold $\overline{\mathcal{M}_{0,n+1}}(\mathbb{R})$ is not formal but is a rational $K(\pi,1)$ space. We give the description of the Lie algebras associated with the lower central series filtration of the pure Cacti groups.

[3] with Evgeny Feigin and Ievgen Makedonskyi
`math.arXiv`:1906.03290 submitted to Compositio

The classical Peter-Weyl theorem describes the structure of the space of functions on a semi-simple algebraic group. On the level of characters (in type A) this boils down to the Cauchy identity for the products of Schur polynomials. We formulate and prove the analogue of the Peter-Weyl theorem for the current groups. In particular, in type A the corresponding characters identity is governed by the Cauchy identity for the products of q-Whittaker functions. We also formulate and prove a version of the Schur-Weyl theorem for current groups. The link between the Peter-Weyl and Schur-Weyl theorems is provided by the (current version of) Howe duality.

[4] with Pedro Tamaroff
to appear

We define derived Poincaré–Birkhoff–Witt maps of dg operads —or derived PBW maps, for brevity. These extend the definition of PBW maps of operads of P. Tamaroff and V. Dotsenko, and show that the map from the homotopy Lie operad to the homotopy associative operad is derived PBW.

As an application, we show that three different known definitions of universal envelope of an L_∞-algebra invented by V.Baranovsky, J.Moreno-Fernandez and M.Markl-T.Lada are all A_∞-quasiisomorphic.

3.1.7 Iosif Krasilshchik

[1] With H. Baran H., M. Marvan, P. Blaschke.
On symmetries of the Gibbons-Tsarev equation, Journal of Geometry and Physics. 2019, **144**, pp. 54–80

We study the Gibbons-Tsarev equation $z_{yy} + z_x z_{xy} - z_y z_{xx} + 1 = 0$ and, using the known Lax pair, we construct infinite series of conservation laws and the algebra of nonlocal symmetries in the covering associated with these conservation laws. We prove that the algebra is isomorphic to the Witt algebra. Finally, we show that the constructed symmetries are unique in the class of polynomial ones.

[2] With A. Sergyeyev.
Integrability of anti-self-dual vacuum Einstein equations with nonzero cosmological constant: an infinite hierarchy of nonlocal conservation laws, Annales Henri Poincare. 2019, **20**, Iss. 8. pp. 2699–2715

We present an infinite hierarchy of nonlocal conservation laws for the Przanowski equation, an integrable second-order PDE locally equivalent to anti-self-dual vacuum Einstein equations with nonzero cosmological constant. The hierarchy in question is constructed using a nonisospectral Lax pair for the equation under study. As a byproduct, we obtain an infinite-dimensional differential covering over the Przanowski equation.

[3] With O.I. Morozov, P. Vojčák.
Nonlocal symmetries, conservation laws, and recursion operators of the Veronese web. Journal of Geometry and Physics, 2019, **146**, 103519.

We study the Veronese web equation $u_y u_{tx} + \lambda u_x u_{ty} - (\lambda + 1) u_t u_{xy}$, $\lambda \in \mathbb{R}$, and using its isospectral Lax pair construct two infinite series of nonlocal conservation laws. In the infinite differential coverings associated to these series, we describe the Lie algebras of the corresponding nonlocal symmetries. Finally, we construct a recursion operator and explore its action on nonlocal shadows. The operator provides a new shadow which serves as a master-symmetry.

3.1.8 Mikhail Lashkevich

[1] With Ya. Pugai
The complex sinh-Gordon model: form factors of descendant operators and current–current perturbations

arXiv:1811.02631, submitted to JHEP.

We study quasilocal operators in the quantum complex sinh-Gordon theory in the form factor approach. The free field procedure for descendant operators is developed by introducing the algebra of screening currents and related algebraic objects. We work out null vector equations in the space of operators. Further we apply the proposed algebraic structures to constructing form factors of the conserved currents T_k and Θ_k. We propose also form factors of current–current operators of the form T_kT_{-l}. Explicit computations of the four-particle form factors allow us to verify the recent conjecture of Smirnov and Zamolodchikov about the structure of the exact scattering matrix of an integrable theory perturbed by a combination of irrelevant operators. Our calculations confirm that such perturbations of the complex sinh-Gordon model and of the $\mathbb{Z}_N$ symmetric Ising models result in extra CDD factors in the S matrix.

3.1.9 Maxim Leyenson

"Notes on Riemann Hypothesys" (in Russian). Accepted to publication at the "Troitsky variant — Nauka", Also

`https://drive.google.com/drive/folders/1TXfyb108LsTlAw8ODojntp01xHyn3CEG?usp=sharing` available online.

We give an introduction into the Riemann hypothesis. First, we introduce the Riemann's zeta-function $\zeta(s)$, and explain the Riemann's hypothesis. We then briefly tell the history of the study of Riemann and Dedekind ζ-functions.

Then we proceed to explain the notion of Artin's ζ-functions for curves over finite fields, the geometric version of the Dedekind ζ-functions; formulate the Weil's formulation of the [geometric version of the] Riemann hypothesis for this zeta - function, and tell about Andre Weil's proof of this geometric version in case of algebraic curves.

Finally, we explain the parallel between arithmetic [of number fields] and geometry of curves, discovered by Dedekind and Kronecker; explain the notion of arithmetic curves, and the precise parallel between Dedekind and Artin zeta-functions.

3.1.10 Alexei Litvinov

Integrable $\mathfrak{gl}(n|n)$ Toda field theory and its sigma-model dual, arXiv:1901.04799, JETP letters 110(2019) 723-726

In this paper we studied the duality between sigma-models and Toda QFT's. We claim that $\mathfrak{gl}(n|n)$ affine Toda field theory behaves in the strong coupling limit as ηdeformed $CP(n1)$ sigma-model plus a free field.

3.1.11 Taras Panov

[1] With Yakov Veryovkin.

On the commutator subgroup of a right-angled Artin group.
J. Algebra, 2019, Vol. 521, pp. 284–298.

We use polyhedral product models to analyse the structure of the commutator subgroup of a right-angled Artin group. In particular, we provide a minimal set of generators for the commutator subgroup, consisting of special iterated commutators of canonical generators.

[2] With Semyon Abramyan.
Higher Whitehead products in moment-angle complexes and substitution of simplicial complexes.
Trudy Matematicheskogo Instituta imeni V. A. Steklova, 2019, Vol. 305, pp. 7–28 (Russian); Proceedings of the Steklov Institute of Mathematics, 2019, Vol. 305, pp. 1–21 (English translation).

We study the question of realisability of iterated higher Whitehead products with a given form of nested brackets by simplicial complexes, using the notion of the moment-angle complex $\mathcal{Z}_\mathcal{K}$. Namely, we say that a simplicial complex $\mathcal{K}$ realises an iterated higher Whitehead product w if w is a nontrivial element of $\pi_*(\mathcal{Z}_\mathcal{K})$. The combinatorial approach to the question of realisability uses the operation of substitution of simplicial complexes: for any iterated higher Whitehead product w we describe a simplicial complex $\partial\Delta_w$ that realises w. Furthermore, for a particular form of brackets inside w, we prove that $\partial\Delta_w$ is the smallest complex that realises w. We also give a combinatorial criterion for the nontriviality of the product w. In the proof of nontriviality we use the Hurewicz image of w in the cellular chains of $\mathcal{Z}_\mathcal{K}$ and the description of the cohomology product of $\mathcal{Z}_\mathcal{K}$. The second approach is algebraic: we use the coalgebraic versions of the Koszul and Taylor complex for the face coalgebra of $\mathcal{K}$ to describe the canonical cycles corresponding to iterated higher Whitehead products w. This gives another criterion for realisability of w.

[3] With Ivan Limonchenko and Georgiy Chernykh.
SU-bordism: structure results and geometric representatives.
Uspekhi Mat. Nauk, 2019, Vol. 74, No. 3, pp. 95–166 (Russian); Russian Math. Surveys, 2019, Vol. 74, No. 3, pp. 461–524 (English translation).

In the first part of this survey we give a modernised exposition of the structure of the special unitary bordism ring, by combining the classical geometric methods of Conner–Floyd, Wall and Stong with the Adams-Novikov spectral sequence and formal group law techniques that emerged after the fundamental 1967 work of Novikov. In the second part we use toric topology to describe geometric representatives in SU-bordism classes, including toric, quasitoric and Calabi-Yau manifolds.

[4] With Roman Krutowski.
Dolbeault cohomology of complex manifolds with torus action.

arXiv:1908.06356

We describe the basic Dolbealut cohomology algebra of the canonical foliation on a class of complex manifolds with a torus symmetry group. This class includes complex moment-angle manifolds, LVM- and LVMB-manifolds and, in most generality, complex manifolds with a maximal holomorphic torus action. We also provide a dga model for the ordinary Dolbeault cohomology algebra. The Hodge decomposition for the basic Dolbeault cohomology is proved by reducing to the transversely Kähler (equivalently, polytopal) case using a foliated analogue of toric blow-up.

3.1.12 Alexei Penskoi

[1] Isoperimetric Inequalities for Higher Eigenvalues of the Laplace-Beltrami Operator on Surfaces

Proc. Steklov Inst. Math., 2019, Vol. 305, pp. 270–286.

In this paper recent advances in isoperimetric inequalities for higher eigenvalues of the Laplace-Beltrami operator on the sphere and on the projective plane are discussed.

[2] With M. A. Karpukhin, N. S. Nadirashvili and I. V. Polterovich

An isoperimetric inequality for Laplace eigenvalues on the sphere.

arXiv:1706.05713, *to appear in Journal of Differential Geometry*

In this paper we show that for any positive integer k, the k-th nonzero eigenvalue of the Laplace-Beltrami operator on the two-dimensional sphere endowed with a Riemannian metric of unit area, is maximized in the limit by a sequence of metrics converging to a union of k touching identical round spheres. This proves a conjecture posed by the second author in 2002 and yields a sharp isoperimetric inequality for all nonzero eigenvalues of the Laplacian on a sphere. Earlier, the result was known only for $k = 1$ (J. Hersch, 1970), $k = 2$ (N. Nadirashvili, 2002; R. Petrides, 2014) and $k = 3$ (N. Nadirashvili and Y. Sire, 2017). In particular, we argue that for any $k \geq 2$, the supremum of the k-th nonzero eigenvalue on a sphere of unit area is not attained in the class of Riemannian metrics which are smooth outsitde a finite set of conical singularities. The proof uses certain properties of harmonic maps between spheres, the key new ingredient being a bound on the harmonic degree of a harmonic map into a sphere obtained by N. Ejiri.

3.1.13 Vladimir Poberezhnyi

[1] With G.F.Helminck and S.V.Polenkova

Strict versions of integrable hierarchies in pseudodifference operators and the related Cauchy problems,

TMF, 198:2 (2019), 225245; Theoret. and Math. Phys., 198:2 (2019), 197214

In the algebra $\text{Ps}\Delta$ of pseudodifference operators, we consider two deformations of the Lie subalgebra spanned by positive powers of an invertible constant first-degree pseudodifference operator Λ_0. The first deformation is by the group in $\text{Ps}\Delta$ corresponding to the Lie subalgebra $\text{Ps}\Delta_{<0}$ of elements of negative degree, and the second is by the group corresponding to the Lie subalgebra $\text{Ps}\Delta_{\leqslant 0}$ of elements of degree zero or lower. We require that the evolution equations of both deformations be certain compatible Lax equations that are determined by choosing a Lie subalgebra depending on Λ_0 that respectively complements the Lie subalgebra $\text{Ps}\Delta_{<0}$ or $\text{Ps}\Delta_{\leqslant 0}$. This yields two integrable hierarchies associated with Λ_0, where the hierarchy of the wider deformation is called the strict version of the first because of the form of the Lax equations. For Λ_0 equal to the matrix of the shift operator, the hierarchy corresponding to the simplest deformation is called the discrete KP hierarchy. We show that the two hierarchies have an equivalent zero-curvature form and conclude by discussing the solvability of the related Cauchy problems.

3.1.14 Yaroslav Pugai

[1] With Michael Lashkevich

The complex sinh-Gordon model: form factors of descendant operators and current-current perturbations. Journal of High Energy Physics 2019 (1), 71

We study quasilocal operators in the quantum complex sinh-Gordon theory in the form factor approach. The free field procedure for descendant operators is developed by introducing the algebra of screening currents and related algebraic objects. We work out null vector equations in the space of operators. Further we apply the proposed algebraic structures to constructing form factors of the conserved currents. We propose also form factors of current-current operators of the form TT. Explicit computations of the four-particle form factors allow us to verify the recent conjecture of Smirnov and Zamolodchikov about the structure of the exact scattering matrix of an integrable theory perturbed by a combination of irrelevant operators. Our calculations confirm that such perturbations of the complex sinh-Gordon model and of the ZN symmetric Ising models result in extra CDD factors in the S matrix.

[2] With Michael Lashkevich, Jun'ichi Shiraishi, Yohei Tutiya.
Lattice models, deformed Virasoro algebra and reduction equation.
arXiv:1911.11412 *Submitted to J.Phys.A.*

We study the fused currents of the deformed Virasoro algebra (DVA). By constructing a homotopy operator we show that for special values of the parameter of the algebra fused currents pairwise coincide on the cohomologies of the Felder resolution. Within the algebraic approach to lattice models these currents are known to describe neutral excitations of the solid-on-solid (SOS) models in the transfer-matrix picture. It allows us to prove the closeness of the system of excitations for a special nonunitary series of restricted SOS (RSOS) models. Though the results of the algebraic approach to lattice models were consistent with the results of other methods, the lack of such proof had been an essential gap in its construction.

3.1.15 Petr Pushkar

[1] Morse theory on manifolds with boundary I. Strong Morse function, cellular structures and algebraic simplification of cellular differential.
arXiv: submit/2968420

Main subject of the paper is a (strong) Morse function on a compact manifold with boundary. We construct a cellular structure and discuss its algebraic properties in this paper.

3.1.16 George Shabat

[1] Coordinate and synthetic geometry in the school (in Russian). Proceedings of the international conference on the occasion of the centenary of the birth of V.T. Basylev. Moscow Pedagogical State University, 2019, pp. 149-150.

The unity of coordinate and synthetic approaches to teaching geometry is declared. As an illustration the problem of conic through six points in the plane and the cubic through ten points are considered.

[2] Square-tiled surfaces and curves over number fields. Proceedings of the international conference on the occasion of the 90-year anniversary of the algebra department of the mechanico-mathematical facuty of Moscow State University, 2019, pp. 63-64.

The paper presents an analog of the old result by the author and V. Voevodsky, according to which a Riemann surface admits a conformal structure, defined by an equilateral triangulation, if and only if the corresponding algebraic curve can be defined over the field of the algebraic numbers; the similar result is obtained for the square-tiled surfaces.

[3] Belyi pairs in the critical filtrations of Hurwitz spaces. https://arxiv.org/pdf/1902.02034.

The stratification of the Hurwitz and the moduli spaces are introduced, the Belyi pairs constituting the zero-dimensional part of it.

[4] Belyi pairs and Fried families. To appear in the volume, dedicated to IR. Shafarevich, published by the Steklov Mathematical Institute of Russian Academy of Sciences.

The number 3 of critical values of the rational functions on algebraic curves, typical for the Belyi functions, is replaced by 4. The emerging objects, unlike the rigid Belyi pairs, can be deformed; they constitute the one-parametric families. The main properties of the corresponding curves in the moduli spaces are established and the relation of these curves with the Belyi pairs is considered. The case of the clean Belyi pairs of genus 2 of the minimal degree 8 is analyzed.

3.1.17 Stanislav Shaposhnikov

[1] Bogachev V.I., Miftakhov A.F., Shaposhnikov S.V. Differential Properties of Semigroups and Estimates of Distances between Stationary Distributions of Diffusions. Doklady Mathematics, 2019, V. 99, N. 2, P. 175–180.

In this paper new estimates are obtained for the total variation distance and the Kantorovich distance between stationary distributions of diffusion processes in terms of certain distances between the diffusion coefficients and drifts of these processes.

[2] Bogachev V.I., Rckner M., Shaposhnikov S.V. On convergence to stationary distributions for solutions of nonlinear Fokker–Planck–Kolmogorov equations. Journal of Mathematical Sciences, 2019, V. 242, N. 1, P. 69–84.

In this paper we obtain conditions under which solutions to a nonlinear Fokker–Planck–Kolmogorov equation with the diffusion matrix depending on the solution converge to the stationary solution.

3.1.18 George Sharygin

[1] Deformation quantization of commutative families and vector fields
XXXVII Workshop on Geometric Methods in Physics, Bialoweza 2018, “Trends in Mathematics”, Springer Nature, 2019, p. 100-120

We describe a series of cohomological obstructions for the deformation of involutive families of functions on a Poisson manifold and for the deformation of Poisson vector fields acting on it.

[2] With V.Retakh and V.Rubtsov:
Noncommutative Cross-Ratio and Schwarz Derivative
arXiv:1905.01366, *to appear in Integrable systems and algebraic geometry (dedicated to Emma Previato), vol 2. LNS 459 p.487-516 (2020)*

We present here a theory of noncommutative cross-ratio, Schwarz derivative and their connections and relations to the operator cross-ratio. We apply the theory to "noncommutative elementary geometry" and relate it to noncommutative integrable systems. We also provide a noncommutative version of the celebrated "pentagramma mirificum".

[3] With Yu.Chernyakov and A.Sorin:
On the invariants of the full symmetric Toda system
arXiv: nlin.SI 1910.04789

In this paper we continue our study of the geometric properties of full symmetric Toda systems from [CSS14,CSS17,CSS19]. Namely we describe here a simple geometric construction of a commutative family of vector fields on compact groups, that include the Toda vector field, i.e. the field, which generates the full symmetric Toda system associated with the Cartan decomposition of a semisimple Lie algebra. Our construction makes use of the representations of the semisimple algebra and does not depend on the splitness of the Cartan pair. It is very close to the family of invariants and semiinvariants of the Toda system associted with SL_n, introduced in [CS].

[4] L_∞-derivations and the argument shift method for deformation quantization algebras
arXiv:1912.00586, *submitted to Acta Mathematica Spalatensia*

The argument shift method is a well-known method for generating commutative families of functions in Poisson algebras from central elements and a vector field, verifying a special condition with respect to the Poisson bracket. In this notice we give an analogous construction, which gives one a way to create commutative subalgebras of a deformed algebra from its center (which is as it is well known describable in the terms of the center of the Poisson algebra) and an L_∞-differentiation of the algebra of Hochschild cochains, verifying some additional conditions with respect to the Poisson structure.

[5] Problem about a sphere in a triangular prism. (in Russian)
to appear in "Kvant" magazine

This paper discusses a problem in elementary solid Geometry, which turns out to be closely related with various important constructions on Euclidean plane, such as the isogonal conjugation.

3.1.19 Arkady Skopenkov

[1] A. Skopenkov and M. Tancer, Hardness of almost embedding simplicial complexes in R^d, Discr. and Comp. Geom. 61:2 (2019), 452463; DOI 10.1007/s00454-018-0013-1, arXiv:1703.06305.

A map $f : K \to \mathbb{R}^d$ of a simplicial complex is an almost embedding if $f(\sigma) \cap f(\tau) = \emptyset$ whenever σ, τ are disjoint simplices of K.

Theorem. Fix integers $d, k \geq 2$ such that $d = \frac{3k}{2} + 1$.

(a) Assume that $P \neq NP$. Then there exists a finite k-dimensional complex K that does not admit an almost embedding in R^d but for which there exists an equivariant map $K \to S^{d1}$.

(b) The algorithmic problem of recognition almost embeddability of finite k-dimensional complexes in R^d is NP hard.

The proof is based on the technique from the Matoušek-Tancer-Wagner paper (proving an analogous result for embeddings), and on singular versions of the higher-dimensional Borromean rings lemma and a generalized van Kampen–Flores theorem.

[2] A. Skopenkov. Classification of knotted tori, Proc. A of the Royal Society of Edinburgh, to appear, `https://doi.org/10.1017/prm.2018.141`, arxiv: 1502.04470.

We describe the group of (smooth isotopy classes of smooth) embeddings $S^p \times S^q \to R^m$ for $p \leq q$ and $m \geq 2p+q+3$. Earlier such a description was known only for $2m \geq 3p+3q+4$. We use a recent exact sequence of M. Skopenkov.

[3] D. Crowley and A. Skopenkov, Embeddings of non-simply-connected 4-manifolds in 7-space, I. Classification modulo knots. Moscow Math. J., to appear, arxiv:1611.04738.

We work in the smooth category. Let N be a closed connected orientable 4-manifold with torsion free H_1, where $H_q := H_q(N; \mathbb{Z})$. The main result is *a complete readily calculable classification of embeddings* $N \to \mathbb{R}^7$, up to equivalence which is isotopy and embedded connected sum with embeddings $S^4 \to \mathbb{R}^7$. Such a classification was earlier known only for $H_1 = 0$ by Boéchat-Haefliger-Hudson 1970. Our classification involves Boéchat-Haefliger invariant $\kappa(f) \in H_2$, Seifert bilinear form $\lambda(f) : H_3 \times H_3 \to \mathbb{Z}$ and β-invariant assuming values in the quotient of H_1 defined by values of $\kappa(f)$ and $\lambda(f)$.

In particular, for $N = S^1 \times S^3$ we define geometrically a 1–1 correspondence between the set of equivalence classes of embeddings and an explicitly defined quotient of $\mathbb{Z} \oplus \mathbb{Z}$.

[4] S. Avvakumov, I. Mabillard, A. Skopenkov and U. Wagner, Eliminating Higher-Multiplicity Intersections, III. Codimension 2, Israel J. Math., to appear, arxiv:1511.03501.

We study conditions under which a finite simplicial complex K can be mapped to $\mathbb{R}^d$ without higher-multiplicity intersections. An *almost r-embedding* is a map $f\colon K \to \mathbb{R}^d$ such that the images of any r pairwise disjoint simplices of K do not have a common point. We show that if r is not a prime power and $d \geq 2r + 1$, then there is a counterexample to the topological Tverberg conjecture, i.e., *there is an almost r-embedding of the $(d+1)(r-1)$-simplex in $\mathbb{R}^d$*. This improves on previous constructions of counterexamples (for $d \geq 3r$) based on a series of papers by M. Özaydin, M. Gromov, P. Blagojević, F. Frick, G. Ziegler, and the second and fourth present author.

The counterexamples are obtained by proving the following algebraic criterion in codimension 2: *If $r \geq 3$ and if K is a finite $2(r-1)$-complex then there exists an almost r-embedding $K \to \mathbb{R}^{2r}$ if and only if there exists a general position PL map $f\colon K \to \mathbb{R}^{2r}$*

such that the algebraic intersection number of the f-images of any r pairwise disjoint simplices of K is zero. This result can be restated in terms of cohomological obstructions or equivariant maps, and extends an analogous codimension 3 criterion by the second and fourth author.

It follows from work of M. Freedman, V. Krushkal, and P. Teichner that the analogous criterion for $r = 2$ is false. We prove a beautiful lemma on singular higher-dimensional Borromean rings, yielding an elementary proof of the counterexample. As another application of our methods, we classify *ornaments* $f\colon S^3 \sqcup S^3 \sqcup S^3 \to \mathbb{R}^5$ up to *ornament concordance*.

[5] S. Avvakumov, I. Mabillard, A. Skopenkov and U. Wagner, Eliminating Higher-Multiplicity Intersections, III. Codimension 2 (extended abstract), Russian Math. Surveys, to appear.

This is an announcement of [4].

[6] A. Skopenkov. Invariants of graph drawings in the plane, Arnold Math. J., to appear, full version: arXiv:1805.10237.

We present a simplified exposition of some classical and modern results on graph drawings in the plane. These results are chosen so that they illustrate some spectacular recent higher-dimensional results on the border of geometry, combinatorics and topology. We define a $\mathbb{Z}_2$-valued *self-intersection invariant* (i.e. the van Kampen number) and its generalizations. We present elementary formulations and arguments accessible to mathematicians not specialized in any of the areas discussed. So most part of this survey could be studied before textbooks on algebraic topology, as an introduction to starting ideas of algebraic topology motivated by algorithmic, combinatorial and geometric problems.

[7] A. Skopenkov. A short exposition of S. Parsa's theorem on intrinsic linking and non-realizability, Discr. and Comp. Geom., to appear, full version: arXiv:1808.08363.

We present a short exposition of the following results by S. Parsa.

Let L be a graph such that the join $L * \{1,2,3\}$ (i.e. the union of three cones over L along their common bases) piecewise linearly (PL) embeds into $\mathbb{R}^4$. Then L admits a PL embedding into $\mathbb{R}^3$ such that any two disjoint cycles have zero linking number.

There is C such that every 2-dimensional simplicial complex having n vertices and embeddable into $\mathbb{R}^4$ contains less than $Cn^{8/3}$ simplices of dimension 2.

We also present corrected statement and proof of the analogue of the second result for intrinsic linking.

[8] A. Skopenkov, High codimension embeddings: classification, to appear in Bulletin of the Manifold Atlas.

`http://www.map.mpim-bonn.mpg.de/High_codimension_embeddings`

This page is intended not only for specialists in embeddings, but also for mathematicians from other areas who want to apply or to learn the theory of embeddings.

This article gives a short guide to the Knotting Problem of compact manifolds N in Euclidean spaces and in spheres. After making general remarks we record some of the dimension ranges where no knotting is possible, i.e. where any two embeddings of N are

isotopic. We then establish notation and conventions and give references to other pages on the Knotting Problem, to which this page serves as an introduction. We conclude by introducing connected sum and make some comments on codimension 1 and 2 embeddings.

[9] A. Skopenkov, Embeddings just below the stable range: classification, to appear in Bull. Man. Atl. `http://www.map.mpim-bonn.mpg.de/Embeddings_just_below_the_stable_range:_classification`

This page is intended not only for specialists in embeddings, but also for mathematician from other areas who want to apply or to learn the theory of embeddings.

Recall the Whitney-Wu Unknotting Theorem: if N is a connected manifold of dimension $n > 1$, and $m \geq 2n+1$, then every two embeddings $N \to \mathbb{R}^m$ are isotopic. In this page we summarize the situation for $m = 2n \geq 6$ and some more general situations.

[10] A. Skopenkov, 3-manifolds in 6-space, to appear in Bull. Man. Atl. `http://www.map.mpim-bonn.mpg.de/3-manifolds_in_6-space`

This page is intended not only for specialists in embeddings, but also for mathematicians from other areas who want to apply or to learn the theory of embeddings.

The classification of 3-manifolds in 6-space is of course a particular case of the classification of n-manifolds in $2n$-space. In this page we recall the general results as they apply when $n = 3$ and we discuss examples and invariants peculiar to the case $n = 3$.

[11] A. Skopenkov, 4-manifolds in 7-space, to appear in Bull. Man. Atl. `http://www.map.mpim-bonn.mpg.de/4-manifolds_in_7-space`

This page is intended not only for specialists in embeddings, but also for mathematician from other areas who want to apply or to learn the theory of embeddings.

Basic results on 4-manifolds in 7-space are particular cases of results on n-manifolds in $(2n-1)$-space for $n = 4$. In this page we concentrate on more advanced results peculiar for $n = 4$.

[12] A. Skopenkov, High codimension links, to appear in Bull. Man. Atl. `http://www.map.mpim-bonn.mpg.de/High_codimension_links`

This page is intended not only for specialists in embeddings, but also for mathematician from other areas who want to apply or to learn the theory of embeddings. We describe classification of embeddings $S^{n_1} \sqcup \ldots \sqcup S^{n_s} \to S^m$ for $m - 3 \geq n_i$.

[13] S. Avvakumov, R. Karasev and A. Skopenkov. Stronger counterexamples to the topological Tverberg conjecture, submitted, arxiv:1908.08731.

Denote by Δ_N the N-dimensional simplex. A map $f\colon \Delta_N \to \mathbb{R}^d$ is an *almost r-embedding* if $f\sigma_1 \cap \ldots \cap f\sigma_r = \emptyset$ whenever $\sigma_1, \ldots, \sigma_r$ are pairwise disjoint faces. A counterexample to the topological Tverberg conjecture asserts that *if r is not a prime power and $d \geq 2r+1$, then there is an almost r-embedding $\Delta_{(d+1)(r-1)} \to \mathbb{R}^d$.* We improve this by showing that *if r is not a prime power and $N := (d+1)r - r\left\lceil \dfrac{d+2}{r+1} \right\rceil - 2$, then there is an almost r-embedding $\Delta_N \to \mathbb{R}^d$.* For the r-fold van Kampen–Flores conjecture we also

produce counterexamples which are stronger than previously known. Our proof is based on generalizations of the Mabillard–Wagner theorem on construction of almost r-embeddings from equivariant maps, and of the Özaydin theorem on existence of equivariant maps.

[14] A. Skopenkov, A short exposition of the Levine-Lidman example of spineless 4-manifolds, arXiv:1911.07330.

A 2018 paper by A. Levine and T. Lidman outlines a proof of the following interesting result in topology of manifolds: there is a compact smooth 4-manifold W with boundary such that W is homotopy equivalent to S^2 but there does not exist an embedding $S^2 \to W$ which is a homotopy equivalence and is simplicial for some triangulations of W and of S^2. We present a shorter (and hopefully clearer) exposition. We reveal that some parts of the proof are missing, and that some results are used in that paper without proof or reference, or even without explicit statement.

Expository publications for university students

[15] A. Skopenkov, Mathematics via problems: from olympiades and math circles to a profession. Algebra. AMS, Providence, to appear.

This is a collection of teaching materials used in several Russian universities, schools, and mathematical circles. Most problems are chosen in such a way that in the course of the solution and discussion a reader learns important mathematical ideas and theories. The materials can be used by both teachers and students.

[16] A. Skopenkov, A user's guide to basic knot and link theory, `https://www.mccme.ru/circles/oim/exalg_eng.pdf`

We define simple invariants of knots or links (linking number, Arf-Casson invariants and Alexander-Conway polynomials) motivated by interesting results whose statements are accessible to a non-specialist or a student. The simplest invariants naturally appear in an attempt to unknot a knot or unlink a link. Then we present certain 'skein' recursive relations for the simplest invariants, which allow to introduce stronger invariants. We state the Vassiliev-Kontsevich theorem in a way convenient for calculating the invariants themselves, not only the dimension of the space of the invariants. No prerequisites are required; we give rigorous definitions of the main notions in a way not obstructing intuitive understanding.

[17] A. Skopenkov, Algebraic Topology From Geometric Viewpoint, MCCME, Moscow, 2nd edition to appear `http://www.mccme.ru/circles/oim/home/combtop13.htm#photo`

In this book we present a 'geometric' approach to algebraic topology. The book is essentially rewritten for the second edition.

[18] A. Skopenkov, Algebraic Topology From Algorithmic Viewpoint, draft of a book, `http://www.mccme.ru/circles/oim/algor.pdf` (some sections are rewritten in 2019)

In this book we present an 'algorithmic' approach to algebraic topology.

3.1.20 Evgeni Smirnov

[1] Multiple flag varieties

Journal of Mathematical Sciences, to appear in 2020

This is a survey of results on multiple flag varieties, i.e. varieties of the form $G/P_1 \times \cdots \times G/P_k$. We provide a classification of multiple flag varieties of complexity 0 and 1 and results on the combinatorics and geometry of B-orbits and their closures in double cominuscule flag varieties. We also discuss questions of finiteness for the number of G-orbits and existence of an open G-orbits on a multiple flag variety.

[2] On quadratic residues, Kvant No. 10 (2019), pp. 2-11 (in Russian).

This is a popular article for schoolchildren majoring in mathematics and undergraduate students. We explain the basics of the theory of quadratic residues and provide several proofs of the quadratic reciprocity law.

3.1.21 Alexei Sossinsky

[1] Topology-1 : Lecture Notes, Independent University of Moscow, 101 pages, 2018 (actually appeared in 2019)

This booklet contains brief lecture notes of a one-semester introductory course in topology given in 2017 at the Independent University of Moscow in the framework of the "Math in Moscow" program.

[2] "Tolerance Spaces Revisited I : Almost Solutions", Mathematical Notes, vol. 106, No. 3., pp. 439-445 (2019)

The paper gives a brief review of tolerance space theory and develops its applications to finding almost solutions (i.e., functions that, substituted into the given equation, satisfy it up to a small numerical error) for equations of different types, providing existence theorems (proved by homological methods) of almost solutions for a wide variety of equations.

[3] "On the equivalence of three models of the hyperbolic plane", Matematicheskoe Prosveschenie, 25, pp.7-17 (2020) (in Russian)

In the paper, it is shown, in two different ways, that the three classical models of the hyperbolic plane (the Cayley–Klein model, Poincare disk and half-plane models) are equivalent. First, by means of physical experiments with a toy (a transparent hemi-sphere) and then rigorously, by specifying isomorphims of the three models regarded as Klein geometries, (i.e., sets with transformation groupos acting on them).

[4] Introduction to Topology : a Lecture Course (in Russian), 240 pages, MCCME Publishers, 235 pages, in print.

The book is a two-semester introductory course in topology given in the form of revised and expanded lectures. The first part, called "Elementary Topology" is a basic introduction to the subject, emphasizing the geometric and algebraic aspects, the second part, called "Introduction to Algebraic Topology" corresponds to a one-semester graduate-level course on the subject.

[5] "Sum of the angles of triangles and the Gauss–Bonnet theorem", 11 pages (in Russian) to appear in Matematicheskoe Prosveschenie

In this paper, we present a discrete version of the Gauss–Bonnet theorem (with a new proof) and show that this version of the theorem is a natural generalization of Euclid's theorem on the sum of angles of a triangle.

[6] (Jointly with A.M.Vinogradov et al), Smooth Manifolds and Observables, Second revised and expanded edition, 443 pages, Submitted to Springer Verlag.

The main objective of this book is to explain how the differential calculus on smooth manifolds is a natural part of commutative algebra. This is achieved by studying the corresponding algebras of smooth functions, which yield a general construction of the differential calculus on various categories of modules over the given commutative algebra. This approach opens the way to numerous applications, ranging from delicate questions of algebraic geometry to the theory of elementary particles.

3.2 Scientific conferences and seminar talks

3.2.1 Vsevolod Adler

[1] Talk (with A.B. Shabat) "Some exact solutions of the Volterra lattice" (L.D. Landau Institute for theoretical physics)

[2] Talk "Nonautonomous symmetries of the KdV equation and step-like solutions" (L.D. Landau Institute for theoretical physics)

3.2.2 Alexander Belavin

[1] International conference "Interaction Between Algebraic Geometry and QFT", Dolgoprudny, MIPT, 24/06 –28/06 2019

Talk "JKLMR conjecture and Batyrev's polytopes";

[2] Trieste , ICTP (International Ccentre of Theoretical Physics), August 17 – September 9 2019

Talk "The (2,2) Supersymmetric Gauged Theories for Calabi-Yau Manifolds of Berglund-Huebsch Class"

3.2.3 Konstantin Fedorovsky

[1] International conference "8th Russian-Armenian Workshop on Mathematical Physics, Complex Analysis and Related Topics", September 16-20, 2019, Moscow, Russia.

Invited talk "On Lip^m- and C^m-reflection of harmonic functions".

[2] The 27th International Conference on Finite and Infinite Dimensional Complex Analysis and Applications, August 12–16, 2019, Krasnoyarsk, Russia.

Invited talk "On Chui's conjecture and approximation by simplest fractions".

[3] 28th St. Petersburg Summer Meeting in Mathematical Analysis, June 25–30, 2019, St. Petersburg, Russia.

Plenary (invited) talk "Approximation by simplest fractions and Chui conjecture".

[4] Workshop "Reproducing Kernels in Function Spaces and Their Applications", June 3–7, 2019, St. Petersburg, Russia.

Plenary (invited) talk "Dimension of boundaries of Nevanlinna domains".

[5] Conference "One-Dimensional Complex Analysis and Operator Theory", May 13–17, 2019, St. Petersburg, Russia.

Invited talk "On Lip^m-reflection of harmonic functions".

3.2.4 Sergei Gorchinsky

[1] "Workshop on Diophantine problems and p-adic period mappings", Alpbach, Austria, June, 30 – July, 5

Talks "Hypersurfaces I", "Hypersurfaces II"

[2] Workshop "Mixed Hodge modules: applications", Tatihou, France, July, 22 – July, 26

Talk " Statement and sketch of proof of the vanishing theorem of Saito"

[3] Conference "VII Escuela IMCA: Algebraic Geometry", Lima, Peru, September, 16 – September, 20

Mini-course of 4 lectures "Chow motives and their applications"

3.2.5 Anton Khoroshkin

[1] Workshop "Props, graph complexes and moduli spaces" held at the University of Luxembourg http://math.uni.lu/propmoduli/programme.html

talk "Cacti groups, real locus of Deligne-Mumford compactification of $\overline{M_{0,n}}$".

[2] Conference "Graph complexes in algebraic geometry and topology" held at the university of Manchester: http://ibykus.sdf.org/graphc/index.html

talk "Real locus of $\overline{M_{0,n}}$, cacti groups and single generated Hopf operads"

3.2.6 Iosif Krasilshchik

[1] Conference “Dynamics, Geometry and Analysis: 20 years of Mathematical Institute in Opava”, 8–13 September 2019, Hradec nad Moravicí, Czech Republic,
http://conferences.math.slu.cz/dga20/

Talk “Geometry of PDEs and symmetry properties of linearly degenerate integrable equations”

[2] Conference “Dynamics in Siberia”, 25.02–2.03 2019, Novosibirsk, Sobolev Institute of Mathematics and Novosibirsk State University,
http://math.nsc.ru/conference/ds/2019/

Talk “2D-reductions of the Mikhalev-Pavlov equation and their nonlocal symmetries”

[3] Conference “16th Conference Mathematics in Technical and Natural Sciences”, 30th June – 5th July, 2019. Kościelisko, Poland,
http://www.wms.agh.edu.pl/konferencje/mntp/

Talk “Nonlocal symmetries of linearly degenerate integrable equations”

[4] Conference “Differential Geometry and its Applications”, Hradec Králové, Czech Republic, September 1 – 7, 2019,
http://dga2019.uhk.cz/

Talk “Nonlocal Schouten and Nijenhuis brackets” (with A. Verbovetsky).

3.2.7 Maxim Leyenson

Andrei Levin’s seminar “Algebra and Geometry” at HSE.

Talk “Geometry and arithmetics of the (2,3,7)-triangle group, following Shimura and Elkies”

3.2.8 Alexei Litvinov

[1] Conference “Probability and quantum field theory: discrete models, CFT, SLE and constructive aspects”, Porquerolles,France, June 10th - 21st 2019

Talk “Introduction to AGT correspondence”

[2] Conference ”Conformal field theory in higher dimensions”, Moscow, September 2019

Talk ”On dual description of η-deformed OSP sigma models”

3.2.9 Taras Panov

[1] Conference “December Readings in Tomsk-2019”, Tomsk, Russia, December, 10–15.

Plenary talk “Polyhedral products, right-angled Coxeter groups, and hyperbolic manifolds”.

[2] International Conference “Toric Topology 2019 in Okayama”, Okayama, Japan, November, 18–21.

Plenary talk “Foliations arising from configurations of vectors, Gale duality, and moment-angle manifolds”.

[3] The 46th Symposium on Transformation Groups, Osaka, Japan, October, 31 – November, 2.

Plenary talk “Higher Whitehead products in moment-angle complexes”.

[4] International Conference “Morse Theory and its Applications” dedicated to the memory and 70th anniversary of Volodymyr Sharko, Kiev, Ukraine, September, 24–29.

Plenary talk “Right-angled polytopes, hyperbolic manifolds and torus actions”.

[5] International Conference “Topology, Geometry, and Dynamics: Rokhlin–100” dedicated to the 100th birthday anniversary of Vladimir Rokhlin, St. Petersburg, Russia, August, 19–23.

Plenary talk “A geometric view on SU-bordism”.

[6] International Conference “Algebra and Geometry”, Jaroslavl, Russia, July, 30–31.

Plenary talk “Higher Whitehead products in moment-angle complexes”.

[7] Yaroslavl Summer Mathematical School “Algebra and Geometry 2019”, Jaroslavl, Russia, July, 24–29.

Invited lectures “Complex geometry of manifolds with torus action”.

[8] Bilateral MSU-SJTU Conference on Dynamics in Finite and Infinite Dimensional System, SJTU, Shanghai, China, May, 13-14.

Invited talk “Foliations arising from configurations of vectors, and topology of nondegenerate leaf spaces”.

[9] FDIS 2019 Finite Dimensional Integrable Systems in Geometry and Mathematical Physics, SJTU, Shanghai, China, May, 7–11.

Plenary talk “Holomorphic foliations on complex moment-angle manifolds”.

3.2.10 Vladimir Poberezhnyi

[1] Seminar “Analitical theory of differential equations” (Steklov Mathematical Institute)

Talk “Cauchy problems for integrable pseudodifference hierarhies”

3.2.11 Yaroslav Pugai

[1] International conference dedicated to the 100th anniversary of I. M. Khalatnikov ”Quantum Fluids, Quantum Field Theory, and Gravity” October, 17-20, 2019 Chernogolovka, Russia.

3.2.12 Petr Pushkar

[1] A participation (without talks) at Homological Mirror Symmetry at HSE
http://hms.mirrorsymmetry.ru/

3.2.13 George Shabat

[1] Invited seminar talk, Birmingham University (UK), 11 February, 2019.
Computer problems posed by dessins d'enfants theory .
[2] Invited lecture, Uxbridge College (London), 10 June, 2019.
Three contemporaries of Newton: Wallis, Taylor and Stirling.
[3] International Conference "Riemann surfaces and Teichmüller theory" Euler International Mathematical Institute, St. Petersburg, July 12, 2019.
The plenary talk On the uniformization of Fried families.

3.2.14 Stanislav Shaposhnikov

[1] International workshop "LSA winter meeting"
(Laboratory of Stochastic Analysis and its Applications, HSE, Moscow, 02.12.2019-06.12.2019)
Talk: Distances between stationary distributions of diffusions.

3.2.15 George Sharygin

[1] Conference "Dynamics in Siberia, 2019", Novosibirsk, February, 25 – March, 2
Talk "Deformation quantization of commutative families"
[2] Conference "Differential geometry and applications", Hradec Kralove, September, 1 – September, 7
Talk "Geometry of full symmetric Toda system on compact groups"
[3] Conference "Geoquant2019", Taipei, September, 9 – September, 13
Talk "Geometry of full symmetric Toda system on compact groups"
[4] Visit to Koper, July
Talk "Combinatoric formulas for the characteristic classes of triangulated bundles", (University Primorska)
[5] Visit to Beijing, September-December
Talk "Geometry of full symmetric Toda system", (workshop on pure Mathematics, Peking University)
Talk "Bruhat order and the full symmetric Toda system", (Mathematics seminar, Peking University)

Talk “Bruhat order and the full symmetric Toda system”, (Mathematics seminar, Shanghai Jiao Tong University)

3.2.16 Arkady Skopenkov

[1] Topology, Geometry, and Dynamics: Rokhlin – 100, St Petersburg, August. Talk ‘Analogue of Whitney trick for eliminating multiple intersections’.

[2] Workshop Geometric, Algebraic, and Topological Combinatorics, Oberwolfach, August. No talk.

[3] Morse theory and its applications, Kiev, September. Talk ‘Analogue of Whitney trick for eliminating multiple intersections’.

[4] Probabilistic Methods in Discrete Mathematics, Petrozavodsk, May. Talk ‘Invariants of graph drawings in the plane’.

[5] 3rd Hungarian-Russian Combinatorics Workshop, Dolgoprudnyi, May. Talk ‘On van Kampen-Flores, Conway-Gordon-Sachs and Radon theorems’.

[6] Conference of Moscow Institute of Physics and Technology, Dolgoprudnyi, November. Talk ‘Stronger counterexamples to the topological Tverberg conjecture’.

[7] Math education at school and university, Kazan, October. Talk ‘Research problems for high-school students’.

[8] Department of Mathematics and Statistics, Faculty of Science, Masaryk University, Brno. Talk ‘Analogue of Whitney trick for eliminating multiple intersections’.

[9] Postnikov memorial seminar, Moscow State University. Talks ‘Embeddability of manifolds in Euclidean spaces’, ‘On van Kampen-Flores, Conway-Gordon-Sachs and Radon theorems’.

[10] Seminar for math teachers at MCCME. Talk ‘Studying derivative in a sequence of olympic problems’.

3.2.17 Evgeni Smirnov

[1] Representation theory of Lie groups, mathematical physics, and combinatorics

Two talks: “Schubert polynomials: from geometry to combinatorics and back”

[2] School “Facets of algebraic geometry”, LUMS, Lahore, Pakistan, April 16-18, 2019

Three talks “Geometry of flag varieties”

[3] COMSATS University, Islamabad, April 23, 2019

Talk “Young diagrams from Euler to our days”

[4] Lomonossov Moscow State University, Mechanics and Mathematics Department, seminar on Lie groups and invariant theory, March 27, 2019

Talk: “Subword complexes and slide polynomials”

[5] Independent University of Moscow, seminar “Riemann surfaces, Lie algebras, and mathematical physics”, March 29, 2019

Talk: “Subword complexes and slide polynomials”

3.3 Teaching

3.3.1 Vsevolod Adler

[1] Classic integrable systems. Independent University of Moscow, II year students, February-May 2019, 2 hours per week.

Program

1. Nonlinear waves and solitons
2. Conservation law and Miura transformation
3. Lax representation and higher flows of the KdV hierarchy
4. Zero curvature representation, examples of integrable equations
5. Commutativity of the Lie algebra of higher symmetries
6. Lagrangian and Hamiltonian structures
7. Multi-soliton and finite-gap solutions
8. Dubrovin equations and the Liouville intgrability
9. Inverse scattering method
10. Classic symmetries and self-similar reductions
11. Linearizable and exactly solvable equations
12. Darboux–Bäcklund transformations, the factorization method
13. Overview of problems in the theory of integrable systems

3.3.2 Alexander Belavin

[1] Introduction to Superstring Theory. Independent University of Moscow, IV-V year students, September-December 2019, 2 hours per week.

Program. Lecture 1.Introduction 1. Why strings? 2. The analogy with the relativistic particle. 3. Theory with the connections.

Lecture 2. Relativistic bosonic string 1. The action of Goto-Nambu. 2. The action of Polyakov. 4. The equivalence of the two approaches.

Lecture 3. Classical dynamics 1.Lagangian. 2.Action. 3.Equations of motion.

Lectures 4. Symmetries of the Bosonic string theory 1. Symmetries, Noether theorem, conserved current. 2. Symmetries on the world surface. 3. Symmetries in the physical space.

Lectures 5. Gauge symmetry 1. Fixing the conformal gauge. Faddev-Popov determinant. 2. Conformal field theory. 3. The free massless bosonic fields theory.

Lectures 6. Quantization of the Bosonic string theory 1. Canonical quantization. 2. Modes, commutation relations. 3. Fock space of states.

Lectures 7. Physical states in NSR String theory. 1. Imposing the physical requirements. 2. Algebra Virasoro. 3. The definition of the space of physical fields.

Lectures 8. Physical states in NSR String theory (continuation). 1. Physical states at levels 0.1 and 2. 2. The positivity of the norms of physical states at d = 26.

Lectures 9. 1. Katz theorem and Physical states. 2. Jacobi triple identity and the number of physical states.

Lectures 10.Neveu-Schwartz-Ramon fermionic string theory. 1. The action in superconformal gauge. 2. NS- and R-sectors of the Fock space. 3. Super-Virasoro algebra.

Lectures 11. 1. Fock space. Spinor vacuum of the R-sector. 2. Physical conditions conditions T=J=0 and N = 1 Super-Virasoro algebra.

Lectures 12. 1. Physical conditions at the lower levels. 2. The positivity of the states norm for a = 1/2(5/8) in the NS(R) sectors and d = 10.

Lectures 13. Katz's theorem 1. Katz theorem for N=1 Super-Virasoro algebra. 2. Counting the number of physical states on an arbitrary level.

Lectures 14. GSO projection 1. GSO projection, Jacobi triple identity and the number of physical states. 2. NS- R-. . . 3. Space-Time supersymmetry in the NSR String theory after GSO projection.

3.3.3 Yurii Burman

[1] Geometry. Independent University of Moscow, I year students, February–May 2019, 4 hours per week (2 hours of lectures and 2 hours of exercise sessions).

Program

1. Affine geometry.

(a) Affine group vs linear group of the parallel space.
(b) Category of affine spaces.
(c) Affine group of a hyperplane is its normalizer in the linear group of the ambient space.
(d) Action of the affine group on the tuples of points.

2. Convex geometry

   (a) Convexity of a ball.
   (b) Convex hull of a set.
   (c) Carathéodory's, Radon's, and Helly's theorems.
   (d) Hyperplane separation theorem (for convex closed sets in $\mathbb{R}^n$).
   (e) Krein–Milman's theorem in $\mathbb{R}^n$.
   (f) Equivalence of two definitions of a compact convex polyhedron.

3. Projective geometry.

   (a) Projective group vs linear group.
   (b) Projective group vs affine group.
   (c) Action of the projective group on the tuples of points.
   (d) Desargues's theorem and its dual.
   (e) Pappus's hexagon theorem, Pascal's theorem and their duals.

4. Bilinear and quadratic forms, quadrics.

   (a) Projective classification of quadratic forms over any field of char $\neq 2$, over $\mathbb{C}$ and over $\mathbb{R}$.
   (b) Affine classification of quadratic forms over any field of char $\neq 2$, over $\mathbb{C}$ and over $\mathbb{R}$.
   (c) Affine classification of quadrics over $\mathbb{C}$ and over $\mathbb{R}$.

5. Lobachevsky's geometry.

   (a) Four standard models of the Lobachevsky's geometry.
   (b) Affine Lobachevsky's group in the four standard models.
   (c) Action of the affine Lobachevsky's group on the set of triples of points on the absolute.

(d) Action of the affine Lobachevsky's group at the set of flags. Existence and uniqueness of the invariant metrics.

(e) Invariant metrics in the four standard models.

(f) Area of the triangle vs its defect.

[2] Topology, Indepenedent University of Moscow, I year students, September–December 2019, 4 hours per week (2 hours of lectures and 2 hours of exercise sessions).

Program.

1. Singular (chain and cochain) complex of a topological space.
2. Homology and cohomology as functors from homotopy category to the category of graded modules. A similar theorem for the relative case. Barycentric subdivision lemma.
3. Bockstein's lemma and the Mayer–Vietoris sequence.
4. Degree of a map. Action of the maps of sphere wedges on their homology.
5. Higher homotopy groups. The n-th homotopy group of the n-sphere is $\mathbb{Z}$.
6. Borsuk's lemma about cell pairs.
7. Cell homology of cell complexes; its equivalence to the singular homology.
8. Excision lemma.
9. Degree of a smooth map as a sum of signs of preimages of a regular value.
10. Smooth manifolds. Homology orientability vs atlas orientability.
11. Higher homology of a compact orientable manifold.
12. Multiplication in cohomology.

[3] Calculus-1. Higher School of Economics, I year students, September–December 2019, 4 hours per week (2 hours of lectures and 2 hours of exercise sessions).

Program.

1. Real numbers. Supremum and infimum. Definition and properties of operations over reals.
2. Topological spaces. Continuous maps. Cartesian products of spaces and maps. Continuity of addition and multiplication, and continuous extension of these operations to infinity.

3. Limits. Limits of monotonic maps. Limits of the composition.
4. Nested intervals theorem. Intermediate value theorem.
5. Fundamental maps into $\mathbb{R}^n$. Fundamentality vs convergence. Basic properties of series: absolute convergence implies convergence and the product of absolutely convergent series converges. Exponential function.
6. Closed sets and limit points.
7. Compacts. Closedness of compacts. A closed subset of a compact is a compact. A continuous image of a compact is a compact. Description of compacts in $\mathbb{R}^n$.
8. Derivatives. Theorems of Rolle, Lagrange and Cauchy. Derivative of the composition and of the functional inverse.
9. Taylor's formula: Peano's and Lagrange's forms of the remainder.
10. Uniform convergence. Limit of a limit vs double limit in the uniform case. A uniform limit of continuous maps is continuous.
11. Power series. Radius of convergence. Absolute uniform convergence on compacts in the convergence disk. Infinite differentiability of the sum.
12. Uniform comtinuity. Uniform continuity of continuous maps of metric compacts.

3.3.4 Konstantin Fedorovsky

[1] Complex Analysis. Independent University of Moscow, II year students, February–May 2019, 2 hours per week.

Program:

1) Complex number, their properties and operations on them. The function e^z and exponential form of complex numbers. Complex plane $\mathbb{C}$ and its compactification $\mathbb{C}_\infty$. Functions of a complex variable and their real and complex differentiability.

2) Holomorphic and harmonic functions. Conformity and its relationship with holomorphicity. Basic elementary functions of a complex variable and their properties. Construction of conformal mappings of simple domains using compositions of elementary functions. Power series, Cauchy–Hadamard formula, disk of convergence, holomorphicity of the sum of a power series.

3) Paths and curves in $\mathbb{C}$. Increment of the argument along a path. Index of a path and its properties. Integral over a path and over a curve by a complex variable and their properties. Goursat lemma. Cauchy integral theorem. Complex primitive, its properties

and Newton–Leibnitz formula. Existence of holomorphic branches of the root function and of the logarithm in simply connected domains in $\mathbb{C} \setminus \{0\}$.

Cauchy integral formula, Cauchy formula for derivatives and infinite differentiability of holomorphic functions. Pompeiu formula. Mean value theorem, maximum modulus principle. Morera theorem. Local uniformly convergence of sequences of holomorphic functions. Weierstrass theorem.

4) Taylor series. Expansion of holomorphic function into a power series. Cauchy inequalities for Taylor coefficients. Liouville theorem. Zeros of holomorphic functions. Uniqueness theorem. Singular points at the boundary of the disk of convergence of a power series. Pringsheim theorem. Approximation of holomorphic functions by polynomials, Runge's theorem.

5) Laurent series. Expansion of holomorphic functions into a Laurent series. Cauchy inequalities for Laurent coefficients. Isolated singularities of holomorphic functions and their classification, Sokhotskii theorem. Infinity as a singular point, entire and meromorphic functions with poles at infinity.

Residues. Cauchy residues theorem. Residue at ∞. Evaluation of residues. Jordan lemma. Evaluation of integrals (including integrals in the sense of principal values) using the method of residues. Logarithmic residue. Argument principle. Rouche theorem. Domain preservation principle.

6) Univalent functions and their basic properties (criteria for local and global univalence, Hurwitz theorem and its corollaries, area theorem and Koebe theorem).

7) Schwarz lemma and conformal automorphisms of the basic domains. Compact families of holomorphic functions. Riemann mapping theorem. Riemann–Schwarz symmetry principle, and its applications for construction of conformal mappings of a given domains. Elliptic sinus and modular function. Picard theorems. Inverse boundary correspondence principle for conformal mapping. Boundary behavior of conformal mappings (Carathéodory theorems, Lindelof theorem, Kellogg–Warschawski theorem).

8) Analytic elements and their analytic continuation. Analytic continuation along a path and by a chain. Theorem about continuation by homotopic paths and monodromy theorem.

Complete analytic function in the sense of Weierstrass, its holomorphic branches. Branch points of analytic functions and their classification. Complete analytic function 'root' and 'logarithm'. The concept of a Riemann surface.

[2] Theory of functions of a complex variable, Bauman Moscow State Technical University, II year students, February–June 2019, 3 hours per week.

Program:

1) Complex numbers, complex plane. Elements of topology of the complex plane. Increment of the argument along a path. Index of a path with respect to a point and its properties.

2) Differentiability of functions of a complex variable. Cauchy–Riemann conditions. Holomorphic functions. The notion of a conformal mapping. Elementary functions of a complex variable and their properties.

3) Integration of complex functions over a curve. Complex primitive. Goursat lemma. Cauchy integral theorem. Cauchy integral formula. Mean value theorem. Cauchy formula for derivatives and infinite differentiability of holomorphic functions. Morera theorem.

4) Power series and their domains of convergence. Holomorphicity of the sum of a power series. Taylor series expansion of a holomorphic function. Cauchy inequalities. Liouville theorem.

5) Loran series and their domains of convergence. Cauchy inequalities for coefficients of Loran series. Isolated singularities of holomorphic functions and their description. Sokhotskii theorem. Infinity as a singular point.

6) Residues (definition, basic properties and formulae for evaluation of residues). Cauchy residue theorem. Residue at the infinity and theorem on the total sum of residues. Residue with respect to a domain. Jordan lemma. Evaluation of integrals using the method of residues.

7) Logarithmic residue and its properties. Argument principle. Rouché theorem. Maximum modulus principle and domain preservation principle.

8) Criteria of univalence and local invertibility. Hurwitz theorem and its corollaries. Properties of univalent functions.

9) The concept of a conformal mapping. Elementary functions and respective conformal mappings. Inverse boundary correspondence principle for conformal mappings. Riemann-Schwarz symmetry principle and its applications.

10) Schwarz's lemma and evaluation of groups of conformal automorphisms of basic domains. Riemann theorem and Carathéodory extension theorem.

11) The concept of an analytic continuation. Weierstrass theory. Analytic functions and their singularities.

12) Laplace transform, and its properties. Using the operational calculus for solving ordinary differential equations.

3.3.5 Sergei Gorchinsky

[1] Around algebraic theory of multiple zeta-values I. Independent University of Moscow, master students and PhD students, September-December 2019, 2 hours per week.

Program

1. Introduction to multiple zeta values: definition, series of relations, statement of conjectures and theorems.

2. Iterated integrals: definition and basic properties, MZV as iterated integrals, holonomy interpretation, relative cohomology intepretation.

3. Hopf algebras: c-vector spaces, coalgebras, bialgebras, general properties, unipotent filtration, prounipotent Hof algebras.
4. Prounipotent completion of groups: existence and uniqueness, Quillen formula, the case of a free group.
5. Symmetric tensor categories: graded vector spaces, complexes, bicomplexes, algebras in symmetric tensor categories.
6. Simplicial sets: general properties, simplicial vector spaces, normalized complex, Dold–Kan correspondence, Eilenberg–Zilber and Alexander–Whitney maps.
7. Bar-complex: the case of a commutative algebra, generalization for commutative dg-algebras, reduced bar-complex, Hopf algebra structure on H^0, relation with relative cohomology and iterated integrals.
8. Chen theorem: statement, Riemann–Hilbert correspondence, proof of Chen theorem, generalization for tensor symmetric abelian categories.

3.3.6 Anton Khoroshkin

[1] Basic algebra for I year bachelor students, Independent university of Moscow, fall 2019, 2hours lecturs + 2 hours exercises class per week Program include:

- Commutative algebraic structures: rings, fields, abelian groups;
- Vector spaces, System of linear equations, linear maps;
- Groups, group action, orbits, stabilizers, normal subgroups;
- p-Groups, Sylov's theorems;
- Jordan-Hölder decomposition, simple, solvable and nilpotent groups;
- Dual vector space, bilinear forms, orthogonal basis;
- multilinear forms and determinants.

[2] Introduction to Commutative Algebra, NRU Higher School of Economics, III-IV year bachelor students and master students, Spring 2019, 2 hours lectures + 2 hours seminars per week.
Program includes the following subjects

- Rings, algebras, ideals and modules

- Noetherian rings
- Unique factorization domains
- Rings and modules of fractions
- Integral dependence and Noethers normalization theorem
- The going-up and going-down theorems
- Limits, colimits and tensor product
- Flat and projective modules
- Hilbert Nullstellensatz
- The spectrum of the ring
- Krull dimension and transcendence degree
- Primary decomposition
- Discrete valuation rings and Dedekind domains
- Dimension theory for noetherian rings
- Hilbert series

[3] Additional chapters of Algebra, NRU Higher School of Economics, II-III year bachelor students, Spring 2019, 2 hours seminars per week (lectures are given by L.Rybnikov). Program includes the following subjects:

- Principal ideal domains; factorian domains; rings of fractions; Gauss theorem;
- Resultant and discriminant; Bezout theorem;
- Modules over rings, Jordan-Hölder and Krull-Schmidt theorems;
- Modules over the rings of principal ideal domain;
- integral extensions and algebraic integers;
- Noetherian rings; Hilbert's basis theorem and Hilbert's theorem on invariants; Hilbert Nullstellensatz.
- modules over the algebras and Schur's lemma.
- double centralizer theorem and classification of semisimple algebras.

- Representations of finite groups, Maschke's theorem, characters.
- Induced representations and Frobenius reciprocity;
- Representations of symmetric groups and Schur-Weyl duality.

3.3.7 Iosif Krasilshchik

[1] Symmetries of differential equations and recursion operators Calculus. Independent University of Moscow, senior students, post-graduates, February-May 2019, 2 hours per week.

Program

1. Jets of fiber bundles. The Cartan distribution.
2. "Geometrization" of PDEs. Solutions.
3. Classical symmetries. Lie-Bácklund theorem.
4. Symmetries of ODEs. Bianchi-Lie theorem.
5. Infinite jets and infinite prolongations. The Cartan distribution.
6. Higher symmetries. Evolutionary vector fields and linearizations.
7. Computational examples. The Lenard recursion operator for symmetries of the KdV equation.
8. Algebraic theory of recursion operators. Nijenhuis bracket.
9. Differential coverings. Conservation laws. Nonlocal symmetries.
10. The tangent covering. Canonical conservation laws. Nonlocal recursion operators.

3.3.8 Mikhail Lashkevich

[1] General Relativity Theory. III year students, September–December 2018, 3 hours per week.

Program

The texts of the lecture can be found at:
`https://homepages.itp.ac.ru/~lashkevi/lectures/gr18/`.

1. Geometry and physics of special relativity theory:
   - nondegenerate symmetric form in a linear space and signature;
   - metric in an affine space, index and indexless notation;
   - principle of least action for particles and fields.

   Seminar: Lorentz transformation, coordinate systems, metric.

2. Main concepts of differential geometry and space-time
   - manifolds, tangent bundles, tensor fields;
   - affine connection and covariant derivatives;
   - metric and signature;
   - Levi-Civita connection.

   Seminar: Physical interpretation of metric: time and space intervals, synchronization of clocks.

3. Riemann curvature. Transformations of tensor fields:
   - Riemann curvature tensor, Ricci tensor and Ricci scalar;
   - properties of the curvature tensor, Bianchi identities;
   - transformation of coordinates and Lie derivative.

   Seminar: Symmetries of metric and Killing vector fields.

4. Particles in a curved space-time:
   - a particle in a curved space-time, geodesics, external electromagnetic field;
   - Hamilton–Jacobi equation.

   Seminar: Hamilton–Jacobi equation, separation of variables and integrability.

5. Fields in a curved space-time and energy-momentum tensor:
   - Action and equations of motion;
   - Canonical energy-momentum tensor in flat space-time;
   - Energy-momentum tensor in curved space-time and its covariant conservation.

   Seminar: Energy-momentum tensor for different physical systems

6. Equations of gravitation field and conservation laws:
   - Hilbert–Einstein action;
   - Einstein equations, their structure, number of degrees of freedom;
   - Energy-momentum pseudotensor and conservation laws.

   Seminar: Total energy of stationary system.

7. Weak gravitational field:
   - linearized Einstein equations, gauge fixing;
   - static solutions to linearized Einstein equations, residual gauge freedom;
   - energy and angular momentum via asymptotics of gravitational field.

   Seminar: Problems on gravitational field in the linear approximation.

8. Gravitational waves:
   - free solutions of homogeneous linearized Einstein equations;
   - plane waves, monochromatic waves, gauge fixing, polarizations;
   - energy-momentum pseudotensor of a plane gravitational wave.

   Seminar: Strong gravitational wave.

9. Emission of gravitation waves:
   - retarded solution to the wave equation, simplification in dimension 4;
   - non-relativistic source: restrictions by the energy-momentum conservation and quadrupole emission;
   - angular distribution and calculation of the total emitted energy.

   Seminar: Interaction of gravitational waves with condensed matter and electromagnetic field.

10. Schwartzschild solution:

- spherically symmetric Einstein equation, its direct solution;
- geodesics in the Schwartzschild metric, incompleteness of the Schwartzschild coordinates;
- Eddington–Finkelstein coordinates, event horizon and singularity;
- Kruskal coordinates and Penrose diagram, maximally extended Kruskal manifold.

Seminar: Gravitational field of static spherically symmetric body, static equilibrium condition.

11. Motion of a particle in the Schwartzschild metric:

- solution of the Hamilton–Jacobi equation;
- four types of motion in the Schwartzschild metric, conditions for their realization.

Seminar: Fall of a layer of dustlike matter on a black hole.

12. Motion in a rather weak gravitational field and experimental checks of general relativity:

- nearly-Newtonian field and perihelion precession;
- deviation of a light beam in a weak gravitational field.

Seminar: Isotropic hypersurfaces: invariance of images.

13. Charged and rotating black holes:

- Reissner–Norstrm solution of Maxwell and Einstein equations;
- singularity, event horizon, Cauchy horizon, Penrose diagram;
- Kerr–Newman solution, ergosphere, ring singularity, Penrose diagram.

Seminar: Isotropic hypersurfaces: evolution of images.

14. Cosmological solutions. Friedmann models:

- homogeneous and isotropic Universe, constant curvature spaces;
- Friedmann equations, their solutions for dustlike matter and ultrarelativistic gas;
- Cosmological constant and accelerating expansion, modern model of expanding Universe, dark matter and dark energy.

Seminar: Difficulties of the Friedmann models. Inflation theory.

3.3.9 Maxim Leyenson

Topics in explicit algebraic geometry – 2. Independent University of Moscow, September – October 2019.

Program

- Topics in the Grassmannian varieties, characteristic classes, and intersection theory
    - ⋆ Intersection theory on Gr(2,4): Schubert classes on Gr(2,4). Trying to intersect them: the idea of the rational equivalence.
    - ⋆ Rational equivalence of cycles on algebraic variety. Properly intersecting cycles, and their intersections. Severi(-Chow) ring of cycles up to the rational equivalence on a smooth algebraic variety.
    - ⋆ Characteristic classes of vector bundles with values in the Sever-Chow ring: Segre's definition, axiomatic definition, and other definitions. Basic properties of characteristic classes.
    - ⋆ Characteristic classes of tautological bundles on Grassmanian varieties. Examples: 27 lines on a cubic surface, via top Chern class; 2875 lines on a quintic threefold.

3.3.10 Alexei Litvinov

[1] Introduction to conformal field theory. Independent University of Moscow, January-May, September-December 2019, 2 hours per week.

1. Scale and conformal invariance. Conformal group in $D-$dimansions. Conformal group in 2 dimensions. Local and global conformal transformations.

2. Basics of classical and quatum Euclidean field theory: equations of motion, symmetries, Noether theorem, Ward identities,operator product expansion

3. Role of stress-energy tensor in two-dimensional conformal field theory, Virasoro algebra, conformal Ward identities, classification of fields

4. OPE in conformal field theory, conformal blocks

5. Degenerate representations of Virasoro algebra, BPZ differential equation, OPE for degenerate fields

6. Analytic structure of conformal blocks, Zamolodchikov's recursion formula

7. Representations of Virasoro algebra with special central charge, minial models of CFT

8. Simplest CFT's: free boson CFT, free fermion CFT, $\beta-\gamma$ system

9. Conformal field theory in a curved space-time, conformal anomaly, Liouville field theory

10. Classical Liouville field theory, simplest solutions, classical conformal block, accessory parameters

11. Quantum Liouville field theory, correlation functions, structure constants of operator algebra

12. Free field representation for correlation functions, Dotsenko-Fateev integrals, Selberg integrals

13. CFT with boundary, boundary states

14. CFT on a torus, partition function, modular properties of partition function

15. CFT's with higher spin symmetries, $W-$algebras

16. Integrable perturbations of minimal models and quantum KdV system

17. AGT basis and explicit formula for conformal blocks

3.3.11 Taras Panov

[1] Topology-3. Independent Independent University of Moscow, II year students, February–May 2019, 2 hours per week.

Program:

1. Topological and smooth manifolds, Poincaré duality.
2. Vector and principal bundles, classifying spaces.
3. Cohomology of classifying spaces.
4. Stiefel–Whitney, Chern and Pontryagin characteristics classes.
5. Characteristic numbers, Hirzebruch genera.
6. Leray spectral sequence of a filtration, Serre spectral sequence of a fibration, multiplicative structure.

[2] Linear algebra and geometry, Faculty of Mathematics and Mechanics, Moscow State University, I year students, February–May 2019, 4 hours per week.

Program:

1. Vector spaces.
2. Linear operators.
3. Geometry of Euclidean and Hermitian spaces.
4. Operators in Euclidean and Hermitian spaces.
5. Bilinear and sesquilinear functions.
6. Tensors.

http://higeom.math.msu.su/people/taras/teaching/panov-linalg.pdf

[3] Introduction to topology, Faculty of Mathematics and Mechanics, Moscow State University, II year students, September-December 2019, 2 hours per week.

Program:

1. Necessary facts from point-set topology.
2. Operations on topological spaces.
3. Homotopy and homotopy equivalence.
4. Cellular (CW) complexes.
5. Fundamental group.
6. Van Kampen Theorem.
7. Fundamental group of cellular complexes.
8. Coverings.
9. Fibrations.
10. Homotopy groups.

http://higeom.math.msu.su/people/taras/teaching/panov-topology1.pdf

[4] Complex cobordism and torus actions. Part II (advanced course), Department of Mathematics and Mechanics, Moscow State University, II–VI year students, February–May 2019, 2 hours per week.

Program:

1. Formal group laws and Hirzebruch genera.
2. Equivariant bordism: geometric and homotopic approaches.
3. Localisation theormes in cobordism theory.
4. Rigidity and fibre multiplicativity of Hirzebruch genera.

3.3.12 Alexei Penskoi

[1] Complex Geometry. National Research University Higher School of Economics, for students starting from the IIIrd year, February-May 2019, 4 hours per week (2 hours lectures + 2 hours exercise classes).

Program.

1. Complex analysis of several variables

2. Complex analytic manifolds and holomorphic vector bundles

3. Sheafs

4. Connections and their curvature

5. Characteristic classes

6. Holomorphic linear bundles and divisors.

7. Harmonic theory on compact manifolds.

8. Hodge decomposition on compact Kähler manifolds.

9. Hodge manifolds.

10. Kodaira theorem.

[2] Complex Geometry. National Research University Higher School of Economics, for students starting from the IIIrd year, September-December 2019, 4 hours per week (2 hours lectures + 2 hours exercise classes).

Program is exactly as in the Spring Semester.

[3] Analytic geometry, Moscow State University, Ist year students, September-December 2019, 8 hours per week (4 hours lectures + 4 hours exercise classes)..

Program.

1. Geometric theory of conics, elementary properties.

2. Line, Plane and Space, vectors and points.

3. Linear independence, basis, frame.

4. Scalar product, cross product, oriented area and volume.

5. Line, Plane

6. Quadrics in plane. Canonical form, canonical frame.

7. Asymptotic directions, diameters, principal directions and diameters, principal axes, tangent lines.

8. Polar correspondence.

9. Pascal and Brianchon theorems.

10. Affine transformations and isometries. Chasles Theorem.

11. Metric and affine classification of planar quadrics.

12. Quadrics in space. Canonical form, canonical frame.

13. Asymptotic directions, diametral planes, principal directions and diametral planes, principal axes, tangent lines and planes.

14. Particular types of quadratic surfaces. Doubly ruled surfaces.

15. Metric and affine classification of space quadrics.

16. Applications of quadrics.

17. Projective geometry of line and plane.

18. Projective transformations.

19. Projective classification of planar quadrics.

[4] Differential Geometry. Independent University of Moscow, II year students, February-May 2019, 4 hours per week (lecture 2 hours + exersise class 2 hours).

Program

1. Curves and surfaces in the plane and the three-dimensional space. Curvature, torsion, Frenet frame. First and second fundamental forms. Principal curvatures, mean curvature and Gauß curvature. Mean curvature normal vector. Euler formula for the normal section curvature.

2. Surfaces in n-dimensional space. First and second fundamental forms. Connections in the tangent and normals bundles on a surface. Second fundamental form and Weingarten operator. Gauß-Weingarten derivational equations. Gauß-Bonnet theorem for surfaces.

3. Basic theory of Lie groups ans algebras.

4. Vector bundles and gluing cocycles. Structure group. Euclidean and hermitian bundles. Natural operations with bundles. Orientable bundles.

5. Connections in vector bundles. Connection local form, Christoffel symbols. Connections in euclidean and hermitian bundles. Connections compatible with metrics and their curvature.

6. Riemannian manifolds. Curvature, torsion. Levi-Civita connection. Symmetries of curvature tensor. Ricci tensor. Scalar curvature.

7. Riemannian manifolds II. Geodesics. Geodesic coordinates. Lagrangian approach to geodesics. Second variation.

8. Submanifolds of Riemannian manifolds. First and second fundamental forms.

9. Laplace-Beltrami operator and minimal submanifolds, Takahashi theorem.

10. Characteristic classes. Chern-Weil construction of characteristic classes. Chern, Pontryagin and Euler classes and their properties.

[5] Analysis on Manifolds, Independent University of Moscow, 2 year students, September-December 2019, 4 hours per week (lecture 2 hours + exersise class 2 hours).

Program.

1. Reminiscences from Calculus: implicit function theorem, inverse function theorem, rank theorem. Surfaces in affine spaces and different ways of their definition.

2. Smooth manifolds. Partition of unity. Maps of manifolds.

3. Tangent vectors and differential of a map. Tangent and cotangent spaces.

4. Immersions, submanifolds, submersions.

5. Vector fields. Commutator of vector fields. Integral curves of a vector filed. One-parametric group generated by a vector field.

6. Tensor fields, differential forms. Riemann metric, volume form. Exterior differential.

7. Lie derivative. Cartan identity. Hodge operation. Relation between d and grad, rot and div.

8. Distributions and Frobenius theorem.

9. Orientation of a manifold. Integration of forms over manifolds.

10. Manifolds with boundary. Stokes theorem for manifolds with boundary. Relation to Green, Stokes and Gauß-Ostrogradsky formulas in calculus.

11. De Rham cohomologies, de Rham cohomologies with compact support. Poincaré lemma. Mayer-Vietoris long exact sequence.

12. Properties of de Rham cohomologies (finite dimension, Künneth formula etc).

13. Basics of Lie groups and algebras.

14. Actions of Lie groups. Homogeneous spaces.

15. Sarde lemma. Transversality. Whitney theorem.

[6] Spectral Geometry. Moscow State University, for students starting from the IIIrd year, September-December 2019, 2 hours per week.

Program

1. Laplace-Beltrami operator on Riemannian manifolds

2. Eigenvalues of Laplace-Beltrami operator (Dirichlet problem, Neumann problem, problem on manifolds without boundary)

3. Variational desription of eigenvalues, Rayleigh quotient

4. Weyl function and its asymptotics

5. Inequalities for eigenvalues, Dirichlet-Neumann bracketing

6. Nodal domains, Courant nodal domain theorem
7. Isoperimetric inequalities, symmetrization
8. Cheeger isoperimetric constant, Cheeger inequality
9. Conformal volume, Yang-Yau inequality
10. Extremal metrics and minimal submanifolds of spheres
11. Examples

[7] Topology-I. "Math in Moscow" program at the Independent University of Moscow for undergraduate students from the U.S. and Canada, February-May 2019, 4 hours per week (lecture 2 hours + exersise class 2 hours).

Program

1. The language of topology. Continuity, homeomorphism, compactness for subsets of $\mathbb{R}^n$ (from the epsilon-delta language to the language of neighborhoods and coverings).
2. The objects of topology: topological and metric spaces, cell spaces, manifolds. Topological constructions (product, disjoint union, wedge, cone, suspension, quotient spaces, cell spaces, examples of fiber bundles).
3. Examples of surfaces (2-manifolds), orientability, Euler characteristic. Classification of surfaces (geometric proof for triangulated surfaces).
4. Homotopy and homotopy equivalence, fundamental groups and their elementary properties.
5. Fundamental group and covering spaces. Algebraic classification of covering spaces (via subgroups of the fundamental group of the base). Branched coverings, Riemann-Hurwitz theorem.
6. Knots and links in 3-space. Reidemeister moves. Polynomial invariants.

[8] Differential Geometry. Math in Moscow program of the Independent University of Moscow for undergraduate students from the U.S. and Canada, September-December 2019, 4 hours per week (lecture 2 hours + exersise class 2 hours).

1. Plane and space curves. Curvature, torsion, Frenet frame.
2. Surfaces in 3-space. Metrics and the second quadratic form. Curvature.
3. Connections in tangent and normal bundles to a k-surfaces in $\mathbf{R}^n$.

4. Parallel translations.

5. Geodesics.

6. Gauß and Codazzi formulas. “Theorema egregium” of Gauß.

7. Gauß-Bonnet theorem.

8. Extremal properties of geodesics. Minimal surfaces.

9. Levi-Civita connection.

10. Exponential map.

[9] Exercise classes for Linear Algebra and Geometry at Moscow State University: February-May 2019, 4 hours per week.

[10] Exercise classes for Topology-I at the National Research University — Higher School of Economics: January-March 2019, 3 hours per week.

[11] Exercise classes for Topology-I at the National Research University — Higher School of Economics: September-December 2019, 3 hours per week.

3.3.13 Vladimir Poberezhnyi

[1] Calculus. Independent University of Moscow, I year students, January-May 2019, 2 hours per week.

Program

1. Series

2. Lebesgue measure

3. Measurable functions

4. Fubini and Radon-Nikodym theorems

5. L_2 space and Fourier series

6. Gravitational potentials and harmonic functions

7. Variational problems

[2] With I.V.Vyugin Geometry and analysis of differential equations, Higher school of economics, III-VI year students, September-December 2019, 2 hours per week.

Program.

1. Complex differential equations
2. Singularities of differential equations
3. Invariants
4. Monodromy
5. Distributions, symmetry and integrability

3.3.14 Yaroslav Pugai

[1] Quantum Field theory of gauge fields and standard model of elementary particles. Independent University of Moscow. A joint course with MIPT. 5th grade students. September-December 2019, 2 hours lecture and 2 hours seminar per week.

Program

- Free fields
- Gauge invariance
- Wilson loop. Yang-Mills Lagrangian
- Lie algebras su(n) and their representations
- Comment on instantonic solutions
- Functional integral. Perturbation theory, Feynman rules
- Quantization. Faddeev-Popov ghosts
- Ward idenitites and their generalizations
- BRST quantization. Ghosts and unitarity
- Feynman diagramms
- One loop computations
- Regularization
- Renormalization
- Running coupling constant, beta function and renormalization group
- Beta function in Yang-Mills theory. Asymptotic freedom

- Selected topics from QCD
- Goldstone theorem and Higgs Mechanism
- Electroweak theory
- Selected topics from standard model
- Chiral anomaly

[2] Integrable models of statistical mechanics. Independent University of Moscow. A joint course with MIPT, 3 year students. February-June 2019, 2 hours lecture and 2 hours seminar per week.

Program.

- Basic notions of statistical mechanics.
- One dimensional Ising mode
- Two dimensional Ising model
- Disorder operator, fermions and energy
- Crammers Wannier duality and Star-Triangle equation
- Transfer Matrix
- Theta functions and solving functional equation
- Jordan-Wigner transform
- Free energy of Ising model
- Spin operator correlation functions
- Scaling limit. Critical exponents
- Six Vertex model. XXZ model
- Yang-Baxter equation. Transfer matrix
- Coordinate Bethe ansatz for XXZ
- Free energy via Bethe ansatz
- Algebraic Bethe ansatz
- Eight vertex model
- Corner transfer matrix for Ising model

3.3.15 Petr Pushkar

[1] Morse Theory, Independent University of Moscow, 2 year students and higher, special course, February-May 2019, 2 hours per week.

Program.

1. Morse functions, Morse lemma

2. Morse inequalities

3 Arnold's problem on estimation of a number of critical points on a compact manifold with boundary.

4. Pairs of complexies, attempts of classifications.

5. Combinatorics.

6. Robin Forman Morse theory.

7. Cerf diagramms and application for contact topology.

8. Morse theory and symplectic topology

[2] Ordinary Differential Equations, 2 year students, September-December 2019, 4 hours per week.

Program

1. Tangent vectors, Vector fields

2. Main theorems

3. Linear systems and equations, exponential function, Quasi-polynomials

4. Linearization and Lyapunov stability

5. Obstruction to integrability of distributions

6. First-order differential equations

7. Symplectic and Contact structures and differential equations

8. Examples.

3.3.16 George Shabat

[1] Algebra-2. Independent University of Moscow, 1 year students, February-May 2019, 2 hours per week.

Program

1. Modules and vector spaces.

2. Algebras

3. Systems of linear equations.

4. Fields.

5. Galois theory.

6. Solubility of polynomial equations by radicals.

[2] Algebra-3. Independent University of Moscow, 2 year students, September-December 2019, 2 hours per week.

Program
0. Categorical foundations of algebra.
1. Abelian categories.
2. Derived functors.
3. Group cohomology.
4. Representations of finite groups.
5. Groups of order ≤ 60.

3.3.17 Stanislav Shaposhnikov

1) Mathematical calculus. Independent University of Moscow, 1 year students, September – December 2019, 4 hours per week.

Program:

1. Sets. Functions. Equivalence relations. Partially or linearly ordered sets. Mathematical induction. Axiom of choice and Zorns lemma.
2. Real numbers: an axiomatic approach. Complex numbers.
3. Sequences and series. The Cauchy sequences.
4. Real numbers are the completion of the rational numbers. p-adic numbers.
5. Topology and metric spaces.
6. Complete metric spaces. Baire category theorem.
7. Compact sets. Hausdorff criteria.
8. Continuous functions. Pointwise and uniform convergence.
9. Differentiable functions. Taylor series.

3.3.18 George Sharygin

[1] K-theory and applications. Independent University of Moscow, special course (3-5 years students), February-May 2019, 2 hours per week.

Program. The Russian version has been sent to the organizers; the English variant is close to the program of [3].

[2] Differential geometry. Moscow State University, 1-st year master students.

Program

1. Theory of curves in $\mathbb{R}^2$ and $\mathbb{R}^3$: curvature, torsion, various formulas for them, Frenet's formulas etc.

2. Theory of surfaces in $\mathbb{R}^3$: the 1st and the 2nd quadratic forms, Menier theorem, normal sections, Euler's formula, Gauss and mean curvature, etc.

3. Gauss-Weingarten equations, Gauss-Petersen-Codazzi-Mainardi equations, Teorema egregium, non-isometric surfaces.

4. Covariant derivatives, parallel transport and geodesic lines, geodesic curvature.
5. Gauss-Bonnet's theorem: polyhedral and traditional.

[3] Mini-course on K-theory and its applications. Tokyo University, Second international undergraduate school on Pure Math, July 29-August 10 2019, 5 lectures

Program.

Lecture 1. We shall first address the general theory of fibred bundles; we shall begin with the definition of locally-trivial bundles, principal bundles and associated bundles and Čech cocycles, that determine them. We shall prove the theorem, identifying the set of all isomorphism classes of principal G-bundles over X with the elements of the first noncommutative Čech cohomology: $\check{H}^1(X, \mathscr{G})$, where $\mathscr{G}$ is the (pre)sheaf of G-valued local functions on X. As a corollary of this description we shall derive the homotopy invariance of the pull-back of a bundle. In addition, we shall obtain the first examples of characteristic classes, the Stiefel-Whitney classes w_1, w_2 and the Chern class c_1 of $\mathbb{C}^*$-bundles.

Lecture 2. Next we shall have a thorough discussion of vector bundles and their properties. We shall begin with the elementary algebraic operation on vector bundles, given by continuous functors on the category of vector spaces, the most important of which are the Whitney sum, and the tensor product. We shall prove the splitting property of the exact sequences of vector bundles and use it to prove the existence of complementary bundles (i.e. such bundles, whose direct sum with the given one is trivial). This result will lead us to two important theorems: first, the Serre-Swan theorem that identifies the categories of vector bundles and the category of projective modules over functions; second, we shall use it to construct the classifying space of $U(n)$-bundles.

Lecture 3. Here we shall discuss the definitions, properties and main constructions of the K-groups of a cell space, $K^*(X)$ including the reduced and relative K-groups $\tilde{K}^*(X)$, $K^*(X, Y)$: we shall give both homotopy theoretic and geometric definitions of these objects. We shall construct the classifying space BU of K-theory and consider few examples of computations in K-theory.

Lecture 4. This lecture shall be dedicated to the discussion of one important property of K-theory, Bott periodicity theorem. We shall describe several different approaches leading to the proof of this result, including the Atiyahs proof, based on the notion of the index of a family of elliptic operators. In the end, if time permits, we shall give a very brief introduction to the theory of characteristic classes.

Lecture 5. After a brief introduction, concerning the definition and major properties of the Adams cohomological operations in K-theory, we shall give the proof of the famous Adams theorem about the dimensions of algebras with division over $\mathbb{R}$.

[5] Modern differential geometry and topology (in English). Moscow State University, 5 year students, February-May 2019, 2 hours per week.

Program.

1. Polyhedra and the discrete Gauss-Bonnet theorem (with and without boundary).
2. Gauss-Bonnet theorem for surfaces in $\mathbb{R}^3$ (with and without boundary).
3. Mapping degree and the index of a singular point of a vector field.
4. Poincré-Hopf theorem.
5. Morse theorem on the index of a vector field on a bounded region in $\mathbb{R}^n$.
6. Gauss-Bonnet theorem for hypersurfaces in $\mathbb{R}^n$
7. Connections on vector bundles over smooth manifolds.
8. Chern-Weil theory of characteristic classes.
9. Gauss-Bonnet-Chern theorem for manifolds.

[6] Deformation quantization, Peking University, September-December 2019 (3-5 years students and post-graduate students)

Program

1. Introduction: pseudodifferential operators, Weyl's quantization formula, Moyal product.
2. Poisson structures on manifolds, Darboux theorem, Lichnerowicz's and Brylinski's Poisson homology and cohomology.
3. The deformation problem. Example: Baker-Campbell-Hausdorff formula and deformation of the product on coadjoint representation space.
4. Hochschild homology and cohomology; Hochschild-Kostant-Rosenberg theorem.
5. Gutt, Lecomte and De Wilde construction: deformation quantization of symplectic manifold.
6. Fedosov quantization. Its complex and algebraic versions. Nest-Tsygan classes.
7. Applications: algebraic index theorem.
8. Higher homotopy structures on algebras. A_∞ and L_∞ algebras. The existence of inverse maps.

9. Kontsevich's theorem for $\mathbb{R}^n$. The first proof (by Kontsevich). Globalization.

10. Algebraic operads and algebras over them. Koszul duality for operads. Examples.

11. Tamarkin's proof of Kontsevich's theorem.

12. Applications: Duflo's isomorphism.

3.3.19 Arkady Skopenkov

A list of university courses taught by A. Skopenkov in 2019

[1] Discrete structures and algorithms in topology, II year students, September-December 2019, 4 hours per week. Moscow Institute of Physics and Technology (DIHT)

Program. It is shown how in the course of solution of interesting geometric problems (close to discrete mathematics and computer science) naturally appear basic notions of algebraic topology (e.g. degree of a map, winding number, Euler characteristics). Thus main ideas of algebraic topology are presented with minimal technicalities.

Detailed program and detailed information in Russian:

`http://www.mccme.ru/circles/oim/home/combtop13.htm#fivt`

[2] Linear algebraic method in topology: homology theory, III-IV year students, September-December 2019, 4 hours per week. Moscow Institute of Physics and Technology (DGAP)

Program. It is shown how in the course of solution of interesting geometric problems (close to discrete mathematics and computer science) naturally appear main notions of algebraic topology (homology groups, obstructions and invariants). Thus main ideas of algebraic topology are presented with minimal technicalities.

Detailed information in Russian: `http://www.mccme.ru/circles/oim/home/combtop13.htm#combtop14`

[3] Vector fields on manifolds and homology theory, Independent University of Moscow, February-May 2019, 2 hours per week.

Program. The main content is exposition of some deep ideas of algebraic topology motivated by applications to vector fields and embeddings.

Detailed program in Russian:

`https://www.mccme.ru/circles/oim/home/combtop13.htm#vefi`

[4] Classification of links, Independent University of Moscow, September-December 2019, 2 hours per week.

Program. Some examples, invariants and classification results for links are presented. We study both the classical case of 1-dimensional links in $\mathbb{R}^3$ and higher-dimensional generalization.

Detailed information in Russian: `http://www.mccme.ru/circles/oim/home/combtop13.htm#link`

[5] Discrete analysis (exercises), II year students, February-December 2019, 2 hours per week. Moscow Institute of Physics and Technology (DIHT)

Program. We study certain topics in combinatorics and graph theory (including random graphs).

Detailed information in Russian:

http://www.mccme.ru/circles/oim/home/discran1314.htm

Other educational activities by A. Skopenkov in 2019

[6] International Summer Conference of Tournament of Towns, Jury member, June-August, Moscow-Arandjelovac. Detailed information:

http://www.turgor.ru/en/lktg/index.php

[7] Moscow Mathematical Conference of High-school Students, Programme Committee member, September-December, Moscow. Detailed information in Russian: http://www.mccme.ru/mmks/index.htm

[8] A course on 'special' mathematics for high-school students, high-school 'Intellectual', January-December. Detailed information in Russian:

http://www.mccme.ru/circles/oim/index.htm#il

[9] Math circle 'Olympiads and Mathematics' for high-school students, MCCME, January-December. Detailed information in Russian:

http://www.mccme.ru/circles/oim/index.htm#oim

[10] Minicources on mathematics for high-school students, Moscow 'olympic' schools, April, June and November, Moscow region.

3.3.20 Evgeni Smirnov

[1] Linear algebra, 1st year, 1st semester, Higher School of Economics, Department of Physics, September–December 2019, 2 hours of lectures and 2 hours of exercise sessions per week

Course outline:

1. Systems of linear equations. Gaussian elimination, row-echelon form.
2. Vector spaces, subspaces. Linear dependence and independence.
3. Linear maps, matrices, multiplication of matrices
4. Determinants.
5. Eigenvectors, eigenvalues, diagonalization of linear operators.
6. Bilinear forms, Jacobi's theorem, Sylvester's criterion.
7. Euclidean and Hermitian spaces.
8. Tensors, tensor product of vector spaces.

[2] Symmetric Functions, Higher School of Economics, course for 2nd–4th year students, September–December 2019, 4 hours per week

Course outline:

1. Symmetric polynomials. Elementary and complete symmetric polynomials, duality. Skew-symmetric polynomials. Schur polynomials.

2. Pieri, Giambelli, Jacobi–Trudi formulas.

3. Combinatorial description of Schur polynomials. Standard and semistandard Young tableaux. Kostka numbers. Hook length formula.

4. Danilov–Koshevoy massifs. Robinson–Schensted–Knuth correspondence via massifs. Littlewood–Richardson rule.

5. Schubert polynomials. Positivity. Pipe dreams, Fomin–Kirillov theorem, Monk's rule. Pipe dream complexes.

[3] Algebra, 1st year, 2nd semester, Higher School of Ecomonics, January–June 2018, 3 hours of lectures and 3 hours of exercise sessions per week

Course outline:

1. Finite Abelian groups
2. Eigenvectors and eigenvalies
3. Jordan and Frobenius normal forms
4. Classification of finitely generated modules over Euclidean rings
5. Bilinear and quadratic forms over R and C. Symmetric/Hermitian, orthogonal/unitary operators
6. Tensors. Tensor algebra. Symmetric and exterior algebras.

[4] Combinatorics, 1st year, 2nd semester, February–May 2019, Independent University of Moscow, 2 hours of lectures and 2 hours of exercise sessions per week

Course outline:

1. Generating functions. Rational generating functions, linear recurrencies.
2. Catalan, Schroeder and Motzkin numbers.
3. q-binomial coefficients.
4. Euler's generating function for Young diagrams. Euler's pentagonal theorem and the triple Jacobi identity.
5. Dirichlet generating functions. Lagrange inversin.
6. Bernoulli-Euler triangle and power sums.
7. Partially ordered sets. Moebius function, Moebius inversion.
8. Lindstroem–Gessel–Viennot theorem. Determinant as the sum over nonintersecting families of paths.
9. Matrix identities: Binet–Cauchy formulas and the Lewis Carroll identity.
10. Matrix tree theorem. Kirchhoff's theorem.

3.3.21 Alexei Sossinsky

[1] GEOMETRY, 3 hours a week, September-December,
First-year IUM students

Program

1. Geometries in the sense of Klein
2. Geometries of Platonic bodies
3. Tilings of the plane and space (Fiodorov groups)
4. Reflection groups and Coxeter geometries
5. Spherical geometry
6. Poincare disk model
7. Poincare half-plane model
8. Main properties of hyperbolic geometry
9. The fifth postulate (history of non-Euclidean geometry)
10. Projective geometry
11. Classical theorems of projective geometry
12. "Projective geometry is all geometry"
13. Finite geometries over Galois fields

TOPOLOGY-1, 3 hours per week, Math in Moscow students, September-December

1. Topology of subsets of Euclidean spaces
2. Abstract topological spaces: main theorems
3. Graphs
4. Surfaces with or without boundary, triangulation, orientation, Euler characteristic, classification theorem
5. Homotopic maps and homotopy equivalent spaces, degree of circle maps and Brouwer's fixed point theorem
6. Vector fields on the plane and on surfaces, Poincare index theorem, "hairy ball theorem"
7. Regular curves in the plane, winding number, Whitney classification theorem, degree of a point w.r.t. a curve, proof of the fundamental theorem of algebra
8. Fundamental group
9. Covering spaces, covering homotopy theorem, coverings with given fundamental group
10. Knots, links and braids, Reidemeister moves, Alexander– Conway polynomial

Yulij Ilyashenko
President of the Independent University of Moscow