The IUM report to the Simons foundation, 2020

Contents

1 Introduction: list of awardees

The Simons foundation supported two programs launched by the IUM:
Simons stipends for students and graduate students;
Simons IUM fellowships.

11 applications were received for the Simons stipends contest. The selection committee consisting of *Yu.Ilyashenko (Chair), G.Dobrushina, G.Kabatyanski, S.Lando, I.Paramonova (Academic Secretary), A.Sossinsky, M.Tsfasman* awarded Simons stipends for 2020 year to the following students and graduate students:

1. Abramyan Semyon Arturovich
2. Dumansky, Ilya Sergeevich
3. Guseva, Lyalya Andreevna
4. Ivanov, Alexei Nikolaevich
5. Konovalov, Andrei Anatolyevich
6. Koshelev, Dmitry Igorevich
7. Kuchumov, Nikolai Igorevich
8. Mishnyakov, Victor Victorovich
9. Sagdeev, Arsenii Alexeevich

17 applications were received for the Simons IUM fellowships contest for the first half year of 2020 and 19 applications were received for the second half year. The selection committee consisting of *Yu.Ilyashenko (Chair), G.Dobrushina, B.Feigin, I.Paramonova (Academic Secretary), A.Sossinsky, M.Tsfasman, V.Vassiliev* awarded

Simons IUM-fellowships for the first half year of 2020 to the following researches:

1. Gorchinsky, Sergei Olegovich
2. Khoroshkin, Anton Sergeevich
3. Loginov, Konstantin Valeryevich
4. Penskoi, Alexei Victorovich
5. Pushkar, Petr Evgenyevich

6. Shabat, George Borisovich
7. Shaposhnikov, Stanislav Valeryevich
8. Sharygin, George Igorevich
9. Skopenkov, Arkadii Borisovich
10. Smirnov, Evgeni Yuryevich
11. Sossinsky, Alexei Bronislavovich
12. Zhgoon, Vladimir Sergeevich

Simons IUM-fellowships for the second half year of 2020 to the following researches:

1. Ayzenberg, Anton Andreevich
2. Burman, Yurii Mikhailovich
3. Elagin, Alexei Dmitrievich
4. Gorodentsev, Alexei Lvovich
5. Khoroshkin, Anton Sergeevich
6. Krasilshchik, Iosif Semenovich
7. Panov, Taras Evgenyevich
8. Penskoi, Alexei Victorovich
9. Pushkar, Petr Evgenyevic
10. Shaposhnikov, Stanislav Valeryevich
11. Sharygin, George Igorevich
12. Smirnov, Evgeni Yurevich
13. Sossinsky, Alexei Bronislavovich

The report below is split in two sections corresponding to the two programs above. The first subsection in each section is a report on the research activities. It consists of the titles of the papers published or submitted in the year of 2020, together with the corresponding abstracts. The second subsection of each section is devoted to conferences and some most important seminar talks. The last subsection of the second section is devoted to the syllabi of the courses given by the winners of the Simons IUM fellowships. Most of these courses are innovative, as required by the rules of the contest for the Simons IUM fellowships.

Due to the COVID-19 pandemic, the second subsection (conferences) is smaller than in the previous reports. If an awarded person attended no conference at all, the corresponding subsection is void.

The Independent University remains one of the most active centers of Moscow Mathematical life. There is no room here to list its main activities. We only mention that, out of the 19 last winners of the very prestigious Moscow Mathematical Society Prize for young mathematicians, 11 are the alumni of the IUM. Amidst them is Natalya Goncharuk, graduated 2012, awarded 2018, the first women ever to receive this Prize.

The support of the Simons foundation have drastically improved the financial situation at the IUM, and the whole atmosphere as well. On behalf of the IUM, I send my deep gratitude and the best New year wishes to Jim Simons, Yuri Tschinkel, and the whole team of the Simons foundation.

Yulij Ilyashenko

President of the Independent University of Moscow

2 Program: Simons stipends for students and graduate students

2.1 Research

2.1.1 Semyon Abramyan

[1] On homology of the MSU spectrum
in preparation

In this paper we compute the structure of $H_*(MSU;\mathbb{F}_p)$ over the dual Steenrod algebra $\mathfrak{A}_p^*$, p is an odd prime. We give a complete proof of the isomorphism $\Omega^{SU} \otimes \mathbb{Z}[\frac{1}{2}][y_2, y_3, \ldots]$, $\deg y_n = 2n$ using the original methods of the Adams spectral sequence. We also describe the Hurewicz map and the divisiblity of characteristic numbers of SU-manifolds.

2.1.2 Ilya Dumansky

[1] With E. Feigin and M. Finkelberg
Beilinson-Drinfeld Schubert varieties and global Demazure modules

arXiv:2003.12930 *submitted to Forum of Mathematics, Sigma*

We compute the spaces of sections of powers of the determinant line bundle on the spherical Schubert subvarieties of the Beilinson- Drinfeld affine Grassmannians. The answer is given in terms of global Demazure modules of the current Lie algebra.

2.1.3 Lyalya Guseva

[1] On the derived category of IGr(3,8)
Sb. Math., 211:7 (2020), 922955

We construct a full exceptional collection of vector bundles in the bounded derived category of coherent sheaves on the Grassmannian IGr(3,8) of isotropic 3-dimensional subspaces in an 8-dimensional symplectic vector space.
[2] On the derived category of the Cayley Grassmannian
In preparation

We construct a full exceptional collection of vector bundles in the bounded derived category of coherent sheaves on the Cayley Grassmannian that parametrizes four dimensional subalgebras of the complex octonions.

2.1.4 Alexei Ivanov (in August changed his family name to Lavrov, the last preprint is signed by the new name)

[1] A new series of moduli components of rank-2 semistable sheaves on P3 with singularities of mixed dimension
Sbornik: Mathematics, 2020, Vol. 211, No. 7, pp. 967986.

We construct a new infinite series of irreducible components of the Gieseker-Maruyama moduli scheme $\mathcal{M}(k),\ k \geq 3$ of coherent semistable rank 2 sheaves with Chern classes $c_1 = 0,\ c_2 = k,\ c_3 = 0$ on $\mathbb{P}^3$ whose general points are sheaves with singularities of mixed dimension. These sheaves are constructed by elementary transformations of stable and properly μ-semistable reflexive sheaves along disjoint union of collections of points and smooth irreducible curves which are rational or complete intersection curves. As a special member of this series we obtain a new component of $\mathcal{M}(3)$.

[2] A new non-reduced moduli component of rank 2 semistable sheaves on $\mathbb{P}^3$
arXiv:2012.05611

In the present paper we describe new component of the Gieseker-Maruyama moduli space $\mathcal{M}(14)$ of coherent semistable rank-2 sheaves with Chern classes $c_1 = 0,\ c_2 = 14,\ c_3 =$

0 on $\mathbb{P}^3$ which is generically non-reduced. The construction of this component is based on the technique of elementary transformations of sheaves and the famous Mumford's example of a non-reduced component of the Hilbert scheme of smooth space curves of degree 14 and genus 24.

2.1.5 Andrei Konovalov

[1] Nilpotent invariance of semi-topological K-theory of dg-algebras and the lattice conjecture
Preprint

We prove a conjecture of Katzarkov-Kontsevich-Pantev about existence of a natural rational structure on periodic cyclic homology in the case of proper connective C-dg-algebras. The main ingredient is derived nilpotent invariance of A. Blancs semi-topological K-theory, which we establish along the way.

2.1.6 Dmitry Koshelev

[1] Non-split toric BCH codes on singular del Pezzo surfaces
arXiv:2003.09828, *to appear in IEEE Transactions on Information Theory.*

In the article we construct low-rate non-split toric q-ary codes on some singular surfaces. More precisely, we consider non-split toric cubic and quartic del Pezzo surfaces, whose singular points are $\mathbb{F}_q$-conjugate. Our codes turn out to be BCH ones with sufficiently large minimum distance d. Indeed, we prove that $d - d^* \geqslant q - \lfloor 2\sqrt{q} \rfloor - 1$, where d^* is the designed minimum distance. In other words, we significantly improve upon BCH bound. On the other hand, the defect of the Griesmer bound for the new codes is $\leqslant \lfloor 2\sqrt{q} \rfloor - 1$, which also seems to be quite good. It is worth noting that to better estimate d we actively use the theory of elliptic curves over finite fields.

2.1.7 Nikolai Kuchumov

Preprint "A variational principle for domino tilings of multiply-connected domains", to appear on arXiv.com.

We study random domino tilings of multiply-connected domains with height functions defined on the universal covering spaces of the domains. For them we prove a large deviation principle that includes a law of large numbers for island height. The main technique of the paper is applying a variational approach in the sense of "A variational principle for domino tilings" by Cohn, Kenyon and Propp.

2.1.8 Victor Mishnyakov

[1] A new symmetry of the colored Alexander polynomial. V. Mishnyakov , A. Sleptsov , N. Tselousov

e-Print: 2001.10596 [hep-th]

Accepted to "Annales Henri Poincar"

We present a new conjectural symmetry of the colored Alexander polynomial, that is the specialization of the quantum sl_N invariant widely known as the colored HOMFLY-PT polynomial. We provide arguments in support of the existence of the symmetry by studying the loop expansion and the character expansion of the colored HOMFLY-PT polynomial. We study the constraints this symmetry imposes on the group theoretic structure of the loop expansion and provide solutions to those constraints. The symmetry is a powerful tool for research on polynomial knot invariants and in the end we suggest several possible applications of the symmetry.

[2] A novel symmetry of colored HOMFLY polynomials coming from $sl(N|M)$ superalgebras. V. Mishnyakov , A. Sleptsov , N. Tselousov

e-Print: 2005.01188 [hep-th]

We present a novel symmetry of the colored HOMFLY polynomial. It relates pairs of polynomials colored by different representations at specific values of NN and generalizes the previously known "tug-the-hook" symmetry of the colored Alexander polynomial. As we show, the symmetry has a superalgebra origin, which we discuss qualitatively. Our main focus are the constraints that such a property imposes on the general group-theoretical structure, namely the $sl(N)$ weight system, arising in the perturbative expansion of the invariant. Finally, we demonstrate its tight relation to the eigenvalue conjecture.

2.1.9 Arsenii Sagdeev

[1] With A. Kupavskii

Ramsey theory in the space with Chebyshev metric

Russian Mathematical Surveys, 2020, Vol. 75.

In this paper we consider the analogue of Euclidean Ramsey theory in the space with Chebyshev metric.

[2] With A. Kupavskii

All finite sets are Ramsey in the maximum norm

arXiv:2008.02008, *to appear in Forum of Mathematics, Sigma.*

For two metric spaces $\mathcal{X}$ and $\mathcal{Y}$, the chromatic number $\chi(\mathcal{X};\mathcal{Y})$ of $\mathcal{X}$ with forbidden $\mathcal{Y}$ is the smallest k such that there is a coloring of the points of $\mathcal{X}$ with no monochromatic copy of $\mathcal{Y}$. In this paper, we show that for each finite metric space $\mathcal{M}$ the value $\chi(\mathbb{R}^n_\infty;\mathcal{M})$ grows exponentially with n. We also provide explicit lower and upper bounds for some special $\mathcal{M}$.

2.2 Scientific conferences and seminar talks

2.2.1 Semyon Abramyan

[1] International Conference "Topology and Geometry of Group Actions Conference", online, November, 18–22

Talk "Substitution complexes and nonrealizability example"

[2] International seminar for young researchers "Algebraic, combinatorial and toric topology", online, December, 17–18

Talk "On Homology of the MSU spectrum"

[3] Course "Spectral Sequences in Topology", NRU HSE, Moscow, Russia, Fall 2020, Lecturer

2.2.2 Ilya Dumansky

[1] Aachen-Bochum-Cologne-Darstellungstheorie seminar, Aachen, July, 29

Talk "Global Demazure modules".

[2] Winter school "Integrable systems and Representation Theory", Bologna, January, 13 – 17

[3] Conference "Lie algebras, algebraic groups and invariant theory", Moscow, January, 27 – February, 1

Poster "Global Demazure Modules and Semi-infinite Veronese Curves"

[4] Lie Algebras and Applications seminar, Moscow (HSE University), January, 28

Talk "Global Demazure modules"

[5] Seminar "Lie Groups and Invariant theory", Moscow (MSU), February, 19

Talk "Global Demazure modules".

[6] Lie Algebras and Applications seminar, Moscow (HSE University), Nov 17

Talk "Global Demazure modules and Beilinson-Drinfeld Schubert varieties".

2.2.3 Lyalya Guseva

[1] Conference "Western algebraic geometry online", online, April, 18 – 19

[2] Conference "Rationality", online, July, 27 – 31

[3] Kickoff Meeting, online, September 4

[4] FRG Workshop on Moduli Spaces and Stability, online, December, 7 – 10

[5] Mini-school on moduli of sheaves on three- and four-folds, online, December, 14 – 15

[6] VIII Workshop and Conference on Lie Algebras, Algebraic Groups, and Invariant Theory, Moscow, January, 27 - February, 1

[7] Conference "Categories and birational geometry", online, December 14 – 18

2.2.4 Alexei Ivanov

[1] Conference "Monodromy and Hypergeometric Functions", Istanbul, February, 17 – February, 21

Talk "Feynman integrals and mirror symmetry"

[2] Weekly seminar of the Laboratory of Algebraic Geometry and Its Applications (HSE), November, 6

Talk "New moduli components of rank 2 semistable sheaves on $\mathbb{P}^3$"

2.2.5 Andrei Konovalov

[1] Talk "Nilinvariance of topological K-theory of dg-algebras" at "Seminar of Laboratory of Algebraic Geometry and its Applications", January, Moscow

[2] Talk "Topological Hochschild homology" at "electronic Computational Homotopy Theory (eCHT) seminar", February, via Zoom

2.2.6 Dmitry Koshelev

[1] Conference "Current Trends in Cryptology", Moscow region, September, 15 – 17

Talk "Double point compression for elliptic curves of j-invariant 0"

[2] Visit to Versailles, May and November

Talk "Hashing to elliptic curves of j-invariant 1728" at "Seminar of the algebraic geometry team" (Universit de Versailles Saint-Quentin-en-Yvelines)

Talk "Hashing to elliptic curves $y^2 = x^3 + b$ provided that b is a quadratic residue" at "Seminar of the team CRYPTO" (Universit de Versailles Saint-Quentin-en-Yvelines)

2.2.7 Nikolai Kuchumov

[1]Conference "Online conference on Statistical Mechanics, Integrable Systems and Probability" April 27 - May 1

Poster "A variational principle for multiply-connected domains"

[2] Talk,"Limit shapes in models of random tilings", Student colloquium, Chebyshev Laboratory at Saint-Petersburg state university via zoom, 7th of may .

[3] Talk "Proof of variational principle for domino tilings", PhD seminar on mathematical physics, Skoltech via zoom, 27th of april

[4]Talk "Introduction to the tangent method", PhD seminar on mathematical physics, Skoltech via zoom, November 16th

2.2.8 Victor Mishnyakov

[1] Conference " Youth conference on theoretical and experimental physics", Moscow, November, 16 - 19

Talk "Exotic A-model on a sphere. Gluing the topological and anti-topological model."

2.2.9 Arsenii Sagdeev

[1] Seminar of the Laboratory of Combinatorial and Geometric Structures, MIPT, Moscow, October 7

Talk "Batons and Ramsey theory problems in the space with Chebyshev metric".

[2] Seminar "Mathematical workshop", MIPT, Moscow, October 27

Talk "On the optimal coverings of the line by batons and its connections with chromatic numbers".

3 Program: Simons IUM fellowships

3.1 Research

3.1.1 Anton Ayzenberg

[1] Dimensions of multi-fan algebras, J. Math. Soc. Japan (2020) 72:3, pp.777–794 (preprint: arXiv:1607.03889).

Given an arbitrary non-zero simplicial cycle and a generic vector coloring of its vertices, there is a way to produce a graded Poincare duality algebra associated with these data. The procedure relies on the theory of volume polynomials and multi-fans. The algebras constructed this way include many important examples: cohomology algebras of toric varieties and quasitoric manifolds, and Gorenstein algebras of triangulated homology manifolds, introduced and studied by Novik and Swartz. In all these examples the dimensions of graded components of such duality algebras do not depend on the vector coloring. It was conjectured that the same holds for any simplicial cycle. We disprove this conjecture by showing that the colors of singular points of the cycle may affect the dimensions. However, the colors of nonsingular points are irrelevant. By using bistellar moves we show that the number of distinct dimension vectors arising on a given 3-dimensional pseudomanifold with isolated singularities is a topological invariant. This invariant is trivial on manifolds, but nontrivial on general pseudomanifolds.

[2] With V. Cherepanov,
Torus actions of complexity one in non-general position
arXiv:1905.04761 *to appear in Osaka J. Math.*.

Let the compact torus T^{n-1} act on a smooth compact manifold X^{2n} effectively with nonempty finite set of fixed points. We pose the question: what can be said about the orbit space X^{2n}/T^{n-1} if the action is cohomologically equivariantly formal (which essentially means that $H^{odd}(X^{2n};\mathbb{Z}) = 0$)? It happens that homology of the orbit space can be arbitrary in degrees 3 and higher. For any finite simplicial complex L we construct an equivariantly formal manifold X^{2n} such that X^{2n}/T^{n-1} is homotopy equivalent to $\Sigma^3 L$. The constructed manifold X^{2n} is the total space of a projective line bundle over the permutohedral variety hence the action on X^{2n} is Hamiltonian and cohomologically equivariantly formal. We introduce the notion of an action in j-general position and prove that, for any simplicial complex M, there exists an equivariantly formal action of complexity one in j-general position such that its orbit space is homotopy equivalent to $\Sigma^{j+2} M$.

[3] Torus action on quaternionic projective plane and related spaces
arXiv:1903.03460 *to appear in Arnold Math. J. (2020) https://doi.org/10.1007/s40598-020-00166-4*

For an effective action of a compact torus T on a smooth compact manifold X with nonempty finite set of fixed points, the number $\frac{1}{2}\dim X - \dim T$ is called the complexity of the action. In this paper we study certain examples of torus actions of complexity one and describe their orbit spaces. We prove that $\mathbb{H}P^2/T^3 \cong S^5$ and $S^6/T^2 \cong S^4$, for the homogeneous spaces $\mathbb{H}P^2 = Sp(3)/(Sp(2) \times Sp(1))$ and $S^6 = G_2/SU(3)$. Here the maximal tori of the corresponding Lie groups $Sp(3)$ and G_2 act on the homogeneous spaces from the left. Next we consider the quaternionic analogues of smooth toric surfaces: they give a class of 8-dimensional manifolds with the action of T^3. This class generalizes $\mathbb{H}P^2$. We prove that their orbit spaces are homeomorphic to S^5 as well. We link this result to Kuiper–Massey theorem and its generalizations studied by Arnold.

[4] with V. M. Buchstaber
Manifolds of isospectral matrices and Hessenberg varieties
arXiv:1803.01132 *to appear in Int. Math. Res. Notices https://doi.org/10.1093/imrn/rnz388*

We consider the space X_h of Hermitian matrices having staircase form and the given simple spectrum. There is a natural action of a compact torus on this space. Using generalized Toda flow, we show that X_h is a smooth manifold and its smooth type is independent of the spectrum. Morse theory is then used to show the vanishing of odd degree cohomology, so that X_h is an equivariantly formal manifold. The equivariant and ordinary cohomology rings of X_h are described using GKM-theory. The main goal of this paper is to show the connection between the manifolds X_h and regular semisimple Hessenberg varieties well known in algebraic geometry. Both spaces X_h and Hessenberg varieties form wonderful families of submanifolds in the complete flag variety. There is a certain symmetry between these families which can be generalized to other submanifolds of the flag variety.

[5] with V. M. Buchstaber
Manifolds of isospectral arrow matrices
arXiv:1803.10449 *to appear in Sbornik: Mathematics, 2021*

An arrow matrix is a matrix with zeroes outside the main diagonal, first row, and first column. We consider the space $M_{St_n,\lambda}$ of Hermitian arrow $(n+1)\times(n+1)$-matrices with fixed simple spectrum λ. We prove this space to be a smooth $2n$-manifold, and its smooth structure is independent on the spectrum. Next, this manifold carries the locally standard torus action: we describe the topology and combinatorics of its orbit space. If $n\geq 3$, the orbit space $M_{St_n,\lambda}/T^n$ is not a polytope, hence $M_{St_n,\lambda}$ is not a quasitoric manifold. However, there is a natural permutation action on $M_{St_n,\lambda}$ which induces the combined action of the semidirect product of Σ_n on T^n. The orbit space of this large action is a simple polytope B^n. The structure of this polytope is described in the paper. In case $n=3$, the space $M_{St_3,\lambda}/T^3$ is a solid torus with boundary subdivided into hexagons in a regular way. This description allows to compute the cohomology ring and equivariant cohomology ring of the 6-dimensional manifold $M_{St_3,\lambda}$ using the general theory developed by the first author. This theory is also applied to a certain 6-dimensional manifold called the twin of $M_{St_3,\lambda}$. The twin carries a half-dimensional torus action and has nontrivial tangent and normal bundles.

[6] with M. Masuda
Orbit spaces of equivariantly formal torus actions
arXiv:1912.11696 *submitted to Moscow Mathematical Journal*

Let a compact torus $T=T^{n-1}$ act on a smooth compact manifold $X=X^{2n}$ effectively, with nonempty finite set of fixed points, and suppose that stabilizers of all points are connected. If $H^{odd}(X)=0$ and the weights of tangent representation at each fixed point are in general position, we prove that the orbit space $Q=X/T$ is a homology $(n+1)$-sphere. If, in addition, $\pi_1(X)=0$, then Q is homeomorphic to S^{n+1}. We introduce the notion of j-generality of tangent weights of torus action. For any action of T^k on X^{2n} with isolated fixed points and $H^{odd}(X)=0$, we prove that j-generality of weights implies $(j+1)$-acyclicity of the orbit space Q. This statement generalizes several known results for actions of complexity zero and one. In complexity one, we give a criterion of equivariant formality in terms of the orbit space. In this case, we give a formula expressing Betti numbers of a manifold in terms of certain combinatorial structure that sits in the orbit space.

[7] with A. Rukhovich
Clique complexes of multigraphs, edge inflations, and tournaplexes
arXiv:2012.07600

In this paper we introduce and study the topology of clique complexes of multigraphs without loops. These clique complexes generalize tournaplexes, which were recently introduced by Govc, Levi, and Smith for the topological study of brain functional networks. We study a general construction of edge-inflated simplicial posets, which generalize clique complexes of multigraphs. The poset fiber theorem of Bjrner, Wachs, and Welker is applied to obtain the homotopy wedge decomposition of an edge-inflated simplicial poset. The homological corollary of this result allows to parallelize the homology computations for edge inflated complexes, in particular, for clique complexes of multigraphs and tournaplexes. We provide functorial versions of some results to be used in computations of persistent homology. Finally, we introduce a general notion of simplex inflations and prove homotopy wedge decompositions for this class of spaces.

3.1.2 Yuri Burman

[1] With R.Froeberg and B.Shapiro
Algebraic relations between harmonic and anti-harmonic moments of plane polygons
arXiv:1908.07621, *to appear in International Mathematics Research Notices*, doi:10.1093/imrn/rnz394

In this paper we describe the algebraic relations satisfied by the harmonic and anti-harmonic moments of simply connected, but not necessarily convex planar polygons with a given number of vertices.
[2] With V.Kulishov
Lie elements and the matrix-tree theorem
arXiv:2011.10340 (submitted in November 2020, despite the arXiv number).

For a finite-dimensional representation V of a group G we introduce and study the notion of a Lie element in the group algebra of G. The set of Lie elements is a Lie algebra and a G-module acting on the original representation V.

Lie elements often exhibit nice combinatorial properties. In particular, if G is a permutation group and V, a permutation representation, we prove a formula for the characteristic polynomial of a Lie element similar to the classical matrix-tree theorem.

3.1.3 Alexei Elagin

[1] With V. Lunts
Thick subcategories on curves
Adv. Math., 2021, Vol. 378, Article 107525.

We classify triangulated categories that are equivalent to finitely generated thick subcategories $T \subset D^b(\operatorname{coh} C)$ for smooth projective curves C over an algebraically closed field.

3.1.4 Sergei Gorchinsky

[1] Orthogonal to principle ideles
Math. Notes, 2020, Vol. 107, No. 3, pp. 425-434.

We describe the orthogonal ${K^*}^{\perp}$ to the group of principal ideles K^* with respect to the global tame symbol pairing on the group of ideles $\mathbb{A}_X^*$ of a smooth projective algebraic curve X over a field k. More precisely, we describe the quotient ${K^*}^{\perp}/(K^* \cdot U)$, where U is the kernel of the global tame symbol pairing. When k is algebraically closed, this quotient maps surjectively to $\operatorname{Pic}^0(X)$ with the kernel being $\operatorname{Hom}\big(\operatorname{Pic}^0(X), k^*\big)$. When k is finite, the quotient is trivial.

[2] With D.V. Osipov
The higher-dimensional Contou-Carrère symbol and commutative group schemes
Russian Math. Surveys, 2020, Vol. 75, No. 3, pp. 572-574.

In this note we investigate symbols on rings of iterated Laurent series with values in commutative group schemes. Namely, we stay that all such symbols factor uniquely through the higher-dimensional Contou-Carrère symbol.

[3] With D.V. Osipov
Iterated Laurent series over rings and Contou-Carrère symbol
Russian Math. Surveys, 2020, Vol. 75, No. 6, pp. 3-84.

This article contains a survey of a new algebro-geometric approach to algebraic loop groups associated with iterated Laurent series over arbitrary commutative rings and its applications to the study of the higher-dimensional Contou-Carrère symbol. In addition to the survey, the article contains also new results related to this symbol.

The higher-dimensional Contou-Carrère symbol arises naturally when one considers a deformation of a flag of algebraic subvarieties in an algebraic variety. The non-triviality of the problem is due to the fact that when $n > 1$ it is not known a representation as an ind-flat scheme over a ring for the group of invertible elements of the algebra of n-iterated Laurent series over this ring. Consequently, substantially new algebro-geometric constructions, notions, and methods are required. As an application of the new methods, we describe continuous homomorphisms between algebras of iterated Laurent series over a ring and provide an invertibility criterion for such endomorphisms. It is proved that the higher-dimensional Contou-Carrère symbol, restricted to algebras over the field of rational numbers, is given by a natural explicit formula and this symbol extends uniquely to all rings. We also give an explicit formula for the higher-dimensional Contou-Carrère symbol in the case of arbitrary rings. A connection to higher-dimensional class field theory is described.

As a new result, we prove that the higher-dimensional Contou-Carrère symbol satisfies a universal property. Namely, fix a torsion-free ring and consider a flat group scheme over this ring such that any two points of the scheme are contained in an affine open subset. Then after restriction to algebras over this ring, all morphisms from the n-iterated algebraic loop group of the Milnor K-group of degree $n+1$ to the above group scheme factor through the higher-dimensional Contou-Carrère symbol.

[4] With D. Bergh, M. Larsen, V. Lunts
Categorical measures for finite group actions
arXiv:1709.00620, *to appear in Journal of Algebraic Geometry.*

Given a variety with a finite group action, we compare its equivariant categorical measure, that is, the categorical measure of the corresponding quotient stack, and the categorical measure of the extended quotient. Using weak factorization for orbifolds, we show that for a wide range of cases, these two measures coincide. This implies, in particular, a conjecture of Galkin and Shinder on categorical and motivic zeta-functions of varieties. We provide examples showing that, in general, these two measures are not equal. We also give an example related to a conjecture of Polishchuk and Van den Bergh, showing that a certain condition in this conjecture is indeed necessary.

3.1.5 Alexei Gorodentsev

No papers during this period.

3.1.6 Anton Khoroshkin

[1] "Quadratic Algebras arising from Hopf operads generated by a single element"
Letters in Mathematical Physics 2020. . 1-30.

The operads of Poisson and Gerstenhaber algebras are generated by a single binary element if we consider them as Hopf operads (i.e. as operads in the category of cocommutative coalgebras). In this note we discuss in details the Hopf operads generated by a single element of arbitrary arity. We explain why the dual space to the space of n-ary operations in this operads are quadratic and Koszul algebras. We give the detailed description of generators, relations and a certain monomial basis in these algebras.

[2] "PBW Property for Associative Universal Enveloping Algebras Over an Operad" International Mathematics Research Notices, rnaa215, https://doi.org/10.1093/imrn/rnaa215

Given a symmetric operad $\mathcal{P}$ and a $\mathcal{P}$-algebra V, the associative universal enveloping algebra $\mathsf{U}_{\mathcal{P}}$ is an associative algebra whose category of modules is isomorphic to the abelian category of V-modules. We study the notion of PBW property for universal enveloping algebras over an operad. In case $\mathcal{P}$ is Koszul a criterion for the PBW property is found. A necessary condition on the Hilbert series for $\mathcal{P}$ is discovered. Moreover, given any symmetric operad $\mathcal{P}$, together with a Gröbner basis G, a condition is given in terms of the structure of the underlying trees associated with leading monomials of G, sufficient for the PBW property to hold. Examples are provided.

[3] "Homotopical rigidity of the pre-Lie operad" with Vladimir Dotsenko `math.arXiv`:2002.12918

The paper collects some facts about preLie operad together with some applications for certain classes of preLie algebras. In particular, we reprove the analogue of the PBW theorem for the universal envelloping PreLie algebra of a Lie algebra. We prove that the notion of a preLie algebra is rigid. That is, we prove that there are no nontrivial deformations of the PreLie operad. Moreover we prove that the PreLie operad is a homotopy fixed point for Twisting. As a corollary we conclude that the only universal structure on a convolution Lie algebra is a structure of a PreLie algebra.

[4] "Derived Poincaré–Birkhoff–Witt theorems" with Pedro Tamaroff `math.arXiv`:2003.06055

We define derived Poincaré–Birkhoff–Witt maps of dg operads or derived PBW maps, for short, which extend the definition of PBW maps between operads of V. Dotsenko and the second author, with the purpose of studying the universal enveloping algebra of dg Lie algebras as a functor on the homotopy category. Our main result shows that the map from the homotopy Lie operad to the homotopy associative operad is derived PBW, which gives us an amenable description of the homology of the universal envelope of an L_∞ algebra in the sense of Lada–Markl. We deduce from this several known results involving universal envelopes of L_∞-algebras of V. Baranovsky and J. Moreno-Fernndez, and extend D. Quillen's classical quasi-isomorphism $\mathcal{C} \longrightarrow BU$ from dg Lie algebras to L_∞-algebras; this confirms a conjecture of J. Moreno-Fernández.

[5] "Gröbner Bases for Coloured Operads" with Vladislav Kharitonov `math.arXiv`:2008.05295

In this work we provide a definition of a coloured operad as a monoid in some monoidal category, and develop the machinery of Gröbner bases for coloured operads. Among the examples for which we show the existance of a quadratic Gröbner basis we consider the seminal Lie-Rinehart operad whose algebras include pairs (functions, vector fields).

3.1.7 Iosif Krasilshchik

[1] Nonlocal conservation laws of PDEs possessing differential coverings. Symmetry, 12 (2020) 11, 1760. https://doi.org/10.3390/sym12111760, arXiv:2009.09489 nlin.SI

In his 1892 paper, L. Bianchi noticed, among other things, that quite simple transformations of the formulas that describe the Bäcklund transformation of the sine-Gordon equation lead to what is called a nonlocal conservation law in modern language. Using the techniques of differential coverings, we show that this observation is of a quite general nature. We describe the procedures to construct such conservation laws and present a number of illustrative examples.

[2] With V. Lychagin
Geometric study of gas behavior in a one-dimensional nozzle (the case of the van der Waals gas), Lobachevskii Journal of Mathematics, 2020, 41 (2020) 12, pp. 2458–2465. DOI: 10.1134/S1995080220120185, arXiv:2004.03896v1.

We construct a three-component system of PDEs describing dynamics of van der Walls gas in one-dimensional nozzle. The group of conservation laws for this system is described. We also compute the Lie algebras of point symmetries and present group classification. Examples of exact invariant solutions are given.

[3] With P. Vojcak
On the algebra of nonlocal symmetries for the 4D Martínez Alonso-Shabat equation.
arXiv:2008.10281 nlin.SI *To appear in J. Geom. and Phys.*

We consider the 4D Martínez Alonso-Shabat equation $u_{ty} = u_z u_{xy} - u_y u_{xz}$ (also referred to as the universal hierarchy equation) and using its known Lax pair construct two infinite-dimensional differential coverings over $\mathcal{E}$. In these coverings, we give a complete description of the Lie algebras of nonlocal symmetries. In particular, our results generalize the ones obtained in [O.I. Morozov, A. Sergyeyev, The four-dimensional Mart nez Alonso-shabat equation: reductions and nonlocal symmetries. J. of Geom. and Phys. 85 (2014), 40–45 (arXiv:1401.7942v2)] and contain the constructed there infinite hierarchy of commuting symmetries as a subalgebra in a much bigger Lie algebra.

[4] Noncomformist (A.M. Vinogradov and his seminar)
To appear in A.M. Vinogradov. Selected works. MCCME, Moscow

We discuss the impact of Alexandre Vinogradov on modern geometrical theory of partial differential equation. The history of Vinogradov's seminar at Moscow State University is described.

3.1.8 Konstantin Loginov

[1] Maximal log Fano manifolds are generalized Bott towers
(joint with J. Moraga)

We prove that maximal log Fano manifolds are generalized Bott towers. As an application, we prove that in each dimension, there is a unique maximal snc Fano variety satisfying Friedman's d-semistability condition.

arXiv e-print, 2012.00266.

[2] Bounding non-rationality of divisors on 3-fold Fano fibrations
(joint with C. Birkar)

In this paper we investigate non-rationality of divisors on 3-fold log Fano fibrations $(X, B) \to Z$ under mild conditions. We show that if D is a component of B with coefficient $\geq t > 0$ which is contracted to a point on Z, then D is birational to $\mathbb{P}^1 \times C$ where C is a smooth projective curve with gonality bounded depending only on t. Moreover, if $t > \frac{1}{2}$, then genus of C is bounded depending only on t.

arXiv e-print, 2007.15754, *submitted to Journal für reine und angewandte mathematik.*

3.1.9 Taras Panov

[1] With H. Ishida and R. Krutowski.

Basic cohomology of canonical holomorphic foliations on complex moment-angle manifolds.

International Mathematics Research Notices, published 07 October 2020, DOI:10.1093/imrn/rnaa252

We describe the basic cohomology ring of the canonical holomorphic foliation on a moment-angle manifold, LVMB-manifold or any complex manifold with a maximal holomorphic torus action. Namely, we show that the basic cohomology has a description similar to the cohomology ring of a complete simplicial toric variety due to Danilov and Jurkiewicz. This settles a question of Battaglia and Zaffran, who previously computed the basic Betti numbers for the canonical holomorphic foliation in the case of a shellable fan. Our proof uses an Eilenberg–Moore spectral sequence argument; the key ingredient is the formality of the Cartan model for the torus action on a moment-angle manifold. We develop the concept of transverse equivalence as an important tool for studying smooth and holomorphic foliated manifolds. For an arbitrary complex manifold with a maximal torus action, we show that it is transverse equivalent to a moment-angle manifold and therefore has the same basic cohomology.

[2] With J. Grbić, Marina Ilyasova and George Simmons.

One-relator groups and algebras related to polyhedral products.
arXiv:2002.11476, *to appear in Proceedings of the Royal Society of Edinburgh Section A: Mathematics.*

We link distinct concepts of geometric group theory and homotopy theory through underlying combinatorics. For a flag simplicial complex K, we specify a necessary and sufficient combinatorial condition for the commutator subgroup RC'_K of a right-angled Coxeter group, viewed as the fundamental group of the real moment-angle complex $\mathcal{R}_K$, to be a one-relator group; and for the Pontryagin algebra $H_*(\Omega\mathcal{Z}_K)$ of the moment-angle complex to be a one-relator algebra. We also give a homological characterisation of these properties. For RC'_K, it is given by a condition on the homology group $H_2(\mathcal{R}_K)$, whereas for $H_*(\Omega\mathcal{Z}_K)$ it is stated in terms of the bigrading of the homology groups of $\mathcal{Z}_K$.

3.1.10 Alexei Penskoi

[1] With N. S. Nadirashvili
Free boundary minimal surfaces and overdetermined boundary value problems
Journal d'Analyse Mathématique, 2020, Vol. 141, No. 1, pp. 323-329. DOI https://doi.org/10.1007/s11
020-0129-0

In this paper we establish a connection between free boundary minimal surfaces in a ball in R^3 and free boundary cones arising in a one-phase problem.

[2] With M. A. Karpukhin, N. S. Nadirashvili and I. V. Polterovich
An isoperimetric inequality for Laplace eigenvalues on the sphere.
arXiv:1706.05713, *to appear in Journal of Differential Geometry*

In this paper we show that for any positive integer k, the k-th nonzero eigenvalue of the Laplace-Beltrami operator on the two-dimensional sphere endowed with a Riemannian metric of unit area, is maximized in the limit by a sequence of metrics converging to a union of k touching identical round spheres. This proves a conjecture posed by the second author in 2002 and yields a sharp isoperimetric inequality for all nonzero eigenvalues of the Laplacian on a sphere. Earlier, the result was known only for $k = 1$ (J. Hersch, 1970), $k = 2$ (N. Nadirashvili, 2002; R. Petrides, 2014) and $k = 3$ (N. Nadirashvili and Y. Sire, 2017). In particular, we argue that for any $k \geq 2$, the supremum of the k-th nonzero eigenvalue on a sphere of unit area is not attained in the class of Riemannian metrics which are smooth outsitde a finite set of conical singularities. The proof uses certain properties of harmonic maps between spheres, the key new ingredient being a bound on the harmonic degree of a harmonic map into a sphere obtained by N. Ejiri.

[3] With M. A. Karpukhin, N. S. Nadirashvili and I. V. Polterovich

Conformally maximal metrics for Laplace eigenvalues on surfaces
arXiv:2003.02871

In this paper we consider the maximization of Laplace eigenvalues on surfaces of given volume with a Riemannian metric in a fixed conformal class. A significant progress on this problem has been recently achieved by Nadirashvili-Sire and Petrides using related, though different methods. In particular, it was shown that for a given k, the maximum of the k-th Laplace eigenvalue in a conformal class on a surface is either attained on a metric which is smooth except possibly at a finite number of conical singularities, or it is attained in the limit while a "bubble tree" is formed on a surface. Geometrically, the bubble tree appearing in this setting can be viewed as a union of touching identical round spheres. We present another proof of this statement, developing the approach proposed by the second author and Y. Sire. As a side result, we provide explicit upper bounds on the topological spectrum of surfaces.

3.1.11 Petr Pushkar

[1] With M. Tyomkin
Enhanced Bruhat decomposition and Morse theory.
arxiv.org: 2012.05307

Consider the set of all rectangular $n \times m$ matrices with entries in a field. Recall that unitriangular group T_n consists of upper triangular matrices with 1's on the diagonal. The product $T_n \times T_m$ naturally acts on the aforementioned set: $X \to AXB^{-1}$. Our first observation is that each orbit of this action contains a unique matrix which has at most one non-zero entry in each row and in each column. Thus these non-zero numbers and their positions are invariants of a matrix under this action. This is a variation of a classical Bruhat decomposition for GL. When applied in the setting of Morse theory, this linear algebraic construction leads to invariants of a strong Morse function f. Namely, positions of non-zero entries correspond to the well-known Barannikov decomposition (also known as persistent homology) of f. The novelty is the values themselves, which correspond to numbers, carried by Barannikov pairs (also known as bars in the barcode). Considering further a complex, constructed from a strong Morse function, we interpret the product of all the numbers as a torsion of chain complex.

3.1.12 George Shabat

[1] Square-tiled surfaces and curves over number fields,
Fundamental and Applied Mathematics, 2020, to appear

In this paper an analog is presented of the old result by the author and V. Voevodsky, according to which a Riemann surface admits a conformal structure, defined by an equilateral triangulation, if and only if the corresponding algebraic curve can be defined over the field of the algebraic numbers; the similar result is obtained for the square-tiled surfaces.

3.1.13 Stanislav Shaposhnikov

[1] Bogachev V.I., Krasovitskii T.I., Shaposhnikov S.V. The Kolmogorov problem on uniqueness of probability solutions of a parabolic equation. Doklady Mathematics. 2020. V. 495. (in print)

We give a solution to the Kolmogorov problem on uniqueness of probability solutions to a parabolic Fokker-Planck-Kolmogorov equation.

[2] Bogachev V.I., Krasovitskii T.I., Shaposhnikov S.V. On uniqueness of probability solutions of the Fokker-Planck-Kolmogorov equation. Sbornik Mathematics, 2020. (accepted)

The paper gives a solution to the long-standing problem of uniqueness of probability solutions to the Cauchy problem for the Fokker-Planck-Kolmogorov equation with an unbounded drift coefficient and the unit diffusion coefficient. It is proved that in the one-dimensional case there holds uniqueness and in all other dimensions it fails. Also the case of non-constant diffusion coefficients is investigated.

3.1.14 George Sharygin

[1] With Yu.Chernyakov, A.Sorin and D.Talalaev:
The Full Symmetric Toda Flow and Intersections of Bruhat Cells, SIGMA 16 (2020), 115, 8 pages

In this short note we show that the Bruhat cells in real normal forms of semisimple Lie algebras enjoy the same property as their complex analogs: *for any two elements* w, w' *in the Weyl group* $W(\mathfrak{g})$*, the corresponding real Bruhat cell* X_w *intersects with the dual Bruhat cell* $Y_{w'}$ *iff* $w \prec w'$ *in the Bruhat order on* $W(\mathfrak{g})$. Here $\mathfrak{g}$ is a normal real form of a semisimple complex Lie algebra $\mathfrak{g}_{\mathbb{C}}$. Our reasoning is based on the properties of the Toda flows rather than on the analysis of the Weyl group action and geometric considerations.

[2] L_∞-derivations and the argument shift method for deformation quantization algebras arXiv:1912.00586, to appear in Acta Mathematica Spalatensia, Vol.1 (2020) 53 – 78

The argument shift method is a well-known method for generating commutative families of functions in Poisson algebras from central elements and a vector field, verifying a special condition with respect to the Poisson bracket. In this notice we give an analogous construction, which gives one a way to create commutative subalgebras of a deformed algebra from its center (which is as it is well known describable in the terms of the center of the Poisson algebra) and an L_∞-differentiation of the algebra of Hochschild cochains, verifying some additional conditions with respect to the Poisson structure.

3.1.15 Arkady Skopenkov

(Papers [8]-[12] from my 2019 report are not listed; they are still to appear in Bulletin of the Manifold Atlas.)

[1] A. Skopenkov. Classification of knotted tori, Proc. A of the Royal Society of Edinburgh, 150:2 (2020), 549-567. Full version: arXiv:1502.04470.

We describe the group of (smooth isotopy classes of smooth) embeddings $S^p \times S^q \to R^m$ for $p \le q$ and $m \ge 2p+q+3$. Earlier such a description was known only for $2m \ge 3p+3q+4$. We use a recent exact sequence of M. Skopenkov.

[2] A. Skopenkov. Invariants of graph drawings in the plane. Arnold Math. J., 6 (2020) 21–55; full version: arXiv:1805.10237.

We present a simplified exposition of some classical and modern results on graph drawings in the plane. These results are chosen so that they illustrate some spectacular recent higher-dimensional results on the border of geometry, combinatorics and topology. We define a $\mathbb{Z}_2$-valued *self-intersection invariant* (i.e. the van Kampen number) and its generalizations. We present elementary formulations and arguments accessible to mathematicians not specialized in any of the areas discussed. So most part of this survey could be studied before textbooks on algebraic topology, as an introduction to starting ideas of algebraic topology motivated by algorithmic, combinatorial and geometric problems.

[3] S. Avvakumov, I. Mabillard, A. Skopenkov and U. Wagner. Eliminating Higher-Multiplicity Intersections, III. Codimension 2 (extended abstract) Russian Math. Surveys 75:6 (2020) 173-174.

This is an announcement of [4].

[4] S. Avvakumov, I. Mabillard, A. Skopenkov and U. Wagner, Eliminating Higher-Multiplicity Intersections, III. Codimension 2, Israel J. Math., to appear, arxiv:1511.03501.

We study conditions under which a finite simplicial complex K can be mapped to $\mathbb{R}^d$ without higher-multiplicity intersections. An *almost r-embedding* is a map $f\colon K \to \mathbb{R}^d$ such that the images of any r pairwise disjoint simplices of K do not have a common point. We show that if r is not a prime power and $d \ge 2r+1$, then there is a counterexample to the topological Tverberg conjecture, i.e., *there is an almost r-embedding of the $(d+1)(r-1)$-simplex in $\mathbb{R}^d$*. This improves on previous constructions of counterexamples (for $d \ge 3r$) based on a series of papers by M. Özaydin, M. Gromov, P. Blagojević, F. Frick, G. Ziegler, and the second and fourth present author.

The counterexamples are obtained by proving the following algebraic criterion in codimension 2: *If* $r \geq 3$ *and if* K *is a finite* $2(r-1)$*-complex then there exists an almost* r*-embedding* $K \to \mathbb{R}^{2r}$ *if and only if there exists a general position PL map* $f\colon K \to \mathbb{R}^{2r}$ *such that the algebraic intersection number of the* f*-images of any* r *pairwise disjoint simplices of* K *is zero.* This result can be restated in terms of cohomological obstructions or equivariant maps, and extends an analogous codimension 3 criterion by the second and fourth author.

It follows from work of M. Freedman, V. Krushkal, and P. Teichner that the analogous criterion for $r = 2$ is false. We prove a beautiful lemma on singular higher-dimensional Borromean rings, yielding an elementary proof of the counterexample. As another application of our methods, we classify *ornaments* $f\colon S^3 \sqcup S^3 \sqcup S^3 \to \mathbb{R}^5$ up to *ornament concordance.*

[5] D. Crowley and A. Skopenkov, Embeddings of non-simply-connected 4-manifolds in 7-space, I. Classification modulo knots. Moscow Math. J., to appear, arxiv:1611.04738.

We work in the smooth category. Let N be a closed connected orientable 4-manifold with torsion free H_1, where $H_q := H_q(N;\mathbb{Z})$. The main result is *a complete readily calculable classification of embeddings* $N \to \mathbb{R}^7$, up to equivalence which is isotopy and embedded connected sum with embeddings $S^4 \to \mathbb{R}^7$. Such a classification was earlier known only for $H_1 = 0$ by Boéchat-Haefliger-Hudson 1970. Our classification involves Boéchat-Haefliger invariant $\varkappa(f) \in H_2$, Seifert bilinear form $\lambda(f) : H_3 \times H_3 \to \mathbb{Z}$ and β-invariant assuming values in the quotient of H_1 defined by values of $\varkappa(f)$ and $\lambda(f)$.

In particular, for $N = S^1 \times S^3$ we define geometrically a 1–1 correspondence between the set of equivalence classes of embeddings and an explicitly defined quotient of $\mathbb{Z} \oplus \mathbb{Z}$.

[6] A. Skopenkov. A short exposition of S. Parsa's theorem on intrinsic linking and non-realizability, Discr. and Comp. Geom., to appear, full version: arXiv:1808.08363. https://link.springer.com/article/10.1007/s00454-019-00158-y

We present a short exposition of the following results by S. Parsa.

Let L *be a graph such that the join* $L * \{1,2,3\}$ *(i.e. the union of three cones over* L *along their common bases) piecewise linearly (PL) embeds into* $\mathbb{R}^4$*. Then* L *admits a PL embedding into* $\mathbb{R}^3$ *such that any two disjoint cycles have zero linking number.*

There is C *such that every 2-dimensional simplicial complex having* n *vertices and embeddable into* $\mathbb{R}^4$ *contains less than* $Cn^{8/3}$ *simplices of dimension 2.*

We also present corrected statement and proof of the analogue of the second result for intrinsic linking.

[7] A. Skopenkov. A user's guide to basic knot and link theory, in: 'Topology, Geometry, and Dynamics: Rokhlin Memorial'. 772, ser. Contemp. Math. AMS, to appear. arXiv:2001.01472.

We define simple invariants of knots or links (linking number, Arf-Casson invariants and Alexander-Conway polynomials) motivated by interesting results whose statements are accessible to a non-specialist or a student. The simplest invariants naturally appear in an attempt to unknot a knot or unlink a link. Then we present certain 'skein' recursive relations for the simplest invariants, which allow to introduce stronger invariants. We state the Vassiliev-Kontsevich theorem in a way convenient for calculating the invariants themselves, not only the dimension of the space of the invariants. No prerequisites are required; we give rigorous definitions of the main notions in a way not obstructing intuitive understanding.

[8] D. Crowley and A. Skopenkov, Embeddings of non-simply-connected 4-manifolds in 7-space, II, arXiv:1612.04776, submitted.

We work in the smooth category. Let N be a closed connected orientable 4-manifold with torsion free H_1, where $H_q := H_q(N;\mathbb{Z})$. Our main result is *a readily calculable classification of embeddings* $N \to \mathbb{R}^7$ *up to isotopy*, with an indeterminacy. Such a classification was only known before for $H_1 = 0$ by our earlier work from 2008. Our classification is complete when $H_2 = 0$ or when the signature of N is divisible neither by 64 nor by 9.

The group of knots $S^4 \to \mathbb{R}^7$ acts on the set of embeddings $N \to \mathbb{R}^7$ up to isotopy by embedded connected sum. In Part I we classified the quotient of this action. The main novelty of this paper is the description of this action for $H_1 \neq 0$, with an indeterminacy.

Besides the invariants of Part I, detecting the action of knots involves a refinement of the Kreck invariant from our work of 2008.

For $N = S^1 \times S^3$ we give a geometrically defined 1–1 correspondence between the set of isotopy classes of embeddings and a certain explicitly defined quotient of the set $\mathbb{Z}\oplus\mathbb{Z}\oplus\mathbb{Z}_{12}$.

[9] S. Avvakumov, R. Karasev and A. Skopenkov. Stronger counterexamples to the topological Tverberg conjecture, submitted, arxiv:1908.08731.

Denote by Δ_N the N-dimensional simplex. A map $f\colon \Delta_N \to \mathbb{R}^d$ is an *almost r-embedding* if $f\sigma_1 \cap \ldots \cap f\sigma_r = \emptyset$ whenever $\sigma_1, \ldots, \sigma_r$ are pairwise disjoint faces. A counterexample to the topological Tverberg conjecture asserts that *if r is not a prime power and $d \geq 2r+1$, then there is an almost r-embedding* $\Delta_{(d+1)(r-1)} \to \mathbb{R}^d$. We improve this by showing that *if r is not a prime power and* $N := (d+1)r - r\left\lceil\dfrac{d+2}{r+1}\right\rceil - 2$*, then there is an almost r-embedding* $\Delta_N \to \mathbb{R}^d$. For the r-fold van Kampen–Flores conjecture we also produce counterexamples which are stronger than previously known. Our proof is based on generalizations of the Mabillard–Wagner theorem on construction of almost r-embeddings from equivariant maps, and of the Özaydin theorem on existence of equivariant maps.

[10] R. Karasev, A. Skopenkov, Some 'converses' to intrinsic linking theorems, submitted, arXiv:2008.02523.

A low-dimensional version of our main result is the following 'converse' of the Conway–Gordon–Sachs Theorem on intrinsic linking of the graph K_6 in 3-space:

For any integer z there are 6 points $1, 2, 3, 4, 5, 6$ in 3-space, of which every two i, j are joint by a polygonal line ij, the interior of one polygonal line is disjoint with any other polygonal line, the linking coefficient of any pair disjoint 3-cycles except for $\{123, 456\}$ is zero, and for the exceptional pair $\{123, 456\}$ is $2z + 1$.

We prove a higher-dimensional analogue, which is a 'converse' of a lemma by Segal–Spież.

[11] A. Skopenkov. Extendability of simplicial maps is undecidable, submitted, arXiv:2008.00492.

We present a short proof of the Čadek-Krčál-Matoušek-Vokřínek-Wagner result from the title (in the following form due to Filakovský-Wagner-Zhechev).

For any fixed integer $l > 1$ there is no algorithm recognizing the extendability of the identity map of $S^l \vee S^l$ to a PL map $X \to S^l \vee S^l$ of given $2l$-dimensional simplicial complex X containing a subdivision of $S^l \vee S^l$ as a given subcomplex.

We also exhibit a gap in the Filakovský-Wagner-Zhechev proof that embeddability of complexes is undecidable in codimension > 1.

[12] S. Parsa and A. Skopenkov. On embeddability of joins and their 'factors', submitted, arXiv:2003.12285

We present a short and clear proof of the following particular case of a 2006 unpublished result of Melikhov-Schepin. *Let K be a k-dimensional simplicial complex and $K * [3]$ the union of three cones over K along their common bases. If $2d \geq 3k + 3$ and $K * [3]$ embeds into $\mathbb{R}^{d+2}$, then K embeds into $\mathbb{R}^d$.* We also present a generalization of this theorem. The proofs are based on the Haefliger-Weber 'configuration spaces' embeddability criterion, equivariant suspension theorem and simple properties of joins and cones.

[13] A. Skopenkov. On some results of S. Abramyan and T. Panov, arXiv:2005.11152.

This note is purely expository and is an extended version of math review to the paper [AP19]=arXiv:1901.07918v3 by S. Abramyan and T. Panov published in Proc. of Steklov Math. Inst. 305 (2019). The authors construct simplicial complexes for whose moment-angle complexes certain homotopy classes are non-trivial. I expose in a shorter and clearer way the main definition and the statement of Theorem 5.1 from [AP19]. The clarification reveals that the main definition used in the statements of the main results is not given in [AP19].

[14] A. Skopenkov, On different reliability standards in current mathematical research, `https://www.mccme.ru/circles/oim/rese_inte.pdf`

In this note I expose the reliability standards I share, and give examples of different reliability standards.

Expository publications for university students

[1] A. Skopenkov, Algebraic Topology From Geometric Viewpoint (in Russian), 2020, MCCME, Moscow (2nd edition) `http://www.mccme.ru/circles/oim/home/combtop13.htm#photo`

In this book we present a 'geometric' approach to algebraic topology. The book is essentially rewritten for the second edition.

[2] A. Skopenkov, Mathematics Through Problems: from olympiades and math circles to profession. Part I. Algebra. AMS, Providence, to appear, `https://www.mccme.ru/circles/oim/algebra_eng.pdf`.

This is a collection of teaching materials used in several Russian universities, schools, and mathematical circles. Most problems are chosen in such a way that in the course of the solution and discussion a reader learns important mathematical ideas and theories. The materials can be used by both teachers and students.

[3] A. Skopenkov. A user's guide to basic knot and link theory (in Russian). Mat. Prosveschenie, to appear. arXiv:2001.01472.

[4] A. Skopenkov. (in Russian). Mat. Prosveschenie, to appear.

[5] V. Retinskiy, A. Ryabichev and A. Skopenkov. Motivated exposition of the proof of the Tverberg Theorem (in Russian). Mat. Prosveschenie, to appear. arXiv:2008.08361.

[6] A. Skopenkov, Algebraic Topology From Algorithmic Viewpoint, draft of a book, `http://www.mccme.ru/circles/oim/algor.pdf` (some sections are rewritten in 2020)

In this book we present an 'algorithmic' approach to algebraic topology.

3.1.16 Evgeni Smirnov

[1] Multiple flag varieties

Journal of Mathematical Sciences, vol. 248, no. 3, 338–373 (2020).

This is a survey of results on multiple flag varieties, i.e. varieties of the form $G/P_1 \times \cdots \times G/P_k$. We provide a classification of multiple flag varieties of complexity 0 and 1 and results on the combinatorics and geometry of B-orbits and their closures in double cominuscule flag varieties. We also discuss questions of finiteness for the number of G-orbits and existence of an open G-orbits on a multiple flag variety.

[2] (with Anna Tutubalina) Slide complexes and subword complexes.

Uspekhi Matematicheskikh Nauk, vol. 75, no. 6 (457), 177–178 (2020) (in Russian).

Short announcement of the results of the preprint [3].

[3] (with Anna Tutubalina) Slide polynomials and subword complexes. Preprint arXiv:2006.16995, 15 pp., 2020. Submitted for publication.

Subword complexes were defined by A. Knutson and E. Miller in 2004 for describing Grbner degenerations of matrix Schubert varieties. The facets of such a complex are indexed by pipe dreams, or, equivalently, by the monomials in the corresponding Schubert polynomial. In 2017 S. Assaf and D. Searles defined a basis of slide polynomials, generalizing Stanley symmetric functions, and described a combinatorial rule for expanding Schubert polynomials in this basis. We describe a decomposition of subword complexes into strata called slide complexes, that correspond to slide polynomials. The slide complexes are shown to be homeomorphic to balls or spheres.

[4] (with Anna Tutubalina) Pipe dreams for Schubert polynomials of the classical groups. Preprint arXiv:2009.14120, 29 pp., 2020. Submitted for publication.

Schubert polynomials for the classical groups were defined by S.Billey and M.Haiman in 1995; they are polynomial representatives of Schubert classes in a full flag variety of a classical group. We provide a combinatorial description for these polynomials, as well as their double versions, by introducing analogues of pipe dreams, or RC-graphs, for the Weyl groups of the classical types.

[5] Number friezes. Kvant no. 5 (2020), 15–24.

We discuss the main properties of Conway and Coxeter's number friezes. This is a popular article for schoolchildren majoring in mathematics.

3.1.17 Alexei Sossinsky

[1] On the equivalence of three models of the Lobachevsky plane [in Russian], Mathematical Enlightenment, Series 3, 25 38-47 (2020).

The equivalence of the three classical models of the hyperbolic plane is demonstrated by formalizing physical experiments involving a transparent half-sphere with half-circles (representing hyperbolic lines) drawn on it.

[2] Sum of the angles of a triangle and the discrete GaussBonnet theorem [in Russian], Mathematical Enlightenment, Series 3, 26 4-11 (2020).

The discrete version of the GaussBonnet theorem, which may be regarded as a generalization of the fact that the sum of angles of a Euclidean triangle is equal to π, is proved.

[3] Geometries [in Russian], Moscow, MCCME Publs. (2020)

The book is a unified exposition of various geometries (Euclidean, hyperbolic, elliptic, projective, discrete, finite) defined as sets of points with various groups acting on them.

[4] Introduction to Topology, a Lecture course [in Russian], Moscow, MCCME Publs. (2020)

The book is a two-semester introductory lecture course in topology, emphasizing its geometric and algebraic aspects, progressively developing homotopy and homology theory in the language of categories and functors.

[5] Elementary theory of the Conway polynomial, Mathematical Notes, Vol. 108, no. 5, 765-767 (2020).

We show that the Alexander–Conway polynomial can be introduced by means of Conway's skein relation and its invariance can be proved by elementary means, without using any homology theory or any nontrivial algebra.

3.1.18 Vladimir Zhgoon

[1] V.S.Zhgoon, F.Knop, On the action of restricted Weyl group on the set of orbits of a minimal parabolic subgroup, Doklady Mathematics, 2020, Vol. 101, No. 1, pp. 25–29.

We construct the action of the restricted Weyl group on the set of principle families of orbits of a minimal parabolic subgroup over algebraically non-closed field. Also we relate this action with the action on polarized cotangent bundle. These results generalize corresponding results of F.Knop on the action of the Weyl group on the families of Borel orbits of the maximal complexity and rank.

[2] V. S. Zhgoon, V. P. Platonov, M.M.Petrunin, On the problem of the periodicity of expansions into a continued fraction of $\sqrt{f}$ for cubic polynomials over number fields,

Doklady Mathematics, 2020, Vol. 102, No. 1, pp. 288–292

We obtain a complete description of the fields $\mathbb{K}$, that are quadratic extensions of $\mathbb{Q}$, and of cubic polynomials $f \in \mathbb{K}[x]$, for which a continued fraction expansion of $\sqrt{f}$ in the field of formal power series $\mathbb{K}((x))$ is periodic. We also prove the finiteness theorem for cubic polynomials $f \in \mathbb{K}[x]$ with periodic decomposition $\sqrt{f}$ over cubic and quartic extensions of $\mathbb{Q}$. These results completes the research in this direction described in the series of papers of the authors, and are a corollary of the description obtained in this paper of the periodic elements $\sqrt{f}$ for the cubic polynomials $f(x)$, which define elliptic curves with the points of order $3 \leq N \leq 22, N = 24$.

[3] V.S.Zhgoon, F.Knop, Complexity of actions over perfect fields, preprint arXiv:2006.11659.

Let G be a connected reductive group over a perfect field k acting on an algebraic variety X and let P be a minimal parabolic subgroup of G. For k-spherical G-varieties we prove finiteness result for P-orbits that contain k-points. This is a consequence of an equality on P-complexities of X and of any P-invariant k-dense subvariety in X, which generalizes a corresponding result of E.B.Vinberg in the case of algebraically closed field k. Also we introduce an action of the restricted Weyl group W on the set of k-dense P-invariant closed subvarieties of X of maximal P-complexity and k-rank in the case of char$k = 0$ and on the set of all k-dense P-orbits in the case of real spherical variety which generalizes the action on B-orbits introduced by F.Knop in the algebraically closed field case. We also introduce a little Weyl group related with this action and describe its generators in terms of the generators of W which generalize the description of M.Brion in algebraically closed field case.

3.2 Scientific conferences and seminar talks

3.2.1 Anton Ayzenberg

[1] International conference "Workshop on Torus Actions in Topology", Toronto, Canada, May 11-15

Talk: "Toric topology of complexity one"

[2] International conference "Topology and geometry of group actions", Moscow, November 18-22.

I was the organizer of the conference (held in Zoom). The website https://cs.hse.ru/ata-lab/tgga/

[3] Talk at Toric Topology Postdoc Seminar at Fields Institute, Toronto, Canada, April 20.

Talk: “Toric topology of complexity one”

[4] Talk at Online Geometry Seminar at ETH Zürich, April 29

Talk: “Multipolytopes, volume polynomials, duality algebras”

[5] Talk at “Geometric topology seminar”, HSE, Moscow, September 18

Talk: “Orbit spaces for actions of complexity one”

[6] Talk at Postnikov memorial seminar “Algebraic topology and its applications”, MSU, Moscow, March 3

Talk: “Toric topology of complexity one”

[7] Three lectures at Autumn online school “Topology, geometry and applications - 2020”, HSE, Moscow, September 11-13

Title of the minicourse: “Topology of properties and group actions”

3.2.2 Alexei Elagin

[1] Visit to Berkeley, USA, March

Talk “Derived categories of coherent sheaves on affine schemes and their thick subcategories” at “Representation Theory and Mathematical Physics seminar” (University of California)

3.2.3 Sergei Gorchinsky

[1] Conference “Geometry days in Novosibirsk – 2020”, Novosibirsk, September, 17–19

Talk “Higher-dimensional Contou-Carrère symbol”

[2] Summer school “Mathematical evenings of the summer school Contemporary mathematics”, Moscow, July, 20 – August, 1

Talk “Panorama of the arithmetic geometry”

[3] Mini-course of 5 lectures “Invitation to arithmetic geometry”, Mathematical Center in Akademgorodok, Novosibirsk, November, 30 – December, 4.

3.2.4 Anton Khoroshkin

This year, due to the pandemic situation in the world all workshops and conferences I supposed to participate/organized were cancelled

3.2.5 Iosif Krasilshchik

[1] Visit to Bologna, April

[2] Talk ‘Nonlocal conservation laws of PDEs possessing differential coverings’ at the seminar on geometry of differential equations, Independent Univ. of Moscow, 30 September 2020

3.2.6 Konstantin Loginov

[1] Online conference “Zoom Algebraic Geometry Marathon”, University of Edinburgh, 1 September 2020, talk “Bounding non-rationality of fibers in Fano fibrations”.

[2] Conference “Escola Transguanabara de Geometria Albebrica”, Instituto de Matematica Pura e Aplicada, 4-7 February 2020, talk “Semistable degenerations of Fano varieties”

[3] Online conference “Zoomerfest”, National Research University Higher School of Economics, 9 April 2020, talk “On the topology of dual complexes’.’

[4] Seminar of the Laboratory of Algebraic Geometry and Homological Algebra, Moscow Institute of Physics and Technology, 8 October 2020, talk “Bounding non-rationality of fibers in Fano fibrations”.

[5] Shafarevich seminar, online, Steklov Mathematical Institute, 20 October 2020, talk “Bounding non-rationality of fibers in Fano fibrations”.

[6] Iskovskikh seminar, online, Steklov Mathematical Institute, 29 October 2020, talk “Maximal log Fano pairs as generalized Bott towers”.

[7] Iskovskikh seminar, Steklov Mathematical Institute, 5 March 2020, talk “Dual complexes of log Calabi-Yau pairs”.

3.2.7 Taras Panov

[1] International Conference “Topology and Geometry of Group Actions”; Faculty of Computer Science, HSE University, Moscow, Russia; November, 18–22.

Talk “Polyhedral products, loop homology, and right-angled Coxeter groups”.

[2] Workshop on Polyhedral Products in Geometric Group Theory; Fields Institute for Research in Mathematical Sciences, Toronto, Canada; May, 25–29

Talk “Polyhedral products, loop homology, and right-angled Coxeter groups”.

[3] Winter Graduate School in Toric Topology, Fields Institute for Research in Mathematical Sciences, Toronto, Canada; January, 13–17

Mini-course on the Topology of Toric Spaces and Moment-angle Manifolds (joint with Anthony Bahri)

[4] Beijing-Moscow Mathematics Colloquium; November, 20 (online).

Talk “Right-angled polytopes, hyperbolic manifolds and torus actions”.

[5] Algebraic Topology Seminar, Princeton University, USA; September, 16 (online);

Talk “A geometric view on SU-bordism”.

[6] Mathematics Department Colloquium, the Ohio State University, USA; March, 5.

Talk “Right-angled polytopes, hyperbolic manifolds and torus actions”.

3.2.8 Alexei Penskoi

[1] Conference “Geometric Measure Theory and Geometric Analysis in Moscow”, Moscow, September 14-18, 2020

Zoom talk “Isoperimetric inequalities for Laplace eigenvalues on the sphere and the real projective plane ”

[2] Novosibirsk-Beijing Seminar ”Geometry, Topology and their Applications”, May, 4, 2020

Zoom talk “Isoperimetric inequalities for Laplace eigenvalues on the sphere and the real projective plane”

[3] Skoltech Center for Advanced Studies (Moscow) Seminar on Mondays, November 30, 2020

Zoom talk “Isoperimetric inequalities for eigenvalues of the Laplace-Beltrami operator”

3.2.9 Petr Pushkar

Talk “Around four verticies” at HSE seminar ”Contact topology and invariants of legendrian knots ”

Talk “Morse theory and legendrian knots ” at HSE seminar ”Contact topology and invariants of legendrian knots ”

3.2.10 George Shabat

[1] Scientific seminar of the department of Mathematical Logic and Algorithm Theory, Lomonosov Moscow State University, Moscow, April, 15

Talk “Verification of long proofs: dreams, plans, reality”

[2] International seminar “Graphs on surfaces and curves over number fields”, Lomonosov Moscow State University, Moscow, Southampton, Bonn, Madrid, May, 27

Talk “On the Belyi height”

[3] International seminar “Graphs on surfaces and curves over number fields”, Lomonosov Moscow State University, Moscow, Southampton, Bonn, Paris, November, 4

Talk “Dessins d’enfants and piecewise-Euclidean metrics on Riemann surfaces”

3.2.11 Stanislav Shaposhnikov

[1] Online International Workshop "LSA Autumn Meeting 2020"
(Laboratory of Stochastic Analysis and its Applications,
HSE, Moscow, 19.10.2020-23.10.2020)
Talk: On the Ambrosio-Figalli-Trevisan superposition principle.

3.2.12 George Sharygin

[1] Conference "Second International conference on Integrable systems and nonlinear dynamics", Yaroslavl, October 19 – 23, 2020 (online talk)

Talk "Symmetries of the full symmetric Toda system on real Lie algebras"

[2] Visit to IHES, December 20, 2019 – February 20, 2020

Talk "Geometry of the full symmetric Toda system on real Lie groups" (Dijon, Université de Bourgogne, seminar on Theoretical Physics, January 23, 2020)

Talk "Homology of higher simplex relations" (Angers, Université d'Angers, seminar in Theoretical Physics and Algebraic Topology)

[3] Yaroslavl seminar on Integrable systems, April 8, 2020 (online talk)

Talk "On noncommutative cross ratios", Yaroslavl center for Integrable systems, April 8, 2020

3.2.13 Arkady Skopenkov

[1] Postnikov memorial seminar, Moscow State University. Talks

'Embeddability of complexes is undecidable in codimension > 1' http://www.mathnet.ru/php/seminars.phtml?option_lang=rus&presentid=26685

'Stronger counterexamples to the topological Tverberg conjecture' http://www.mathnet.ru/php/seminars.phtml?option_lang=rus&presentid=26776

[2] Seminar 'Discrete and computational geometry', The Institute for Information Transmission Problems

Talk 'Embeddability of complexes is undecidable in codimension > 1'

[3] Seminar of Lab of theoretical informatics, Faculty of Computer Science, Higher School of Economics, https://cs.hse.ru/big-data/tcs-lab/seminar

Talk 'Embeddability of complexes is undecidable in codimension > 1'

[4] Seminar of International Laboratory of algebraic topology and its applications, Faculty of Computer Science, Higher School of Economics, https://cs.hse.ru/en/ata-lab/seminars

Talk 'Extendability of simplicial maps is undecidable'

3.2.14 Evgeni Smirnov

[1] School and conference "Lie algebras, invariant theory and representation theory", Lomonossov Moscow State University, January 27, 2020

Talk: Involutions in symmetric groups: geometry and combinatorics

[2] Combinatorics and geometry seminar, Chebyshev Laboratory, State University of Saint Petersburg, April 6, 2020, via Zoom

Talk: Subword complexes and slide polynomials

[3] Representation theory and dynamical systems, PDMI RAS, Saint Petersburg, September 23 and October 7, 2020 (via Zoom)

Talk: Schubert polynomials for the classical groups

[4] Integrable systems and related topics, University of Yaroslavl, September 30, 2020 (via Zoom)

Talk: Schubert polynomials for the classical groups

[5] Combinatorics and geometry seminar, Chebyshev Laboratory, State University of Saint Petersburg, October 5, 2020

Talk: Symmetric polynomials and arrays

[6] Lie algebras and applications, HSE University, Moscow, October 13, 2020

Talk: Subword complexes and slide polynomials

3.2.15 Vladimir Zhgoon

[1] Conference Lie algebras, algebraic groups and invariant theory (Moscow) Talk: On the action of the Weyl group on the Borel orbits of submaximal rank.

[2] Seminar of laboratory of Algebraic geometry HSE Moscow

Talk: On complexity of algebraic varieties over algebraically non-closed fields

3.3 Teaching

3.3.1 Anton Ayzenberg

[1] Topology extras. Independent University of Moscow, II-V year students, September-December 2020, 2 hours per week.

Program.

1. Nerve theorem (introductory lecture to check audience background).

2. Partially ordered sets, their geometrical realizations.

3. Diagrams of topological spaces: colimits and homotopy colimits.

4. Configurations of vector subspaces, Goresky–MacPherson formula and its homotopy version.
5. Simple homotopy equivalence. Morse theory and discrete Morse theory.
6. Filtrations and persistence modules. Structure theorem for persistence modules, persistence diagrams. Computational algorithm: conjugate simplices in a filtration.
7. Zigzag persistent homology. Quivers and their representations, diagrams of vector spaces. Remak–Krull–Schmidt decomposition. Gabriel theorem.
8. Shellings of simplicial complexes. Matroids and their independence complexes.
9. Introduction to spectral sequences. Spectral sequence associated with a filtration.
10. Mayer–Vietoris spectral sequence and Serre spectral sequence.
11. Finite Alexandrov topologies. McCord theorems. Core of a poset (of a topology). Morphisms of posets and Quillen's theorem A.
12. Homological theory of sheaves on finite posets.

[2] Calculus-2, Higher School of Economics, II year students, January-June 2020, 4 hours per week.

Program.

1. Fourier series, Hilbert spaces, orthogonal systems.
2. Fourier transform.
3. Multivariable calculus: local and global extrema.
4. Submanifolds and smoothness conditions.
5. Conditional extrema. Lagrange multipliers.
6. Karush–Kuhn–Tucker conditions, basics of linear programming.
7. Green's theorem.
8. Grassmann variables. Differential forms on open subsets of $\mathbb{R}^n$, integration of a form over a submanifold.
9. Exterior derivative. Stokes's theorem and its particular instances: Fundamental theorem of calculus, Green, Gauss–Ostrogradskiy (divergence theorem).
10. Functions of a complex variable, the basics: Cauchy–Riemann equations, path integration and its homotopy invariance for holomorphic functions, poles and residues.

3.3.2 Yuri Burman

[1] Linear Algebra and Geometry. Independent University of Moscow, I year students, September–Devember 2020, 2 hours per week + exercises.

Program

1. Vectors.

    1.1. Vector spaces and bases.

    Vector space, subspace, linear map, quotient space. Finite-dimensional (finitely generated) space; any space of finite dimension has a basis, a basis in a subspace can be extended to a basis in the whole space. Linear hull; relation between dimensions of subspaces, their intersection and their sum.

    1.2. Linear maps.

    Linear maps as a transformation group. Kernel, image and their dimensions. Dual subspace, dual basis and conjugate map. A finite-dimensional space is isomorphic to its dual and double dual. The action of the group $\mathrm{GL}(n)$ on the set of bases is transitive fixed-point-free; matrix of a linear map. The matrix of a composition, of the dual operator, and of the inverse operator.

2. Affine geometry

    2.1. Affine group.

    Affine space, simplices and barycentric coordinates. Linear group is a quotient of the affine group by the subgroup of translations. Normalized of a hyperplace in a linear group is the affine group. The affine group action on the set of simplices of full dimension is transitive and fixed-point-free. Semi-affine maps and field automorphisms.

    2.2. Convex sets.

    Carathéodory's theorem, Radon's theorem and Helly's theorem.

3. Projective geometry.

    3.1. Projective group.

    The projective group is a quotient of the linear group by the subgroup of dilations. Affine group is a normalizer of a hyperplane in the projective group. The action of the projective group on the set of $(n+2)$-tuples of points in general position is transitive and fixed-point-free.

    3.2. Projective grometry of small dimensions.

    A transformation of a line is projective if and only if it preserves the cross-ratio. Action of the group S_4 on cross-ratios. The theorem of Desargues.

3.3. Basic projective duality.

Duality for the annulators and for projective subspaces.

4. Euclidean geometry.

4.1. Determinant.

Determinant is a polylinear fully skew-symmetric function of lines of a square matrix. Laplace expansion formula. Determinant of the transposed matrix. Formula for the inverse matrix and for solution of a system of linear equations. Determinant of a linear operator.

4.2. Bilinear forms and scalar products.

Bilinear forms, quadratic forms. Matrix of a bilinear form and its dependence on the basis. Scalar product, Cauchy–Schwartz inequality, triangle inequality. Darboux basis for a skew-symmetric form.

4.3. Volumes.

Definition of a volume, its invariance under translations. Signed volume of a cuboid is polylinear and skew-symmetric and is proportional to the determinant. Orientation in $\mathbb{R}^n$.

4.4. Quadrics.

Diagonalization of a quadratic form over any field. More precise results over complex numbers and over reals. Projective classification of quadrics over complex numbers and over reals.

3.3.3 Alexei Elagin

[1] Homological methods in representation theory of finite dimensional algebras. Independent University of Moscow, III-V year students, September-December 2020, 2 hours per week.

Program

1. **Basics from representation theory**
   Additive categories. Krull-Schmidt theory. Indecomposable modules. The radical of an additive category. Irreducible morphisms, Auslander-Reiten quiver.
   Abelian categories, module categories over algebras. Characterization of module categories. Morita-equivalence. Basic algebras. Quivers and their representations. Path algebras. Quivers with relations and their representations. Theorems of Gabriel about Morita-equivalence and about isomorphism. Projective, injective and simple modules. k-linear and A-linear duality, Nakayama functor.

2. **Basics from homological algebra**
   The category of complexes, its homotopy and derived category. Their properties. Truncations. Morphisms between pure complexes and Ext groups. Bounded versions.
   Triangulated categories. Triangulation of homotopy and derived categories. Long exact sequences of Homs. Acyclic, h-projective and h-injective complexes. Projective and injective resolutions. Derived categories of modules over algebras. Idempotent completeness of some derived categories. The category of perfect complexes.
   Projective, injective and global dimension. Global dimension of some path algebras. Derived category of a hereditary category.
   Exact functors on triangulated categories. The left and the right derived functors of an exact functor. Derived functors $RHom$ and $\otimes^L$, their adjunction. Derived Morita-equivalence and Rickard's theorem. Tilting objects.
   Semiorthogonal decompositions, admissible subcategories. Their mutations. Exceptional collections and their mutations. Equivalences of derived categories coming from strong exceptional collections. The Serre functor.
   Grothendieck group and Euler form. The Grothendieck group of a finite dimensional algebra.

3. **Homological methods applied in representation theory.**
   Reflection functors as equivalences between derived categories of quiver representations. Admissible sequences of vertices and the corresponding compositions of reflections. Regular, preprojective and preinjective modules. Independence of the derived category of a quiver on the orientation. Coxeter functor and Serre functor.
   Classification of quivers with finite representation type. Tits quadratic form. Dynkin quivers. Roots and Weyl groups. Bijection between roots and indecomposable modules.

3.3.4 Sergei Gorchinsky

[1] Around algebraic theory of multiple zeta-values II. Independent University of Moscow, master students and PhD students, February-May 2020, 2 hours per week.

Program

1. Torsors, groupoids, and Hopf algebroids. Generalization of Chen theorem for paths.

2. Pure and mixed Hodge structures. Mixed Hodge structure on the regular algebra of the prounipotent completion of the fundamental group of a complex algebraic variety.

3. Tangential points. Generalization of the prounipotent completion of the fundamental groupoid for tangential points. Interpretation of MZV's as periods of mixed Hodge structures.

4. Tannakian categories. The case of the extension of a multiplicative group by a unipotent group.

5. Voevodsky motives. Mixed Tate motives. Motivic structure on the regular algebra of the prounipotent completion of the fundamental group of an algebraic variety.

6. Mixed Tate motives over $\mathbb{Z}$. The structure of a mixed Tate motive on the regular algebra of the prounipotent completion of the fundamental group of the variety $\mathbb{P}^1\backslash\{0, 1, \infty\}$. Proof of the Goncharov–Terasoma theorem.

[2] Differential Galois theory. Steklov International Mathematical Center, master students and PhD students, February-May 2020, 2 hours per week.

Program

1. Differential rings.

2. Differential modules.

3. Differential modules and differential equations.

4. Picard–Vessiot theory.

5. Applications of the differential Galois theory.

3.3.5 Alexei Gorodentsev

[1] Algebra-1. Independent University of Moscow, I year students, September–December 2020, 4 hours per week.
Site: `http://gorod.bogomolov-lab.ru/ps/stud/algebra-1/2021/list.html`.
Lecture notes:
`http://gorod.bogomolov-lab.ru/ps/stud/algebra-1/2021/lec_total.pdf`

Program:

1. Sets and maps. Multinomial coefficients, Young diagrams and other combinatorics. Binary relations. Posets and well ordered sets, the Zorn lemma.

2. Definitions and basic properties of abelian groups, commutative rings, fields and their homomorphisms. Direct products. Rings and fields $\mathbb{Z}/(m)$. GCD, coprime elements, and the Chinese remainder theorem in $\mathbb{Z}$. The simple subfield, characteristic, Frobenius mapping.

3. Formal power series and polynomials, formal differntial calculus. GCD, coprime elements, and the Chinese remainder theorem in $\Bbbk[x]$. Roots of polynomials. Rings and fields $\Bbbk[x]/(f)$. The complex numbers. Finite fields. Quadratic reciprocity.

4. Localisation in commutative rings, rings and fields of fractions. The Laurent series. Rational functions: the partial fraction expansion, the power series expansion, linear recursions. The exponent, logarithm, binomial. The Todd series and the Bernoulli numbers.

5. Ideals and quotient rings. Noetherian rings, Hilbert's theorem on a basis of ideal. Principal ideal domains, Euclidean rings. Factorial rings, the Gauss lemma, the polynomial ring over a factorial ring is factorial. Factorization in $\mathbb{Z}[x]$.

6. Modules over commutative rings. Generators and relations, the torsion, a decomposability. Free modules, the rank. A submodule of finitely generated free module over a principal ideal domain is free. Homomorphisms of modules.

7. Vector spaces, bases, the dimnsion. Dimensions of sums, intersections, and fibers of linear maps. An exaple: the polynomial interpolation of jets. Direct sums vs direct products. Quotient spaces and linear spans.

8. The matrix formalism for linear expansions of collections of vectors and for linear maps. The multiplication of matrices. Inverse matrices and base changes. Algebras over a field, the matrix algebra, the algebra of linear endomorphisms. Matrices over non-commutative rings, the inverse for an upper unitriangular matrix.

9. Multilinear antisymmetric forms. The sign of a permutation. The determinant. Cramer's rules and the adjugate matrix. The Cayley-Hamilton identity. Grassmannian polynomials and minors, the Laplace relations. Taylor's expansion for the determinant of a pencil of matrices.

10. The Gauss elimination over a principal ideal domain. The Smith normal form and the invariant factors theorem. Invariant factors vs elementary divisors, the elementary divisors theorem. The canonical decomposition for a finitely generated module over a principal ideal domain. Examples: the classification of finitely generated abelian groups, compatible lattices in $\mathbb{Q}^n$ and their discriminants.

[2] Linear Algebra and Geometry, the National Research University 'Higher School of Economics', Faculty of Math., I year students, September-December 2020, 6 hours per week.
Site: `http://gorod.bogomolov-lab.ru/ps/stud/geom_ru/2021/list.html`.
Lecture notes:
`http://gorod.bogomolov-lab.ru/ps/stud/geom_ru/2021/lec_total.pdf`

Program.

1. Vector spaces. The vector space $\Bbbk^2$: proportional vectors, the determinant 2×2, bases and coordinates, Cramer's rule, the oriented area of parallelogram. The affine space $\Bbbk^2$: affine coordinate systems,collinear points, equations for lines, oriented areas of triangles and polygons. The center of masses and barycentric coordinates.

2. Linear and affine transformations of plane, the differential of an affine map. Linear and affine changes of coordinates, the matrix formalism. Semiaffine and semilinear transformations (the maps sending a line to a line).

3. Inner products and orthonormal bases in a real plane, the Cauchy–Bunyakovsky–Schwartz inequality, the triangle inequality. The Gram determinant. The oriented angle between vectors. Metric properties of line equations, bisectors, angles between lines. Trigonometric identities, geometric descriprion for the complex number multiplication. Euclidean plane = complex line. The similarity transformations of Euclidean plane.

4. Higher dimensional vector spaces: generators and linear relations, the exchange lemma, bases and the dimension. Linear maps, the kernel and the image. Dimensions of sums, intersections, and fibers of linear maps. Direct sums and direct products of vector spaces and subspaces. Affine spaces, the geometry of mutual arrangement of a pair of affine subspaces. Quotient spaces.

5. The matrix formalism for linear expansions of collections of vectors, for linear maps, and for systems of linear equations. The multiplication of matrices. Inverse matrices and base changes. The rank of a matrix. The qualitative theory of linear equations: the Fredholm alternative, the Kronecker–Capelli test, the dimension of solution space. Algebras over a field, the matrix algebra, the algebra of linear endomorphisms. Matrices over non-commutative rings, the inverse for an upper unitriangular matrix.

6. The Gauss elimination over a field: finding a basis for a subspace $U \subset \Bbbk^n$ and for the quotient $\Bbbk^n/U$, computing the rank of matrix, finding the inverse matrix, solving systems of linear equations, finding bases in the kernel and image of a linear map. The combinatorial type of a subspace in $\Bbbk^n$ and uniqueness of a basis with a reduced echelon matrix of coordinates.

7. The dual space, examples of linear forms. The embedding $V \hookrightarrow V^{**}$, an advanced example: polynomials and power series, the umbral calculus. The isomorphism $V \simeq V^{**}$ for a finite-dimensional V, dual bases, coordinates of a linear form in the dual basis. The dual space to the linear span of a given set of vectors, the rank of matrix revisited. The dual space to a subspace. Annihilators, the bijection $U \leftrightarrow \operatorname{Ann} U$ inverts the inclusions and transforms the sums to the intersections and vice versa. Dual linear mappings. The systems of linear equations revisited. Finding a basis and/or an affine frame in an intersection and sum of vector and/or affine subspaces.

8. The volume of an oriented parallelepiped, multilinear skew-symmetric and sign-alternating forms, the space of skew-symmetric n-linear forms on an n-dimensional space has dimension 1. The sign of a permutation, an exapmple: the sign of a shuffle permutation. The determinant of a matrix, its simplest properties: multilinearity, invariance under transposition, multiplicativity, the row and column expansions. Techniques for calculating determinants. The ratio of the volume of a simplex to the volume of a parallelepiped (over $\mathbb{R}$).

9. Cramer's rules, the adjunct matrix, the identity $A \cdot A^{\vee} = \det A \cdot E$. Matrices with elements in the algebra of polynomials = polynomials with coefficients in the algebra of matrices, the Cayley–Hamilton identity. Grassmann polynomials over a field, linear changes of variables and minors, an expansion of a determinant via a set of rows or columns (the Laplase relations).

10. Eigenvalues and eigensubspaces of a linear operator. The characteristic and minimal polynomials. Decomposition of a space into a direct sum of invariant subspaces by the factorization of an annihilating polynomial. Criteria for diagonalizability. Root subspace decomposition. Evaluation of analytic functions at a linear operator via polynomial interpolation of jets at the eigenvalues. Nilpotent operators, a cyclic basis and the cyclic type of nilpotent linear operator. Commuting operators: common eigenvector and simultaneous diagonalization, working example: finite groups of linear transformations. The Jordan decomposition of a linear operator and the Jordan normal form of a matrix over an algebraically closed field.

3.3.6 Anton Khoroshkin

[1] Basic algebra for I year bachelor students (spring semester), Independent university of Moscow, spring 2020, 2hours lecturs + 2 hours exercises class per week

Program include:

- Classification of finite abelian groups,
- Modules over principal ideal domains,
- Frobenius and Jordan normal forms,
- Hermitian and Unitary operators,
- Symmetric functions, resultant and discriminant,
- Tensor product of vector spaces and modules,

- Basics of representations of finite groups:
  Maschke's theorem, characters, orthogonality, quasiregular and induced representations.

[2] Basic algebra for II year bachelor students (fall semester), Independent university of Moscow, fall 2020, 2hours lecturs + 2 hours exercises class per week

Program include:

- Noetherian rings and modules, Hilbert basis theorem and Hilbert invariant theorem.
- Gauss's Lemma, unique factorization domains.
- Algebraic extensions of rings and fields; Doubling the cube, Angle trisection.
- Splitting field and algebraic closures.
- Prime and maximal ideals, Hilbert Nullstellensatz for $\mathbb{C}$.
- Normal, separable and Galois extensions.
- Fundamental theorem of Galois theory.
- Cyclotomic polynomials and cyclotomic extensions, Abel's theorem.
- Rings of integers.
- The Jacobson density theorem (Double centralizer theorem) and the structure theory of semisimple algebras.
- Morita equivalence. The Brauer group. Division rings over $\mathbb{R}$.

[3] Standard basic algebra classes for I year bachelor students, spring 2020, 2 hours seminars per week, NRU Higher School of Economics

[4] Lie theory for III year bachelor students, fall 2020, 2 hours seminars per week, NRU Higher School of Economics

[5] Sheaf theory for IV year bachelor students, fall 2020, 2 hours seminars per week, NRU Higher School of Economics

[6] I am currently a scientific supervisor at NRU HSE of

- 3 bachelor students of the forth grade,
- 2 bachelor student of the third grade,
- 1 bachelor student of the second grade,
- 1 bachelor student of the first grade,
- 2 master student,
- 1 phd student,

3.3.7 Iosif Krasilshchik

[1] Nonlocal geometry of PDEs. Independent University of Moscow, 5 year students, postdocs, postgraduates, September-December 2020, 2 hours per week.

Program

1. Introduction: infinite jets and infinite prolongations. The Cartan distribution. Symmetries. Cosymmetries and conservation laws.

2. Nonlocal constructions in integrable systems with infinite number of degree of freedom. Examples. Discussion.

3. Differential coverings. Definition, motivating examples. Main types of coverings. Abelian coverings.

4. Nonlocal symmetries. Defining equations. Reconstruction theorem. Examples of computations.

5. Bäcklund transformations. Examples.

6. The tangent covering. Definition and basic properties.

7. Nonlocal recursion operators for symmetries and variational symplectic structures.

8. The cotangent covering. Definition and basic properties.

9. Nonlocal recursion operators for cosymmetries and variational Poisson structures.

References

Bocharov, A.V.; Chetverikov, V.N.; Duzhin, S.V.; Khor'kova, N.G., Krasil'shchik, I.S., Samokhin, A.V., Torkhov, Yu.N., Verbovetsky, A.M., Vinogradov, A.M., Symmetries of Differential Equations in Mathematical Physics and Natural Sciences; Vinogradov, A.M., Krasil'shchik, I.S., Eds.; Factorial Publ. House: Moscow, 1997; (In Russian)

Krasil'shchik, I.S., Verbovetsky, A.M., Vitolo, R., The Symbolic Computation of Integrability Structures for Partial Differential Equations; Texts & Monographs in Symbolic Computation; Springer: Berlin, 2017.

Krasil'shchik, I.S., Verbovetsky, A.M., Geometry of jet spaces and integrable systems. J. Geom. Phys. 61:1633-1674, 2011, DOI: 10.1016/j.geomphys.2010.10.012. arXiv:1002.0077 [math.DG]

3.3.8 Konstantin Loginov

[1] Topology II. Math in Moscow, Independent University of Moscow, lectures and problem sessions, fall 2020, 3 hours per week.

1. Homology functors. Brouwer's fixed point theorem.

2. CW-complexes. Examples. Cellular approximation theorem. Fiber bundles.

3. Homotopy groups. Exact homotopy sequence for fiber bundles. The Hopf bundle.

4. Simplicial homology. Chain complexes, cycles, boundaries.

5. Properties of simplicial homology. Relative homology groups. The exact sequence of a pair of topological spaces.

6. Cellular homology. Computing the homology groups of some topological spaces. Mayer-Vietoris exact sequence.

7. Singular homology. Steenrod-Eilenberg axioms.

8. Applications of homology. Connectedness, orientability, Euler characteristic. Lefschetz fixed point theorem.

9. Cohomology. Multiplication in cohomology. Steenrod-Eilenberg axioms for cohomology.

10. Poincaré duality. The cap product

11. Obstruction theory. The Eilenberg-Mac Lane spaces.

12. Vector bundles and G-bundles. Milnor's construction.

[2] Topology 1, Math in Moscow, Independent University of Moscow, lectures and problem sessions, spring 2020, 3 hours per week.

1. Continuity, homeomorphism, compactness for subsets of $\mathbb{R}^n$.

2. Topological and metric spaces, cell spaces, manifolds.

3. Topological constructions (product, disjoint union, wedge, cone, suspension, quotient spaces, cell spaces, examples of fiber bundles).

4. Examples of surfaces (2-manifolds), orientability, Euler characteristic.

5. Classification of surfaces (geometric proof for triangulated surfaces).

6. Homotopy and homotopy equivalence, the homotopy groups and their main properties.

7. Vector fields on the plane. Generic singular points. The index of a plane vector field. Vector fields on surfaces. The Poincaré index theorem.

8. Curves in the plane, degree of a point with respect to a curve, Whitney index, the "fundamental theorem of algebra".

9. Degree of a map of a circle into itself. Brouwer fixed point theorem.

10. Fundamental group, covering spaces. Algebraic classification of covering spaces (via subgroups of the fundamental group of the base).

[3] Seminars, Algebra I, National Research University Higher School of Economics, I year students, 1.5 hours per week.

[4] Introduction to Algebraic Geometry and Commutative Algebra, Moscow Institute of Physics and Technology, fall 2020, 1.5 hours per week.

1. Affine space, algebraic subsets, Zariski topology, Noetherian rings, Hilbert's basis theorem.

2. Regular functions, affine and quasi-affine varieties, isomorphism of varieties, pullback on regular functions.

3. Hilbert's Nullstellensatz.

4. Categories and functors. Ideal - subvarieties correspondence. Radical of an ideal. Equivalence of categories of affine algebras and affine varieties.

5. Projective and quasi-projective varieties. Image of a projective variety is closed.

6. Rational maps and a field of rational functions. Rationality.

3.3.9 Taras Panov

[1] Toric Geometry and Topology, Independent University of Moscow, advanced course, September-December 2020, 2 hours per week.

Program

1. The classical construction of toric varieties from rational fans.
2. Projective toric varieties and polytopes.
3. Cohomology of toric manifolds.
4. The algebraic quotient construction.
5. Hamiltonian torus actions and symplectic reduction.
6. Moment-angle manifolds as levels sets for moment maps and intersections of Hermitian quadrics.
7. The Atiyah–Guillemin–Sternberg convexity theorem and the Delzant theorem on Hamiltonian toric manifolds.

[2] Duality in Algebraic Topology. Moscow State University, advanced course, September-December 2020, 2 hours per week.

Program.

1. Combinatorial Poincaré duality.
2. Simplicial Alexander duality.
3. Poincaré duality for topological manifolds.
4. Alexander duality.
5. Lefschetz duality.

6. Atiyah duality.
7. Eckmann–Hilton duality and basics of the model category theory.

3.3.10 Alexei Penskoi

[1] Differential Geometry. Independent University of Moscow, II year students, February-May 2020, 2 hours per week (lecture 2 hours).

Program

1. Curves and surfaces in the plane and the three-dimensional space. Curvature, torsion, Frenet frame. First and second fundamental forms. Principal curvatures, mean curvature and Gauß curvature. Mean curvature normal vector. Euler formula for the normal section curvature.

2. Surfaces in n-dimensional space. First and second fundamental forms. Connections in the tangent and normals bundles on a surface. Second fundamental form and Weingarten operator. Gauß-Weingarten derivational equations. Gauß-Bonnet theorem for surfaces.

3. Basic theory of Lie groups ans algebras.

4. Vector bundles and gluing cocycles. Structure group. Euclidean and hermitian bundles. Natural operations with bundles. Orientable bundles.

5. Connections in vector bundles. Connection local form, Christoffel symbols. Connections in euclidean and hermitian bundles. Connections compatible with metrics and their curvature.

6. Riemannian manifolds. Curvature, torsion. Levi-Civita connection. Symmetries of curvature tensor. Ricci tensor. Scalar curvature.

7. Riemannian manifolds II. Geodesics. Geodesic coordinates. Lagrangian approach to geodesics. Second variation.

8. Submanifolds of Riemannian manifolds. First and second fundamental forms.

9. Laplace-Beltrami operator and minimal submanifolds, Takahashi theorem.

10. Characteristic classes. Chern-Weil construction of characteristic classes. Chern, Pontryagin and Euler classes and their properties.

[2] Calculus on manifolds. "Math in Moscow" program at the Independent University of Moscow & NRU HSE for undergraduate students from the U.S. and Canada, February-May 2020, 4 hours per week (lecture 2 hours + exersise class 2 hours).

Program

1. Definition and examples of smooth manifolds.

2. Orientability and orientation.

3. Tangent vectors and tangent space to a manifold at a point. Tangent bundles. Vector fields.

4. Skew-symmetric forms on linear spaces. Wedge product.

5. Differential forms on manifolds. Exterior differential.

6. Smooth maps of manifolds. Diffeomorphisms. The transformation rule under coordinate change for functions, vector fields and differential forms.

7. Integration. Coordinate change in the integral. Integration of differential forms. Stokes theorem. Green's formula, Gauss-Ostrogradskii divergence theorem, Stokes formula for a surface in $\mathbb{R}^3$.

8. Closed and exact forms. The Poincare lemma. De Rham cohomology.

[3] Differential geometry and topology, Moscow State University, 3 year students, September-December 2020, 4 hours per week (lecture 2 hours + exersise class 4 hours).

Program.

1. Reminiscences from Calculus: implicit function theorem, inverse function theorem, rank theorem. Surfaces in affine spaces and different ways of their definition.

2. Smooth manifolds. Partition of unity. Maps of manifolds.

3. Tangent vectors and differential of a map. Tangent and cotangent spaces.

4. Immersions, submanifolds, submersions.

5. Vector fields. Commutator of vector fields. Integral curves of a vector filed. One-parametric group generated by a vector field.

6. Tensor fields, differential forms. Riemann metric, volume form. Exterior differential.

7. Relation between d and grad, rot and div.

8. Orientation of a manifold. Integration of forms over manifolds.

9. Manifolds with boundary. Stokes theorem for manifolds with boundary. Relation to Green, Stokes and Gauß-Ostrogradsky formulas in calculus.

10. Levi-Civita connection.

11. Curvature operator, curvature tensor, Ricci tensor, scalar curvature
12. Parallel transport.
13. Geodesics, exponential map.
14. Whitehead Theorem.

[4] Analysis on Manifolds, Independent University of Moscow, 2 year students, September-December 2020, 2 hours per week (lecture 2 hours).

Program.

1. Reminiscences from Calculus: implicit function theorem, inverse function theorem, rank theorem. Surfaces in affine spaces and different ways of their definition.
2. Curvature, torsion, Frenet frame. First and second fundamental forms. Principal curvatures, mean curvature and Gauß curvature. Mean curvature normal vector. Euler formula for the normal section curvature.
3. Surfaces in n-dimensional space. First and second fundamental forms. Connections in the tangent and normals bundles on a surface. Second fundamental form and Weingarten operator. Gauß-Weingarten derivational equations. Gauß-Bonnet theorem for surfaces.
4. Smooth manifolds. Partition of unity. Maps of manifolds.
5. Tangent vectors and differential of a map. Tangent and cotangent spaces.
6. Immersions, submanifolds, submersions.
7. Vector fields. Commutator of vector fields. Integral curves of a vector filed. One-parametric group generated by a vector field.
8. Tensor fields, differential forms. Riemann metric, volume form. Exterior differential.
9. Lie derivative. Cartan identity. Hodge operation. Relation between d and grad, rot and div.
10. Distributions and Frobenius theorem.

[5] Exercise classes for Topology-I at the National Research University — Higher School of Economics: January-March 2020, 2 hours per week (2 hours exercise classes).

[6] Analytic geometry, Moscow State University, Ist year students, September-December 2020, 4 hours per week (4 hours exercise classes).

3.3.11 Petr Pushkar

[1] Complex Analysis. Independent University of Moscow, 2 year students, January-May 2020, 4 hours per week.

Program

1. Complex-valued functions. Holomorphic functions. CauchyRiemann equations.
2. Holomorphic forms. Cauchy Theorem. Expansion as a convergent power series.
3. Meromorphic functions. Loran series.
4. Maximum principle, Cauchy's argument principle, open mappings.
5. Residues.
6. Isolated singlar points. The CasoratiWeierstrass theorem
7. Schvarz lemma. Automorphisms.
8. Uniformization theorem.
9. Holomorphic and harmonic functions.
10. Riemmanian surfaces. Elements of elliptic functions theory. Abel theorem.

[2] Ordinary Differential Equations, 2 year students, September-December 2020, 4 hours per week.

Program

1. Tangent vectors, Vector fields
2. Main theorems
3. Linear systems and equations, exponential function, Quasi-polynomials
4. Linearization and Lyapunov stability
5. Obstruction to integrability of distributions
6. First-order differential equations
7. Symplectic and Contact structures and differential equations
8. Examples.

[3] Calculus and Algebraic topology classes at HSE. Exercises.

3.3.12 George Shabat

[1] Intersections in the Moduli Spaces of Curves. Independent University of Moscow, 3-5 year students, February-December 2020, 2 hours per week.

Program.

1. Moduli spaces of curves $\mathcal{M}_g(\Bbbk)$

1.0. Naive definition and cases of small genera g

1.1. Functor of families and stacks $\mathfrak{M}_g(\Bbbk)$

1.2. Rigidification and spaces $\mathcal{M}_g^{\Gamma}(\Bbbk)$

1.3. Transcendent definition of the spaces $\mathcal{M}_g(\mathbb{C})$

1.4. Deligne-Mumford compactification $\overline{\mathcal{M}_g}(\Bbbk)$

2. The spaces $\overline{\mathcal{M}_{g,N}}(\mathbb{C})$ and their combinatorial models

2.0. Stable curves and marked points on them
2.1. Metrized ribbon graphs
2.2. Strebel-Penner-... theory
2.3. Combinatorial interpretation of fibrations Li and their Chern classes
2.4. Volumes of moduli spaces and their Laplace transforms
2.5. Discrete version of the theory and arithmetic
3. Kontsevich's proof of the Witten conjecture
3.0. Witten's generating function for the intersection numbers
3.1. Matrix intergrals
3.2. The main Kontsevich's combinatorial identity
3.3. Matrix Airy function and its asymptotic behavior
3.4. Applications, generalizations and new ideas
4. Other proofs of the Witten conjecture
4.0. Algebro-geometric proof by Kazarian-Lando
4.1. On the Mirzahani theory
4.2. On the Okun'kov-Pandaripande and Kim-Lu proofs
4.3. Overview of the constructions used

[2] Advanced Algebra, Independent Univeristy of Moscow, Math in Moscow, I year students, February - May 2020, 3 hours per week.

Program.

1. Basic group theory. Cosets, quotients, normal subgroups.
2. Existence of the elements of prime order.
3. Actions of the finite groups on sets. Orbits, stabilizers.
4. Actions of p-groups on finite sets and the number of fixed points.
5. Sylow's theorems. Existence, conjugacy, number of Sylow subgroups.
6. Simple groups. Solvable groups. Nilpotent groups.
7. Algebraic extensions of fields.
8. Separable field extensions. Normal extensions.
9. Galois extensions. Galois correspondence.
10. Solvability of roots of the polynomials in radicals.
11. Complements to the course on basic representation theory.
12. Irreducible representations. Schur's lemma.
13. Semisimple algebras and modules.
14. Group algebra. The Maschke theorem.
15. Application of the theory of characters to the structure of finite groups. Solvability of the group of order $p^n q^m$.

3.3.13 Stanislav Shaposhnikov

1) Mathematical calculus. Independent University of Moscow, 1 year students, September – December 2020, 4 hours per week.

Program:

1. Sets. Functions. Mathematical induction. Axiom of choice and Zorns lemma.
2. Real numbers: an axiomatic approach. Complex numbers.
3. Sequences and series. The Cauchy sequences.
4. Real numbers are the completion of the rational numbers. p-adic numbers.
5. Topology and metric spaces.
6. Complete metric spaces. Baire category theorem.
7. Compact sets. Hausdorff criteria.
8. Continuous functions. Fixed-point theorems.
9. Differentiable functions. Taylor series.

3.3.14 George Sharygin

[1] Differential geometry. Moscow State University, February-May 2020, 1-st year master students.

Program

1. Theory of curves in $\mathbb{R}^2$ and $\mathbb{R}^3$: curvature, torsion, various formulas for them, Frenet's formulas etc.

2. Theory of surfaces in $\mathbb{R}^3$: the 1st and the 2nd quadratic forms, Menier theorem, normal sections, Euler's formula, Gauss and mean curvature, etc.

3. Gauss-Weingarten equations, Gauss-Petersen-Codazzi-Mainardi equations, Teorema egregium, non-isometric surfaces.

4. Covariant derivatives, parallel transport and geodesic lines, geodesic curvature.

5. Gauss-Bonnet's theorem: polyhedral and traditional.

[2] Modern differential geometry and topology (in English). Moscow State University, 5 year students, February-May 2020, 2 hours per week.

Program.

1. Polyhedra and the discrete Gauss-Bonnet theorem (with and without boundary).

2. Gauss-Bonnet theorem for surfaces in $\mathbb{R}^3$ (with and without boundary).

3. Mapping degree and the index of a singular point of a vector field.

4. Poincré-Hopf theorem.
5. Morse theorem on the index of a vector field on a bounded region in $\mathbb{R}^n$.
6. Gauss-Bonnet theorem for hypersurfaces in $\mathbb{R}^n$
7. Connections on vector bundles over smooth manifolds.
8. Chern-Weil theory of characteristic classes.
9. Gauss-Bonnet-Chern theorem for manifolds.

[3] Deformation quantization, Independent University in Moscow, February-May 2020 (5 years students and post-graduate students)

Program

1. Introduction: pseudodifferential operators, Weyl's quantization formula, Moyal product.
2. Poisson structures on manifolds, Darboux theorem, Lichnerowicz's and Brylinski's Poisson homology and cohomology.
3. The deformation problem. Example: Baker-Campbell-Hausdorff formula and deformation of the product on coadjoint representation space.
4. Hochschild homology and cohomology; Hochschild-Kostant-Rosenberg theorem.
5. Gutt, Lecomte and De Wilde construction: deformation quantization of symplectic manifold.
6. Fedosov quantization. Its complex and algebraic versions. Nest-Tsygan classes.
7. Applications: algebraic index theorem.
8. Higher homotopy structures on algebras. A_∞ and L_∞ algebras. The existence of inverse maps.
9. Kontsevich's theorem for $\mathbb{R}^n$. The first proof (by Kontsevich). Globalization.
10. Algebraic operads and algebras over them. Koszul duality for operads. Examples.
11. Tamarkin's proof of Kontsevich's theorem.
12. Applications: Duflo's isomorphism.

[4] Linear algebra and Geometry. Moscow State University, September-December 2020, 1-st year master students.

Program

1. Linear spaces, basises, coordinates, matrices.
2. Linear operators, eigenvalues, diagonalization and Jordan's normal form, Hamilton-Cayley formula.
3. Bilinear forms, index and rank of a form, Lagrange's method, Gram-Schmidt process, Sylvester criterion.
4. Gram determinants and volumes, Cayley-Menger determinant, self-adjoint and unitary operators, normalization of a pair of hermitian matrices.
5. Elements of tensor theory.

[5] Differential geometry and index theorems. Independent University in Moscow, September-December 2020 (4-5 year students, master students)

Program

1. Clifford algebras and their representations.
2. $Spin_n$ and $Spin_n^{\mathbb{C}}$ groups.
3. Atiyah-Bott-Shapiro theorem.
4. Spin-structures on manifolds.
5. Dirac operator. Index of a (Clifford-linear) Dirac operator.
6. Bochner-type formulas and applications.
7. Differential and pseudo-differential operators. Elliptic operators.
8. Index of an elliptic operator, its properties.
9. Heat kernel and its properties.
10. Local index theorems.

3.3.15 Arkady Skopenkov

[1] Linear algebraic method in topology: homology theory, III year students, September-December, 4 hours per week. Moscow Institute of Physics and Technology (DGAP)

Program. It is shown how in the course of solution of interesting geometric problems (close to discrete mathematics and computer science) naturally appear main notions of algebraic topology (homology groups, obstructions and invariants). Thus main ideas of algebraic topology are presented with minimal technicalities.

Detailed information in Russian: http://www.mccme.ru/circles/oim/home/combtop13.htm#combtop14

[2] Homotopical topology from algorithmic point of view, Moscow Institute of Physics and Technology (DGAP) and Independent University of Moscow, February-May, 2 hours per week.

Program. Topological concepts are exposed in the way interesting and accessible to non-specialists, in particular, to computer science students.

Detailed program in Russian:

https://www.mccme.ru/circles/oim/home/combtop13.htm#superfluid

[3] Algorithms for recognition of realizability of graphs and hypergraphs, Independent University of Moscow, September-December, 2 hours per week.

Program. The main content is exposition of some deep ideas of algebraic topology motivated by algorithmic problems on realizability of graphs and hypergraphs in Euclidean space.

Detailed program in Russian: http://www.mccme.ru/circles/oim/home/combtop13.htm#nmuspr15

[4] Discrete analysis (exercises), II year students, February-December, 2 hours per week. Moscow Institute of Physics and Technology (DIHT)

Program. We study certain topics in combinatorics and graph theory (including random graphs).

Detailed information in Russian:

http://www.mccme.ru/circles/oim/home/discran1314.htm

Other educational activities by A. Skopenkov in 2020

[1] International Summer Conference of Tournament of Towns, Jury member, June-August. Detailed information:

http://www.turgor.ru/en/lktg/index.php

[2] Moscow Mathematical Conference of High-school Students, Programme Committee member, September-December, Moscow. Detailed information in Russian: http://www.mccme.ru/mmks/index.htm

[3] An optional course on mathematics for high-school students, high-school 'Intellectual', January-October. Detailed information in Russian:

http://www.mccme.ru/circles/oim/index.htm#il

[4] Math circle 'Olympiads and Mathematics' for high-school students, MCCME, January-October. Detailed information in Russian:

http://www.mccme.ru/circles/oim/index.htm#oim

[5] Minicources on mathematics for high-school students, Moscow 'olympic' schools, May.

3.3.16 Evgeni Smirnov

[1] Combinatorics, 1st year, 2nd semester, February–May 2020, Independent University of Moscow, 2 hours of lectures and 2 hours of exercise sessions per week

Course outline:

1. Generating functions. Rational generating functions, linear recurrencies.
2. Catalan, Schroeder and Motzkin numbers.
3. q-binomial coefficients.
4. Euler's generating function for Young diagrams. Euler's pentagonal theorem and the triple Jacobi identity.
5. Dirichlet generating functions. Lagrange inversion.
6. Bernoulli-Euler triangle and power sums.
7. Partially ordered sets. Moebius function, Moebius inversion.
8. Lindstroem–Gessel–Viennot theorem. Determinant as the sum over nonintersecting families of paths.
9. Matrix identities: Binet–Cauchy formulas and the Lewis Carroll identity.
10. Matrix tree theorem. Kirchhoff's theorem.

[2] Reflection groups, Independent University of Moscow, September–December 2020, 2 hours of lectures and 2 hours of exercises (jointly with Nikolay Bogachev) per week

Course outline:

1. Finite Reflection Groups
2. Root systems
3. Generators and Relations
4. Classification of finite reflection groups
5. Crystallographic reflection groups
6. Classification of regular polytopes
7. Polynomial invariants
8. Chevalley–Shephard–Todd theorem

3.3.17 Alexei Sossinsky

[1] Knot Theory, Math in Moscow students, 3 hours per week, Fall 2020

Program:

1. Oriented knots and links: formal definition as closed polygonal curves, equivalence (= ambient isotopy), knot diagrams, Reidemeister moves, Conway axioms for the Alexander polynomial (without proof of existence), examples of computations.

2. Boxed knots, connected sum (=composition) of knots, the knot semigroup (commutativity and absence of inverse elements), prime knots, unique prime decomposition theorem (without proof).

3. Some simple knot invariants: stick number, crossing number, and unknotting number, tricolorability, Seifert surfaces and the genus of knots.

4. Kauffman bracket of nonoriented knots and links, its properties and its behavior w.r.t. Reidemeister moves.

5. Definition of the Jones polynomial via the Kauffman bracket, main properties, computation via that definition.

6. Axioms for the Jones polynomial, uniqueness, computation via the axioms, knot tables.

7. The braid group: geometric definition, braid generators, pure braids. Group presentations. Artin's theorem on the algebraic presentation of the braid group.

8. Properties of braids, braid ordering, decidability of the word and conjugacy problem for braids.

9. Closure of a braid, Alexande's theorem (any knot is the closure of some braid), Markov moves and Markov's theorem (without full proof).

10. Examples of finite type invariants. Thom-Arnold theory of discriminants, singular knots, the four-term relation, axioms for Vassiliev invariants, examples of computations.

11. The Kontsevich integral and some ideas underlying the proof of the existence of Vassiliev invariants.

[2] Topology-2, IUM second year students, 3 hours per week, fall 2020

Program:

1. Homology functors

2. CW-complexes

3. Homotopy groups

4. Cellular homology

5. Simplicial homology

6. Properties of simplicial homology

7. Singular homology

8. Applications of homology

9. Cohomology

10. Poincare duality

11. Obstruction theory

12. Vector bundles and G-bundles

3.3.18 Vladimir Zhgoon

[1] Algebraic Geometry: Start Up Course. Independent University of Moscow, Math in Moscow, September - December 2020, 3 hours per weeks

1) Projective spaces. Geometry of projective quadrics. Spaces of quadrics.
2) Lines, conics. Rational curves and Veronese curves. Plane cubic curves. Additive law on the points of cubic curve.
3) Grassmannians, Veronese's, and Segre's varieties. Examples of projective maps coming from tensor algebra.
4) Integer elements in ring extensions, finitely generated algebras over a field, transcendence generators, Hilbert's theorems on basis and on the set of zeros.
5) Affine Algebraic Geometry from the viewpoint of Commutative Algebra. Maximal spectrum, pullback morphisms, Zariski topology, geometry of ring homomorphisms.
6) Finite morphisms. Projections.
7) Algebraic varieties, separateness.
8) Projective varieties, properness. Rational functions and maps.
9) Dimension. Dimensions of subvarieties and fibers of regular maps. Dimensions of projective varieties.
10) Linear spaces on quadrics. Lines on cubic surface. Chow varieties.

TEXTBOOKS
1) A.L.Gorodentsev, Algebra II. Textbook for Students of Mathematics. Springer, Ch. 1, 2, 10, 11, 12 2) A.L.Gorodentsev, Algebraic Geometry Start Up Course, MCCME. 3) J.Harris, Algebraic Geometry. A First Course, Springer. 4) D.Mumford, Red book of varieties and schemes, Springer LNM 1358.

[2] Introduction to algebraic number theory. Higher school of economics, September-December 2020, 2 hours per week.

1) Galois theory. Finite fileds.
2) Fields of algebraic numbers. Dedekind domains. Norm and trace.
3) Ramification theory. Different and discriminant.
4) Dirichlet unit theorem. Class group. Minkowskiy constant.
5) Adels and Idels.
6) Galois cohomology.
7) Class field theory.

[3] Introduction to algebraic groups and invariant theory. Higher school of economics, Spring 2020, 2 hours per week.

1) Algebraic groups and Lie algebras. Actions on algebraic varieties.
2) Orbits, stabilizers, homogenious spaces. Chevalley embedding theorem.

3) Jordan decomposition. Solvable, unipotent and nilpotent subgroups.
4) Borel and Cartan subroups. Maximal tori.
5) Borel fixed point theorem. Flag varieties.
6) Reductive groups. Root systems.
7) Actions of reductive groups on affine varieties. Categorical quotient.
8) Actions of unipotent groups on affine varieties.
9) Linearization of a line bundle.

[4] Algebraic Geometry: A First Geometric Look. Higher school of economics, Spring 2020, 2 hours per week.

- Projective spaces. Geometry of projective quadrics. Spaces of quadrics.
- Lines, conics. Rational curves and Veronese curves. Plane cubic curves. Additive law on the points of cubic curve.
- Grassmannians, Veronese's, and Segre's varieties. Examples of projective maps coming from tensor algebra.
- Integer elements in ring extensions, finitely generated algebras over a field, transcendence generators, Hilbert's theorems on basis and on the set of zeros.
- Affine Algebraic Geometry from the viewpoint of Commutative Algebra. Maximal spectrum, pullback morphisms, Zariski topology, geometry of ring homomorphisms.
- Agebraic manifolds, separateness. Irreducible decomposition. Projective manifolds, properness. Rational functions and maps.
- Dimension. Dimensions of subvarieties and fibers of regular maps. Dimensions of projective varieties.
- Linear spaces on quadrics. Lines on cubic surface. Chow varieties.
- Vector bundles and their sheaves of sections. Vector bundles on the projective line. Linear systems, invertible sheaves, and divisors. The Picard group.
- Tangent and normal spaces and cones, smoothness, blowup. The Euler exact sequence on a projective space and Grassmannian.

Yulij Ilyashenko
President of the Independent University of Moscow