The IUM report to the Simons foundation, 2021

Contents

1 Introduction

The Simons foundation supported two programs launched by the IUM:
Simons stipends for students and graduate students;
Simons IUM fellowships.

12 applications were received for the Simons stipends contest. The selection committee consisting of *Yu.Ilyashenko (Chair), G.Dobrushina, G.Kabatyanski, S.Lando, I.Paramonova (Academic Secretary), A.Sossinsky, M.Tsfasman* awarded Simons stipends for 2021 year to the following students and graduate students:

1. Abasheva, Anna Eduardovna
2. Abramyan, Semyon Arturovich
3. Adler, Dmitrii Vsevolodovich
4. Demidovich, Yurii Alexandrovich
5. Konovalov, Andrei Anatolyevich
6. Osipov, Pavel Sergeevich
7. Safonkin, Nikita Alexandrovich
8. Spiridonov, Igor Alexandrovich

16 applications were received for the Simons IUM fellowships contest for the first half year of 2021 and 21 applications were received for the second half year. The selection committee consisting of *Yu.Ilyashenko (Chair), G.Dobrushina, B.Feigin, I.Paramonova (Academic Secretary), A.Sossinsky, M.Tsfasman, V.Vassiliev* awarded

Simons IUM-fellowships for the first half year of 2021 to the following researches:

1. Burman, Yurii Mikhailovich
2. Gorodentsev, Alexei Lvovich
3. Kalmynin, Alexander Borisovich
4. Loginov, Konstantin Valeryevich
5. Lvovski, Sergei Mikhailovich
6. Medvedev, Vladimir Olegovich

7. Penskoi, Alexei Victorovich
8. Pushkar, Petr Evgenyevich
9. Shaposhnikov, Stanislav Valeryevich
10. Smirnov, Evgeni Yuryevich
11. Sossinsky, Alexei Bronislavovich
12. Zhgoon, Vladimir Sergeevich

Simons IUM-fellowships for the second half year of 2021 to the following researches:

1. Bogachev, Vladimir Igorevich
2. Burman, Yurii Mikhailovich
3. Elagin, Alexei Dmitrievich
4. Fonarev, Anton Vyacheslavovich
5. Gorodentsev, Alexei Lvovich
6. Kalmynin, Alexander Borisovich
7. Panov, Taras Evgenyevich
8. Pushkar, Petr Evgenyevich
9. Samokhin, Alexander Valeryevich
10. Shabat, George Borisovich
11. Shaposhnikov, Stanislav Valeryevich
12. Sossinsky, Alexei Bronislavovich
13. Timorin, Vladlen Anatolyevich

The report below is split in two sections corresponding to the two programs above. The first subsection in each section is a report on the research activities. It consists of the titles of the papers published or submitted in the year of 2021, together with the corresponding abstracts. The second subsection of each section is devoted to conferences and some most important seminar talks. The last subsection of the second section is devoted to the syllabi of the courses given by the winners of the Simons IUM fellowships. Most of these courses are innovative, as required by the rules of the contest for the Simons IUM fellowships.

Due to the COVID-19 pandemic, the second subsection (conferences) is smaller than in the previous reports. If an awarded person attended no conference at all, the corresponding subsection is void.

The Independent university of Moscow remains one of the crystallization points of Moscow mathematical life. This fall it celebrates 30 years of its activity. Let me briefly summarize it. In the hard time of 90's when the Steklov Institute and Lomonosov State University suffered a lot because of the brain drain, the IUM was growing up, accepting to its faculty young mathematicians that came back to Russia from the West. Among them were M.Finkelberg, D.Kaledin, M.Verbitsky. They are now actively working mathematicians in Russia (Verbitsky partly works in IMPA, Brasil). All three became later invited speakers of the International Congress of Mathematicians. In the 90's IUM was the most attractive place for capable young people in Moscow where they got top level mathematical education. Later on, the skills obtained by the faculty of the IUM were applied in the organization of the new mathematical department "Faculty of Mathematics" of the Higher School of Economics, launched in 2008. The logistics and funding came from HSE; the mathematics from IUM. Now the Department of Mathematics is the best in the ARWU ranking, which takes into account the number of papers published in leading journals, amidst Russian mathematical departments.

Two invited speakers of the forthcoming ICM-2022, E.Feigin and F.Efimov, are alumni of IUM. A plenary speaker of the same Congress A.Kuznetsov is an alumnus of the graduate school of the IUM, as well, as the Fields medalist of the ICM-2006 A.Okounkov.

The IUM launches several contests for young mathematicians whose winners are awarded with stipends for several years. These are August Möbius contest since 1997, for students and graduate students; Young Mathematics of Russia contest aimed mainly at recent PhD holders, started as the Pierre Deligne contest in 2005 and largely extended since then. The winners of these contests are of very high level. The contests help them to stay and to continue their mathematical careers in Russia.

The IUM also launches two contests "Leader" and "Junior Leader" designed to support active mathematicians of all ages with a small team of their followers.

The IUM launches a Study Abroad program called "Math in Moscow", mainly aimed at American and Canadian students. The program brings together Russian and American mathematical cultures, and is highly estimated by its participants.

In the reality of growing political pressure, IUM remains an oasis of freedom. It is open to everybody who wants to come and attend the lectures, and to everyone who wants and is capable to teach a lecture course of the high enough level.

The IUM is addressed to a small number of students, about 50 in total, but it gives the highest level education in contrast to those universities that admit hundreds of mathematical students.

The life in Moscow becomes more and more expensive and the support of the Simons foundation is crucial for the very survival of IUM. During the previous years, it affected positively not only the financial situation, but also the whole atmosphere at IUM making it fruitful and attractive. On behalf of all the faculty and students, I send our deep gratitude and the best Christmas and New Year wishes of the IUM to Jim Simons, David Spergel, Yuri Tschinkel, and the whole team of the Simons foundation.

Yulij Ilyashenko

President of the Independent University of Moscow

2 Program: Simons stipends for students and graduate students

2.1 Research

2.1.1 Anna Abasheva

[1] Feix–Kaledin metric on the total spaces of cotangent bundles to Kähler quotients
Int. Math. Res. Not., rnab047, May 2021, arXiv: 2007.05773

In this paper, we study the geometry of the total space Y of a cotangent bundle to a Khler manifold N where N is obtained as a Khler reduction from C^n. Using the hyperkhler reduction, we construct a hyperkhler metric on Y and prove that it coincides with the canonical FeixKaledin metric. This metric is in general non-complete. We show that the metric completion $\widetilde{Y}$ of the space Y is equipped with a structure of a stratified hyperkhler space (in the sense of [31]). We give a necessary condition for the FeixKaledin metric to be complete using an observation of R. Bielawski. Pick a complex structure J on $\widetilde{Y}$ induced from the quaternions. Suppose that $J \neq \pm I$ where I is the complex structure whose restriction to $Y = T^*N$ is induced by the complex structure on N. We prove that the space $\widetilde{Y}_J$ admits an algebraic structure and is an affine variety.

[2] With M. Verbitsky
Algebraic dimension and complex subvarieties of hypercomplex nilmanifolds
Awaiting response from Adv. Math., arXiv: 2103.05528

A nilmanifold is a (left) quotient of a nilpotent Lie group by a cocompact lattice. A hypercomplex structure on a manifold is a triple of complex structure operators satisfying the quaternionic relations. A hypercomplex nilmanifold is a compact quotient of a nilpotent Lie group equipped with a left-invariant hypercomplex structure. Such a manifold admits a whole 2-dimensional sphere S^2 of complex structures induced by quaternions. We prove that for any hypercomplex nilmanifold M and a generic complex structure $L \in S^2$, the complex manifold (M, L) has algebraic dimension 0. A stronger result is proven when the hypercomplex nilmanifold is abelian. Consider the Lie algebra of left-invariant vector fields of Hodge type (1,0) on the corresponding nilpotent Lie group with respect to some complex structure $I \in S^2$. A hypercomplex nilmanifold is called abelian when this Lie algebra is abelian. We prove that all complex subvarieties of (M, L) for generic $L \in S^2$ on a hypercomplex abelian nilmanifold are also hypercomplex nilmanifolds.

[3] With V. Rogov
Shafarevich–Tate groups of holomorphic Lagrangian fibrations
In preparation

We define the Shafarevich–Tate group of a Lagrangian fibration on a hyperkähler manifold. This notion generalizes earlier constructions of Shafarevich–Tate groups for elliptic fibrations and certain Lagrangian fibrations on manifolds of $K3^{[n]}$-type. We use it to get a new interpretation of degenerate twistor deformations of Lagrangian fibrations and prove that under some generality assumptions all members of a degenerate twistor family are Kähler. We also get a characterization of algebraic members of degenerate twistor families in terms of the Shafarevich–Tate group. Finally, we apply the technique of Shafarevich–Tate groups to obtain new topological obstructions for the existence of sections on Lagrangian fibrations.

2.1.2 Semyon Abramyan

On homology of the MSU spectrum
avilable at arXiv:2108.12713

In this paper we compute the structure of $H_*(MSU; \mathbb{F}_p)$ over the dual Steenrod algebra $\mathfrak{A}_p^*$, p is an odd prime. We give a complete proof of the isomorphism $\Omega^{SU} \otimes \mathbb{Z}[\frac{1}{2}][y_2, y_3, \ldots]$, $\deg y_n = 2n$ using the original methods of the Adams spectral sequence. We also describe the Hurewicz map and the divisiblity of characteristic numbers of SU-manifolds.

2.1.3 Dmitrii Adler

With V. A. Gritsenko
Weak Jacobi forms for the D_n-type root systems and modular differential equations
Working title, in preparation.

In this paper we construct the tower of generators of the bigraded polynomial ring of the weak Jacobi modular forms invariant with respect to the Weyl group for root systems of C_n and D_n type. Except explicit construction of generators we provide some differential equations for the generators of index 1. In particular, we obtain an analogue of the Ramanujan's system of differential equations for Eisenstein series..

2.1.4 Yurii Demidovich

[1] With M. Zhukovskii

Cycle Saturation in Random Graphs

Nešetřil J., Perarnau G., Rué J., Serra O. (eds) Extended Abstracts EuroComb 2021. Trends in Mathematics, vol 14. Birkhäuser, Cham., pp 811-816.

For a fixed graph F, the minimum number of edges in an edge-maximal F-free subgraph of G is called the F-saturation number. The asymptotics of the F-saturation number of the Erdös–Rényi random graph $G(n,p)$ for constant $p \in (0,1)$ was established for any complete graph and any star graph. In this paper we obtain the asymptotics of the C_m-saturation number of $G(n,p)$ for $m \geq 5$. Also we prove non-trivial linear (in n) lower bounds and upper bounds for the C_4-saturation number of $G(n,p)$ for some fixed values of p.

[2] With M. Zhukovskii

Cycle saturation in random graphs

arXiv:2109.05758, *submitted to SIAM Journal on Discrete Mathematics.*

For a fixed graph F, the minimum number of edges in an edge-maximal F-free subgraph of G is called the F-saturation number. The asymptotics of the F-saturation number of the binomial random graph $G(n,p)$ for constant $p \in (0,1)$ is known for complete graphs $F = K_m$ and stars $F = K_{1,m}$. This paper is devoted to the case when the pattern graph F is a simple cycle C_m. We prove that, for $m \geq 5$, whp $\mathrm{sat}\left(G\left(n,p\right), C_m\right) = n + \Theta\left(\frac{n}{\ln n}\right)$. Also we find $c = c(p)$ such that whp $\frac{3}{2}n(1+o(1)) \leq \mathrm{sat}\left(G\left(n,p\right), C_4\right) \leq cn(1+o(1))$. In particular, whp $\mathrm{sat}\left(G\left(n,\frac{1}{2}\right), C_4\right) \leq \frac{27}{14}n(1+o(1))$.

[3] with M. Zhukovskii

Tight asymptotics of clique-chromatic numbers of dense random graphs

arXiv:2012.03210, *submitted to Journal of Graph Theory*

The clique chromatic number of a graph is the minimum number of colors required to assign to its vertex set so that no inclusion maximal clique is monochromatic. McDiarmid, Mitsche and Prałat proved that the clique chromatic number of the binomial random graph $G\left(n,\frac{1}{2}\right)$ is at most $\left(\frac{1}{2}+o(1)\right)\log_2 n$ with high probability. Alon and Krivelevich showed that it is greater than $\frac{1}{2000}\log_2 n$ with high probability and suggested that the right constant in front of the logarithm is $\frac{1}{2}$. We prove their conjecture and, beyond that, obtain a tight concentration result: whp $\chi_c\left(G\left(n,1/2\right)\right) = \frac{1}{2}\log_2 n - \Theta\left(\ln\ln n\right)$.

2.1.5 Andrei Konovalov

Nilpotent invariance of semi-topological K-theory of dg-algebras and the lattice conjecture
arXiv:2102.01566

We show existence of a natural rational structure on periodic cyclic homology, conjectured by L. Katzarkov, M. Kontsevich, T. Pantev, for several classes of dg-categories, including proper connective C-dg-algebras and dg-categories of local systems. The main ingredient is derived nilpotent invariance of A. Blanc's semi-topological K-theory, which we establish along the way.

2.1.6 Pavel Osipov

[1] Selfsimilar Hessian manifolds
arXiv:1908.01731 *submitted to Journal of Geometry and Physics*

A selfsimiar manifold is a Riemannian manifold (M, g) endowed with a homothetic vector field ξ. We characterize global selfsimilar manifolds and describe the structure of local selfsimilar manifolds. We prove that any selfsimilar manifold with a potential homothetic vector field is a conical Riemannian manifold or a Eucledean space. A radiant Hessian manifold is selfsimilar Hessian manifold (M, ∇, g, ξ) such that $\nabla\xi = \lambda \mathrm{Id}$. We prove that any selfsimilar Hessian manifold with a potential homothetic vector field is locally isomorphic to a product radiant Hessian manifolds and describe the local structure of radiant selfsimialar Hessian manifolds.

[2] Selfsimilar Hessian and conformally Kähler manifolds
arXiv:2012.03791 *submitted to Differential Geometry and its Applications*

Let (M, ∇, g) be a Hessian manifold. Then the total space of the tangent bundle TM can be endowed with a Kähler structure (I, g^{r}). We say that a homogeneous Hessian manifold is a Hessian manifold (M, ∇, g) endowed with a transitive action of a group G preserving ∇ and g. We construct by a Hessian (special Kähler) structure on a simply connected manifold with a certain condition a Kähler (hyeper-Kähler) structure on the tangent (cotangent) bundle. A selfsimilar Hessian (Kähler) manifold is a Hessian manifold endowed with a homothetic vector field ξ. We construct by a selfsimilar Hessian (Kähler) structure on a simply connected manifold with a certain condition a conformally Kähler (hyeper-Kähler) structure on the tangent (cotangent) bundle.

[3] Statistical Lie algebras of a constant curvature and locally conformally Kähler Lie algebras
arXiv:2112.06686

We show that a statistical manifold manifold of a constant non-zero curvature can be realised as a level line of Hessian potential on a Hessian cone. We construct a Sasakian structure on $TM \times \mathbb{R}$ by a statistical manifold of a constant non-zero curvature on M. By a statistical Lie algebra of a constant non-zero Lie algebra we construct a l.c.K Lie algebra.

2.1.7 Nikita Safonkin

[1] Semifinite harmonic functions on branching graphs
Zapiski Nauchnykh Seminarov POMI, 2021, Vol. 507, pp. 114-139.

In this paper we study semifinite harmonic functions on arbitrary branching graphs. We give a detailed exposition of an algebraic method which allows one to classify semifinite indecomposable harmonic functions on some multiplicative branching graphs. It was suggested by A. Wassermann in terms of operator algebras, but we rephrase, clarify, and simplify the main arguments working only with combinatorial objects. This work was inspired by the theory of traceable factor representations of the infinite symmetric group $S(\infty)$.

[2] Semifinite harmonic functions on the zigzag graph
arXiv:2110.01508

We study semifinite harmonic functions on the zigzag graph, which corresponds to Pieri's rule for the fundamental quasisymmetric functions $\{F_\lambda\}$. The main problem, which we solve here, is to classify the indecomposable semifinite harmonic functions on this graph. We describe the set of classification parameters and an explicit construction that produces a semifinite indecomposable harmonic function out of every point of this set. We also establish a semifinite analog of the Vershik-Kerov ring theorem.

2.1.8 Igor Spiridonov

On the structure of the top homology group of the Johnson kernel
arXiv:2111.10568

The Johnson kernel is the subgroup $\mathcal{K}_g$ of the mapping class group $\mathrm{Mod}(\Sigma_g)$ of a genus g oriented closed surface Σ_g generated by all Dehn twists about separating curves. In this paper we study the structure of the top homology group $\mathrm{H}_{2g-3}(\mathcal{K}_g, \mathbb{Z})$. For any collection of $2g-3$ disjoint separating curves on Σ_g one can construct the corresponding abelian cycle in the group $\mathrm{H}_{2g-3}(\mathcal{K}_g, \mathbb{Z})$; such abelian cycles will be called simplest. In this paper we describe the structure of $\mathbb{Z}[\mathrm{Mod}(\Sigma_g)/\mathcal{K}_g]$-module on the subgroup of $\mathrm{H}_{2g-3}(\mathcal{K}_g, \mathbb{Z})$ generated by all simplest abelian cycles and find all relations between them.

2.2 Scientific conferences and seminar talks

2.2.1 Anna Abasheva

[1] Conference “Junior Global Poisson Workshop II”, Zoom, May, 3 – 6, 2021
Talk “Non-algebraicity of hypercomplex nilmanifolds”

[2] Talk “Non-algebraicity of hypercomplex nilmanifolds” at “Seminar of Laboratory of Algebraic geometry” (Higher School of Economics, Moscow), February, 26, 2021

[3] Talk “From Feix-Kaledin metric on T^*X to algebraic geometry” at “KCL/UCL Junior Geometry Seminar” (Zoom, King’s College London/ University College London), May, 27, 2021

[4] Talk “Smooth and holomorphic sections of elliptic surfaces” at “Geometric structures on manifolds” (Higher School of Economics, Moscow), September, 2, 2021

2.2.2 Semyon Abramyan

[1] International Polyhedral Products Seminar, online, Febrary 18
Talk “Substitution complexes and nonrealizability example”

[2] International conference “Topology of Torus Actions and Related Topics”, Sochi, Russia, Octobver, 25–29
Talk “Whitehead products and substition of simplicial complexes”

[3] Course “Introduction to Cobordism theory”, NRU HSE, Moscow, Russia, Fall 2021, Lecturer

[4] Course “Unstable homotopy theory”, IUM, Moscow, Russia, Fall 2021, Lecturer

[5] Course “Introduction ot Algebraic Topology”, NRU HSE, Moscow, Russia, co-organiser

[6] Course “Topology-1”, MiM, Moscow, Russia, Teacher assistant

2.2.3 Dmitrii Adler

[1] Zykin Memorial Conference, Moscow, 24 June, 2021.
Talk “Jacobi forms and root systems”

[2] Talk “Jacobi forms and their applications” at seminar “Automorphic forms and applications”, Moscow, NRU HSE, Department of Mathematics.

2.2.4 Yurii Demidovich

Conference “EuroComb 2021”, Barcelona, September, 6 – 10
Talk “Cycle saturation in random graphs.”

2.2.5 Andrei Konovalov

[1] Graduate Student Topology and Geometry Conference 2021, Zoom, April 9-11, 2021
Talk "Semi-topological K-theory of dg-algebras and the lattice conjecture"
[2] Young Topologists Meeting 2021, Zoom, July 5-16, 2021
Talk "Semi-topological K-theory of dg-categories and lattice conjecture" (poster session)
[3] Geometry and Homological Mirror Symmetry conference, Sochi, December 11 - 14, 2021
Talk "Topological K-theory of dg-algebras and the lattice conjecture"
[4] "Cdh-methods, Weibel's conjecture, and Vorst's conjecture", 2 talks at the Geometric Structures seminar, Moscow, March 2021
[5] Talk "The Hermitian $\mathcal{Q}$- and ρ-constructions" at the eCHT Foundations of Hermitian K-theory Reading Seminar, Zoom, October 2021

2.2.6 Pavel Osipov

Talk "Selfsimilar Hessian Manifolds" at "Geometric Structures on Manifolds" (High School of Economics), 09.09.2021.

2.2.7 Nikita Safonkin

[1] Visit to Saint Petersburg, December
[2] Lie Algebras and Applications seminar, Moscow (Skoltech, HSE University), March, 2
Talk "Semifinite harmonic functions on the Gnedin-Kingman graph"
[3] Joint seminar of SIMC and HSE by A. I. Bufetov, A. V. Dymov, A. V. Klimenko, M. Mariani and G. I. Olshanskii Representations and Probability
Talks:
"Harmonic functions on the Pascal, Kingman and Young graphs 1", February, 4
"Harmonic functions on the Pascal, Kingman and Young graphs 2", February, 11
"Monomial quasisymmetric functions and the Gnedin-Kingman graph", February, 18
"Harmonic functions on the Gnedin-Kingman graph and Kerov's construction", March, 1
"The zigzag graph and the path space of a branching graph", March, 15
"Harmonic functions on the zigzag graph", March, 22
"Semifinite harmonic functions", March, 29.
[4] Working Seminar on Mathematical Physics of HSE University and Skoltech Center for Advanced Studies
Talk "Macdonald polynomials, Kerov's construction, and Matveev's theorem", February 24.

Talk "Harmonic functions on the Kingman and Young graphs", February 10.

[5] St. Petersburg Seminar on Representation Theory and Dynamical Systems (St. Petersburg Department of Steklov Mathematical Institute of the Russian Academy of Sciences)

Talk "Semifinite harmonic functions on the Young and zigzag graphs 1", December, 8

Talk "Semifinite harmonic functions on the Young and zigzag graphs 2"

2.2.8 Igor Spiridonov

[1] International Hybrid Conference "Geometry and Homological Mirror Symmetry", Sochi, Russia, December 11 - 14

Talk "On the homology of Torelli groups"

[2] International Conference "Algebraic Topology and Applications", Moscow, Russia (online), December 20 21

Talk "On the top homology of the Torelli group and the Johnson kernel"

[3] Joint Seminar on Mathematical Physics of National Research University HSE and Skoltech Center for Advanced Studies, Moscow, Russia, October 6 and 13

Talk "Mapping class groups, Teichmller spaces, and moduli spaces of curves"

3 Program: Simons IUM fellowships

3.1 Research

3.1.1 Vladimir Bogachev

[1] Bogachev V.I., Röckner M., Shaposhnikov S.V. On the Ambrosio–Figalli–Trevisan superposition principle for probability solutions to Fokker–Planck–Kolmogorov equations. Journal of Dynamics and Differential Equations. 2021. V. 33, N 2. P. 715-739.

We prove a generalization of the known result of Trevisan on the Ambrosio–Figalli–Trevisan superposition principle for probability solutions to the Cauchy problem for the Fokker–Planck–Kolmogorov equation, according to which such a solution $\{\mu_t\}$ with initial distribution ν is represented by a probability measure P_ν on the path space such that P_ν solves the corresponding martingale problem and μ_t is the one-dimensional distribution of P_ν at time t.

[2] Bogachev V., Kosov E., Popova S. On distributions of homogeneous and convex functions in Gaussian random variables. Izvestiya Mathematics. 2021. V. 85. N 5. P. 25-57.

New results are obtained on distributions of homogeneous and convex functions in Gaussian random variables, in particular, conditions for the boundedness of distribution densities are given.

[3] Bogachev V.I. On sequential properties of spaces of measures. Mat. Zametki. 2021. V. 110, N 3, 459-464 (Russian).

Topological properties of spaces of measures equipped with the weak topology are studied. In particular, some sequential properties are considered.

[4] Bogachev V., Krasovitskii T., Shaposhnikov S. On uniqueness of probability solutions of the Fokker-Planck-Kolmogorov equation. Sbornik Mathematics. 2021. V. 212, N 6. P. 745-781.

A long-standing open problem of uniqueness of probability solutions to the Fokker-Planck-Kolmogorov equation is solved positively in the one-dimensional case and negative in higher dimensions.

[5] Bogachev V., Krasovitskii T., Shaposhnikov S. On nonuniqueness of probability solutions to the Cauchy problem for the Fokker-Planck-Kolmogorov equation. Doklady Mathematics. 2021. V. 103, N 3. P. 108-112.

New uniqueness results for probability solutions to the Cauchy problem for the Fokker-Planck-Kolmogorov equation are obtained.

[6] Bogachev V. Chebyshev-Hermite polynomials and distributions of polynomials in Gaussian random variables. Theory Probab. Appl. 2021. V. 66, N 4 (in Russian).

A survey about Chebyshev-Hermite polynomials and distributions of polynomials in Gaussian random variables is given.

[7] Bogachev V. On approximation of measures by their finite-dimensional images. Functional Analysis and its Applications. 2021. V. 55, N 3. P. 75-81 (in Russian).

Several new problems connected with finite-dimensional projections of measures on infinite-dimensional spaces are posed and discussed.

[8] Alekseev G.A., Afonin K.A., Bogachev V.I. On Gaussian transition operators. Doklady Mathematics. 2021. V. 500 (in Russian).

A generalization of Holevo's theorem about Gaussian transition operators is obtained.

[9] Bogachev V.I., Shaposhnikov S.V. Uniqueness of a probability solution to the Kolmogorov equation with a diffusion matrix satisfying Dini's condition. Doklady Mathematics. 2021. V. 501 (in Russian).

Sufficient conditions for uniqueness of probability solutions to the Kolmogorov equation with diffusion matrices satisfying Dini's condition are obtained.

[10] Bogachev V.I., Shaposhnikov A.V., Wang F.-Y. Sobolev–Kantorovich inequalities under $CD(0, \infty)$ condition. Communications in Contemporary Mathematics. DOI 10.1142/S0219199721500279

Sobolev–Kantorovich inequalities are established for diffusion semigroups satisfying certain curvature conditions.

[11] Bogachev V.I., Kosov A.D., Shaposhnikov A.V. Regularity of solutions to Kolmogorov equations with perturbed drifts. Potential Analysis.
DOI 10.1007/s11118-021-09954-9

New results on regularity of solutions to Kolmogorov equations with perturbed drifts are obtained, in particular, some extensions of logarithmic Sobolev inequalities are proved.

3.1.2 Yurii Burman

[1] With R. Froeberg and B. Shapiro
Algebraic relations between harmonic and anti-harmonic moments of plane polygons.
International Mathematics Research Notices. 2021. No. 14. P. 11140–11168.

In this paper we describe the algebraic relations satisfied by the harmonic and anti-harmonic moments of simply connected, but not necessarily convex planar polygons with a given number of vertices.

[2] With V. Kulishov
Lie elements and the matrix-tree theorem.
Submitted to Moscow Mathematical Journal; preprinted version arXiv:2011.10340.

For a finite-dimensional representation V of a group G we introduce and study the notion of a Lie element in the group algebra of G. The set of Lie elements is a Lie algebra and a G-module acting on the original representation V.

Lie elements often exhibit nice combinatorial properties. In particular, if G is a permutation group and V, a permutation representation, we prove a formula for the characteristic polynomial of a Lie element similar to the classical matrix-tree theorem.

[3] With R. Fesler
Surgery in dimension $1 + 1$ *and Hurwitz numbers*
Preprint arXiv:2107.13861.

Surgery in dimension $1 + 1$ describes how to obtain a surface with boundary (compact, not necessarily oriented) from a collection of disks by joining them with narrow ribbons attached to the boundary. Counting the ways to do it gives rise to a “twisted” version of the classical Hurwitz numbers and of the cut-and-join equation.

3.1.3 Alexei Elagin

[1] Alexey Elagin, Valery A. Lunts
Thick subcategories on curves
Adv. Math., 378 (2021), article 107525.

We classify triangulated categories that are equivalent to finitely generated thick subcategories $T \subset D^b(\operatorname{coh} C)$ for smooth projective curves C over an algebraically closed field.

[2] Alexey Elagin, Valery A. Lunts
Derived categories of coherent sheaves on some zero-dimensional schemes
Journal of Pure and Applied Algebra, 226:6 (2022), article 106939.

Let $R_N = k[y_1, \ldots, y_N]/(y_1, \ldots, y_N)^2$ be the truncated polynomial algebra. Geometrically, it is the algebra of functions on the second infinitesimal neighborhood of a closed point in N-dimensional affine space. In this note we study $D^b(R_N{-}\mathrm{mod})$, the bounded derived category of finitely generated R_N-modules. We show that for $N \geq 2$ the lattice of triangulated subcategories in $D^b(R_N{-}\mathrm{mod})$ has a rich structure (which is probably wild), in contrast to the case of zero-dimensional complete intersections. Our homological methods produce some applications to universal localizations of free associative algebras. These applications are based on a relation between triangulated subcategories in $D^b(R_N{-}\mathrm{mod})$ and universal localizations of a free graded associative algebra in N variables.

[3] Alexey Elagin
Calculating dimension of triangulated categories: path algebras, their tensor powers and orbifold projective lines
Journal of Algebra, 592 (2022), 357-401.

This is a companion paper of "Three notions of dimension for triangulated categories" by me and V. Lunts, where different notions of dimension for triangulated categories are discussed. Here we compute dimensions for some examples of triangulated categories and thus illustrate and motivate material from loc. cit. Our examples include path algebras of finite ordered quivers, orbifold projective lines, some tensor powers of path algebras in Dynkin quivers of type A and categories, generated by an exceptional pair.

3.1.4 Anton Fonarev

Dual exceptional collections on Lagrangian Grassmannians, in preparation.

3.1.5 Alexei Gorodentsev

No papers during this period.

3.1.6 Alexander Kalmynin

With R. Dietmann, C. Elsholtz, S.V. Konyagin and J.A. Maynard: Longer gaps between values of binary quadratic forms (mix of preprints arXiv:1906.09100 and arXiv:1810.03203)

submitted to International Mathematics Research Notices

We prove new lower bounds on large gaps between integers which are sums of two squares, or are represented by *any* binary quadratic form of discriminant D, improving results of Richards. Let $s_1, s_2, \ldots$ be the sequence of positive integers, arranged in increasing order, that are representable by *any* binary quadratic form of fixed discriminant D, then

$$\limsup_{n\to\infty} \frac{s_{n+1}-s_n}{\log s_n} \geq \frac{|D|-1}{2\varphi(|D|)(\log|D| + O((\log\log|D|)^3))},$$

improving a lower bound of $\frac{1}{|D|}$ of Richards. In the special case of sums of two squares, we improve Richards's bound of $1/4$ to $\frac{390}{449} = 0.868\ldots$.

We also generalize Richards's result in another direction: If d is composite we show that there exist constants C_d such that for all integer values of x none of the values $p_d(x) = C_d + x^d$ is a sum of two squares. Let d be a prime. For all $k \in \mathbb{N}$ there exists a smallest positive integer y_k such that none of the integers $y_k + j^d, 1 \leq j \leq k$, is a sum of two squares. Moreover,

$$\limsup_{k\to\infty} \frac{k}{\log y_k} \gg \frac{1}{\sqrt{\log d}}.$$

3.1.7 Konstantin Loginov

[1] Bounding non-rationality of divisors on 3-fold Fano fibrations

(joint with C. Birkar)

In this paper we investigate non-rationality of divisors on 3-fold log Fano fibrations $(X, B) \to Z$ under mild conditions. We show that if D is a component of B with coefficient $\geq t > 0$ which is contracted to a point on Z, then D is birational to $\mathbb{P}^1 \times C$ where C is a smooth projective curve with gonality bounded depending only on t. Moreover, if $t > \frac{1}{2}$, then genus of C is bounded depending only on t.

Journal für die reine und angewandte Mathematik (Crelles Journal), vol. 2021, no. 779, 2021, pp. 167-188.

[2] On the derived length of finite groups acting on rationally connected threefolds. In preparation.

(joint with E. Yasinskiy)

We study boundedness properties of finite subgroups of Cremona group of rank at most 3, and more generally, of the group of automorphisms of rationally connected complex projective varieties of dimension at most 3. We consider the case of solvable sugroups. Our aim is to to compute the exact upper bound for the derived length of such subgroups.

3.1.8 Sergei Lvovski

On non-projective small resolutions
arXiv:2104.11901, *submitted to Euoropean Journal of Mathematics.*

We construct a large class of projective threefolds with one node (aka non-degenerate quadratic singularity) such that their small resolutions are not projective.

3.1.9 Vladimir Medvedev

On the index of the critical Möbius band in $\mathbb{B}^4$
arXiv:2112.04883

In this paper we prove that the Morse index of the critical Möbius band in the $4-$dimensional Euclidean ball $\mathbb{B}^4$ equals 5. It is conjectured that this is the only embedded non-orientable free boundary minimal surface of index 5 in $\mathbb{B}^4$. One of the ingredients in the proof is a comparison theorem between the spectral index of the Steklov problem and the energy index. The latter also enables us to give another proof of the well-known result that the index of the critical catenoid in $\mathbb{B}^3$ equals 4.

3.1.10 Taras Panov

[1] With R. Krutowski.
Dolbeault cohomology of complex manifolds with torus action.
In "Topology, Geometry, and Dynamics: Rokhlin Memorial". Contemporary Mathematica, vol. 772; DOI:10.1090/conm/772/15489; American Mathematical Society, Providence, RI, 2021, pp. 173–187.

We describe the basic Dolbeault cohomology algebra of the canonical foliation on a class of complex manifolds with a torus symmetry group. This class includes complex moment-angle manifolds, LVM- and LVMB-manifolds and, in most generality, complex manifolds with a maximal holomorphic torus action. We also provide a dga model for the ordinary Dolbeault cohomology algebra. The Hodge decomposition for the basic Dolbeault cohomology is proved by reducing to the transversely Kähler (equivalently, polytopal) case using a foliated analogue of toric blow-up.

[2] With J. Grbić, M. Ilyasova, G. Simmons.

One-relator groups and algebras related to polyhedral products.

Proceedings of the Royal Society of Edinburgh Section A: Mathematics, published online by Cambridge University Press: 15 January 2021, pp. 1–20;
DOI:10.1017/prm.2020.101

We link distinct concepts of geometric group theory and homotopy theory through underlying combinatorics. For a flag simplicial complex K, we specify a necessary and sufficient combinatorial condition for the commutator subgroup RC'_K of a right-angled Coxeter group, viewed as the fundamental group of the real moment-angle complex $\mathcal{R}_K$, to be a one-relator group; and for the Pontryagin algebra $H_*(\Omega\mathcal{Z}_K)$ of the moment-angle complex to be a one-relator algebra. We also give a homological characterisation of these properties. For RC'_K, it is given by a condition on the homology group $H_2(\mathcal{R}_K)$, whereas for $H_*(\Omega\mathcal{Z}_K)$ it is stated in terms of the bigrading of the homology groups of $\mathcal{Z}_K$.

[3] With G. Chernykh.
SU-linear operations in complex cobordism and the c1-spherical bordism theory.
arXiv:2106.11876; *submitted to Transactions of the Americal Math. Society.*

We study the SU-linear operations in complex cobordism and prove that they are generated by the well-known geometric operations ∂_i. For the theory W of c_1-spherical bordism, we describe all SU-linear multiplications on W and projections $MU \to W$. We also analyse complex orientations on W and the corresponding formal group laws F_W. The relationship between the formal group laws F_W and the coefficient ring $\varOmega^W$ of the W-theory was studied by Buchstaber in 1972. We extend his results by showing that for any SU-linear multiplication and orientation on W, the coefficients of the corresponding formal group law F_W do not generate the ring $\varOmega^W$, unlike the situation with complex bordism.

3.1.11 Alexei Penskoi

[1] With M. Karpukhin, N. Nadirashvili, I. Polterovich
An isoperimetric inequality for Laplace eigenvalues on the sphere.
J. Differential Geom, 2021, Vol. 118, No. 2, pp. 313–333.

In this paper it is proved that for any positive integer k, the k-th nonzero eigenvalue of the Laplace-Beltrami operator on the two-dimensional sphere endowed with a Riemannian metric of unit area, is maximized in the limit by a sequence of metrics converging to a union of k touching identical round spheres. This proves a conjecture posed by the second author in 2002 and yields a sharp isoperimetric inequality for all nonzero eigenvalues of the Laplacian on a sphere. Earlier, the result was known only for $k = 1$ (J. Hersch, 1970), $k = 2$ (N. Nadirashvili, 2002; R. Petrides, 2014) and $k = 3$ (N. Nadirashvili and Y. Sire, 2017). In particular, it is proved that for any $k \geq 2$, the supremum of the k-th nonzero eigenvalue on a sphere of unit area is not attained in the class of Riemannian metrics which are smooth outsitde a finite set of conical singularities. The proof uses certain properties of harmonic maps between spheres, the key new ingredient being a bound on the harmonic degree of a harmonic map into a sphere obtained by N. Ejiri.

[2] With M. Karpukhin, N. Nadirashvili, I. Polterovich
Conformally maximal metrics for Laplace eigenvalues on surfaces.
arXiv:2003.02871, *to appear in Surveys in Differential Geometry, Vol 24 (2019)* (not yet published, expected in 2021)

The paper is concerned with the maximization of Laplace eigenvalues on surfaces of given volume with a Riemannian metric in a fixed conformal class. A significant progress on this problem has been recently achieved by Nadirashvili–Sire and Petrides using related, though different methods. In particular, it was shown that for a given k, the maximum of the k-th Laplace eigenvalue in a conformal class on a surface is either attained on a metric which is smooth except possibly at a finite number of conical singularities, or it is attained in the limit while a "bubble tree" is formed on a surface. Geometrically, the bubble tree appearing in this setting can be viewed as a union of touching identical round spheres. An another proof of this statement is presented, developing the approach proposed by the second author and Y. Sire. As a side result, we provide explicit upper bounds on the topological spectrum of surfaces.

3.1.12 Petr Pushkar

[1] With M. Tyomkin
On matrix of differential in a Morse complex.
To appear in Russian Mathematical Surveys.

In this paper we consider a construction of Bruhat numbers assigned to each pair of Barannikov critical points. Also we get a new restrictions on a matrix of a Morse differential of a strong Morse function on a closed manifold.

[2] With M. Tyomkin
Enhanced Bruhat decomposition and Morse theory.
arXiv:2012.05307v2 [math.AT] 28 Jun 2021. Paper was completely rewritten, 17 pages added. *submitted to International Mathematics Research Notices*

Consider the set of all rectangular nm matrices with entries in a field. Recall that unitriangular group T_n consists of upper triangular matrices with 1's on the diagonal. The product $T_n \times T_m$ naturally acts on the aforementioned set: $X \mapsto AXB^{-1}$. Our first observation is that each orbit of this action contains a unique matrix which has at most one non-zero entry in each row and in each column. Thus these non-zero numbers and their positions are invariants of a matrix under this action. This is a variation of a classical Bruhat decomposition for GL. When applied in the setting of Morse theory, this linear algebraic construction leads to invariants of a strong Morse function f. Namely, positions of non-zero entries correspond to the well-known Barannikov decomposition (also known as persistent homology) of f. The novelty is the values themselves, which correspond to numbers, carried by Barannikov pairs (also known as bars in the barcode). Considering further a complex, constructed from a strong Morse function, we interpret the product of all the numbers as a torsion of chain complex.

3.1.13 Alexander Samokhin

[1] With R.Gonzales, C.Pech, N.Perrin
Geometry of horospherical varieties of Picard rank one
to appear in International Mathematics Research Notices,
https://doi.org/10.1093/imrn/rnaa331

We study the geometry of smooth non-homogeneous horospherical varieties of Picard rank one. These have been classified by Pasquier and include the well-known odd symplectic Grassmannians. We focus our study on quantum cohomology, with a view towards Dubrovin's conjecture. We start with describing the cohomology groups of smooth horospherical varieties of Picard rank one. We show a Chevalley formula for these and establish that many Gromov–Witten invariants are enumerative. This enables us to prove that in many cases the quantum cohomology is semisimple. We give a presentation of the quantum cohomology ring for odd symplectic Grassmannians. In the last sections, we turn to derived categories of coherent sheaves. We first discuss a general construction of exceptional bundles on horospherical varieties. We work out in detail the case of the horospherical variety associated to the exceptional group G_2 and construct a full rectangular Lefschetz exceptional collection in the derived category.

[2] On the D–affinity of flag varieties in positive characteristic, II
Preprint

We compute the cohomology of the sheaf of small differential operators $\mathcal{D}^{(1)}_{G/B}$ on the flag variety of the exceptional group G_2 and for $p \geq 11$ show the cohomology vanishing above the degree one. On the contrary, the first cohomology group turns out to be non-zero. We discuss possible implications of this non-vanishing result for the failure of D-affinity of the flag variety G_2/B relative to the sheaf of differential operators with divided powers.

3.1.14 George Shabat

[1] On generalizations of Chebyshev polynomials and Catalan numbers (with B.S. Bychkov) Ufimsk. Mat. Zh., 2021, Volume 13, Issue 2, Pages 1117

We provide possible directions of generalizations of earlier found relations between the Chebyshev polynomials and the Catalan numbers arising in studying commuting difference operators. These generalizations are mostly related with ideas proposed by N.H. Abel in his publication in 1826, which then were reproduced by many authors in a modern language. As generalization of Chebyshev polynomials, we propose to consider polynomials with exactly two critical values well-studied in a so-called theory of dessins d'enfants. The Catalan numbers are located in the first column of the table of HarerZagier numbers related with the distribution by genus of orientable sewing of polygons with even number of sides. The commuting difference operators are implicitly contained in the Abel theory, who studied quasi-elliptic integrals, namely, the elliptic integrals of 3rd kind integrable in terms of logarithms. In the present work we formulate conjectures on relation between the main Abel theorem and commuting semi-infinite matrices. In the work we provide calculations supporting the conjectured relations.

[2] Vladimir Voevodsky on mathematics and life.
Biosystems (ELSEVIER) Volume 205, July 2021, pp. 5-11.

A short non-technical review of Voevodsky's mathematical work is presented together with the critical analysis of his attempts to apply his skills to biology. Some details of Voevodsky general views on sciences and life, based on the close personal contacts, are mentioned.

3.1.15 Stanislav Shaposhnikov

[1] With V.I.Bogachev and T.I.Krasovitskii

On Nonuniqueness of Probability Solutions to the Cauchy Problem for the Fokker–Planck–Kolmogorov Equation. Dokl. Math. 2021, Vol. 103, pp. 108–112.

In this paper we give a positive answer to the question about the possibility of existence of several probability solutions to the FokkerPlanckKolmogorov equation for all initial conditions: we construct the first example of an equation with a unit diffusion matrix and a smooth drift coefficient for which the Cauchy problem with every probability initial condition has an infinite-dimensional simplex of probability solutions.

[2] With V.I.Bogachev and T.I.Krasovitskii

On uniqueness of probability solutions of the Fokker–Planck–Kolmogorov equation. Mat. Sb., 2021, Vol. 212, N 6, pp. 3–42.

The paper gives a solution to the long-standing problem of uniqueness for probability solutions to the Cauchy problem for the Fokker-Planck-Kolmogorov equation with an unbounded drift coefficient and unit diffusion coefficient. It is proved that in the one-dimensional case uniqueness holds and in all other dimensions it fails. The case of nonconstant diffusion coefficients is also investigated.

[3] With V.I.Bogachev
Elliptic equations degenerating at infinity and uniqueness of probability solutions to the Kolmogorov equation. Rev. Roumaine de Math. Pures et App. 2021, Vol. 6, N 1, pp. 67–81.

We obtain new sufficient conditions for nonuniqueness of probability solutions to stationary Kolmogorov equations. We also study associated second order elliptic equations for the densities of different solutions with respect to a fixed solution and in the case where the fixed solution satisfies the Poincare inequality we prove a criterion for the existence of a nonconstant solution in the weighted Sobolev space with respect to the fixed solution.

[4] With V.I.Bogachev and M. Rockner
On the Ambrosio–Figalli–Trevisan Superposition Principle for Probability Solutions to Fokker–Planck–Kolmogorov Equations. J. Dyn. Diff. Equat., 2021, Vol. 33, pp. 715–739.
We prove a generalization of the known result of Trevisan on the Ambrosio–Figalli–Trevisan superposition principle for probability solutions to the Cauchy problem for the Fokker–Planck–Kolmogorov equation.

3.1.16 Evgeni Smirnov

With Anna Tutubalina. Slide polynomials and subword complexes. Sbornik: Mathematics, 212 (10), 2021, 131–151 (in Russian).
Subword complexes were defined by A. Knutson and E. Miller in 2004 for describing Grbner degenerations of matrix Schubert varieties. The facets of such a complex are indexed by pipe dreams, or, equivalently, by the monomials in the corresponding Schubert polynomial. In 2017 S. Assaf and D. Searles defined a basis of slide polynomials, generalizing Stanley symmetric functions, and described a combinatorial rule for expanding Schubert polynomials in this basis. We describe a decomposition of subword complexes into strata called slide complexes, that correspond to slide polynomials. The slide complexes are shown to be homeomorphic to balls or spheres.

3.1.17 Alexei Sossinsky

[1] A.B. Sossinsky, Erratum to: Elementary theory of the Alexander–Conway polynomial, Math. Notes, vol. 109, no. 6, 995 (2021)

The elementary proof of the existemce of the Alexander–Conway polynomial contains a *lacuna* that tha author is able to fill in only by non-elementary means.

[2] A.B. Sossinsky, Introduction to Knot Theory, Lecture Notes, 157 pages, MCCME Publications, 2021 (in print)

The book is a lecture course in the theory of knots, links, and their invriants, containing an elementary, but rigorous, exposition of the main recent results of the theory, in particular the Alexander–Conway polynomial, the Kauffman bracket, the Jones polynomial, the Vassiliev invariants, and the Kontsevich integrals. The prerequisites for the course are minimal, so it is accessible to undergraduate math majors, but contains a great deal of serious material that will interest graduate students and more advanced mathematicians. The book is richly illustrated (over a 120 figures), the lectures are supplied with numerous exercises, and contain several interesting open problems.

3.1.18 Vladlen Timorin

[1] A. Blokh, A. Cheritat, L. Oversteegen, V. Timorin *Location of Siegel capture polynomials in parameter spaces*, Nonlinearity 2021. Vol. 34. No. 4. pp. 2430–2453.

A cubic polynomial with a marked fixed point 0 is called an *IS-capture polynomial* if it has a Siegel disk D around 0 and if D contains an eventual image of a critical point. We show that any IS-capture polynomial is on the boundary of a unique bounded hyperbolic component of the polynomial parameter space determined by the rational lamination of the map and relate IS-capture polynomials to the cubic Principal Hyperbolic Domain and its closure.

[2] A. Blokh, L. Oversteegen, V. Timorin, *Slices of the parameter space of cubic polynomials*, to appear in the Transactions of the AMS. DOI: https://doi.org/10.1090/tran/8519

In this paper, we study slices of the parameter space of cubic polynomials, up to affine conjugacy, given by a fixed value of the multiplier at a non-repelling fixed point. In particular, we study the location of the *main cubioid* in this parameter space. The *main cubioid* is the set of affine conjugacy classes of complex cubic polynomials that have certain dynamical properties generalizing those of polynomials $z^2 + c$ for c in the filled main cardioid.

[3] A. Blokh, L. Oversteegen, V. Timorin, *Cutpoints of Invariant Subcontinua of Polynomial Julia Sets*, to appear in Arnold Mathematical Journal, https://link.springer.com/article/10.1007/s40598-021-00186-8

We prove fixed point results for branched covering maps f of the plane. For complex polynomials P with Julia set J_P these imply that periodic cutpoints of some invariant subcontinua of J_P are also cutpoints of J_P. We deduce that, under certain assumptions on invariant subcontinua Q of J_P, every Riemann ray to Q landing at a periodic repelling/parabolic point $x \in Q$ is isotopic to a Riemann ray to J_P relative to Q.

[4] A. Blokh, L. Oversteegen, A. Shepelevtseva, V. Timorin, *Modeling core parts of Zakeri slices I*, arXiv: 2102.07466, *submitted to Moscow Mathematical Journal*.

The paper deals with cubic 1-variable polynomials whose Julia sets are connected. Fixing a bounded type rotation number, we obtain a slice of such polynomials with the origin being a fixed Siegel point of the specified rotation number. Such slices as parameter spaces were studied by S. Zakeri, so we call them *Zakeri slices*. We give a model of the central part of a slice (the subset of the slice that can be approximated by hyperbolic polynomials with Jordan curve Julia sets), and a continuous projection from the central part to the model. The projection is defined dynamically and is coherent with the dynamical-analytic parameterization of the Principal Hyperbolic Domain by Petersen and Tan Lei.

3.1.19 Vladimir Zhgoon

[1] V. P. Platonov, M.M.Petrunin, V. S. Zhgoon, On the problem of periodicity of continued fraction expansions of $\sqrt{f}$ for cubic polynomials over number fields accepted for publication in Mat.Sbornik

We obtain a complete description of the fields K, which are extensions of Q of degree less or equal than 3, and cubic polynomials $f \in K[x]$, for which the expansion is $\sqrt{f}$ in a continuous fraction in the field of formal power series $K((x))$ is periodic. We prove the finiteness theorem for cubic polynomials $f \in K[x]$ with periodic decomposition $\sqrt{f}$ for extensions of Q degree less than 6. A description of the periodic elements is obtained cubic polynomials f(x) defining elliptic curves with points of the order of 3 6 N 6 42, N = 37, 41.

[2] Zhgoon V.S., But N.A. On higher dimensional residues and the generalisations of Cayley-Bacharash theorem. preprint Trydi NIISI RAS

Abstract. This paper is dedicated to an attempt to give an algebraic proof of the Mu Lin Li theorem, which is a generalisation of the Cayley-Bacharach Theorem on the case of an irregular section. We developed a method on which the proof can be based, also we managed to prove some necessary technical results for the algebraisation.

[3] R.S.Avdeev, V.S.Zhgoon On existence of B-root subgroups in affine spherical varieties.

preprint, to appear in Doklady Mathematics, 2018.

Let X be an irreducible affine algebraic variety that is spherical with respect to the action of a connected reductive group G. This paper presents sufficient conditions formulated in terms of combinatorics of weights for the existence on X of one-parameter additive actions normalized by the Borel subgroup $B \subset G$. As an application, it is proved that any G-invariant simple divisor in X can be moved to an open G-orbit using a suitable B-normalizable one-parameter additive action.

3.2 Scientific conferences and seminar talks

3.2.1 Vladimir Bogachev

[1] International Conference "Theory of Probability and Its Applications: P.L. Chebyshev - 200", 17-22 May 2021. Title: Chebyshev-Hermite polynomials and distributions of polynomials in Gaussian random variables.

[2] International Conference New Frontiers in High-Dimensional Probability and Applications to Machine Learning, Sochi, 11-14 May 2021 Title: Spaces of measures: topology, geometry and low-dimensional approximations

[3] Online International Workshop on Stochastic Analysis and Hermite Sobolev Spaces, Freiburg, Germany, 21-26 June 2021 Title: Fractional Sobolev classes on infinite-dimensional spaces.

[4] United Seminar of the Department of Probability Theory of Lomonosov Moscow State University, 22 September 2021 Title: Distributions of homogeneous functions of Gaussian random vectors.

3.2.2 Alexander Kalmynin

[1] Seminar "Contemporary Problems in Number Theory", (Moscow, Russia), April 8

Talk "Quadratic characters with positive partial sums"

[2] XIX International Conference Algebra, Number Theory, Discrete Geometry And Multiscale Modeling Modern Problems, Applications And Problems Of History Dedicated To The Bicentennial Of Academician P. L. Chebyshev (Tula, Russia), May 18-22

Talk "Quadratic characters with nonnegative partial sums"

[3] A seminar on geometric structures on manifolds, (Moscow, Russia), May 27

Talk "Random multiplicative functions and zeros of Dirichlet L-functions"

[4] Conference of International Mathematical Centers of World Level, (Sirius, Russia) August 9-13,

Talk “Positivity of character sums and random multiplicative functions”

[5] Memorial Conference on Analytic Number Theory and Applications Dedicated to the 130th Anniversary of I. M. Vinogradov (Moscow, Russia), September 13-17

Talk “Positivity of character sums and random multiplicative functions”

[6] Seminar “Functional analysis and noncommutative geometry”, (Moscow, Russia), October 5

Talk “Zeros of the Riemann zeta function and spectral triples”

[7] Seminar “Automorphic forms and applications”, (Moscow, Russia), October 11

Talk “Cubic Gauss sums”

3.2.3 Konstantin Loginov

[1] Conference “SIMC Welcomes Postdocs-2020”, 22–23 April 2021, Steklov Mathematical Institute, Moscow. Talk “Log Fano varieties and their applications”

[2] Conference “Multidimensional Residues and Tropical Geometry”, 14–18 June 2021, Mathematical Center ”Sirius”, Sochi, Russia. Talk “Boundedness of divisors on Fano fibrations”

[3] Conference “LUTSINOfest”, Lutsino, Moscow region, Russia, 4–6 July 2021. Talk “Boundedness of irrationality on Fano fibrations”.

[4] Conference “Primorie Mathematical Fair”, Vladivostok, Russia, 21–26 July 2021. Talk “Boundnedness of divisors on Calabi-Yau fibrations”.

[5] Conference “Geometry and Homological Mirror Symmetry”, Mathematical Center ”Sirius”, Sochi, Russia, 11-14 December 2021. Talk “On K-stability of three-dimensional log Fano varieties”.

[6] Seminar of the Laboratory of Algebraic Geometry and Homological Algebra, Moscow Institute of Physics and Technology, 8 April 2021. Talk “On the geometry of logarithmic Fano varieties”.

[7] Iskovskikh seminar, Steklov Mathematical Institute, 15 April 2021, talk “A finiteness theorem for elliptic Calabi-Yau threefolds”.

[8] Iskovskikh seminar, Steklov Mathematical Institute, 18 November 2021, talk “Termination of flips and fundamental groups (after Joaquin Moraga)”.

3.2.4 Vladimir Medvedev

[1] Conference ”Young Researchers in Spectral Geometry”, Montreal, September, 13

Talk ”On the index of the critical Moebius band in the 4-ball”

[2] Moscow, November, 5

Talk "On free boundary minimal submanifolds" at "Laboratory of algebraic geometry: weekly seminar" (Higher School of Economics)

[3] Moscow, October, 2

Talk "On the index of the critical Moebius band in the 4-ball-I" at "Seminar on Spectral Geometry" (Independent University of Moscow and Interdisciplinary Scientific Center J.-V. Poncelet, ISCP, UMI 2615)

[4] Moscow, October, 9

Talk "On the index of the critical Moebius band in the 4-ball-II" at "Seminar on Spectral Geometry" (Independent University of Moscow and Interdisciplinary Scientific Center J.-V. Poncelet, ISCP, UMI 2615)

3.2.5 Taras Panov

[1] International School "Toric Topology and Combinatorics", Sirius Mathematics Center, Sochi, Russia; November, 1–5.

Two lectures "Foliations arising from configurations of vectors, Gale duality, and moment-angle manifolds".

[2] International Workshop "Advances in Homotopy Theory", University of Southampton, UK (virtual); September, 15–17.

Talk "Double cohomology of moment-angle complexes".

[3] International Conference "Primorie Mathematical Fair", Far Eastern Federal University, Vladivostok, Russia; July, 20–25.

Talk "SU-bordism: geometric representatives, operations, multiplications and projections"

[4] Mini-workshop "Algebraic groups: the White Nights season", EIMI, PDMI, St Petersburg, Russia; July 12–16.

Talk "Complex geometry of moment-angle manifolds".

[5] 8th International Youth Summer School-Conference on Geometric Methods of Mathematical Physics, Krasnovidovo, Moscow Region, Russia; June, 28–July, 3.

Mini-course (3 lectures) "Complex geometry of manifolds with torus actions"

[6] International Conference "Toric Topology 2021 in Osaka" (online); March, 24–26.

Talk "SU-bordism: geometric representatives, operations, multiplications and projections".

[7] kpa70 Conference celebrating the 70th birthday of Prof. Krzysztof Pawalowski (online); January 11–13.

Talk "Holomorphic foliations on complex moment-angle manifolds".

3.2.6 Alexei Penskoi

Seminar on Topology, Geometry and Mathematical Physics, Moscow State University, November 24 and December 1, 2021

Talk “Maximisation of Laplace-Beltrami operator eigenvalues: conical points and bubbling of maximal metrics”

3.2.7 Petr Pushkar

[1] Talk “Morse theory and unitriangular geometry I ” at HSE seminar ”Contact topology and invariants of legendrian knots ”

[2] Talk “Morse theory and unitriangular geometry II ” at HSE seminar ”Contact topology and invariants of legendrian knots ”

3.2.8 Alexander Samokhin

[1] Mini-workshop ”Algebraic groups – the White Nights season” (St. Petersburg, July 12–16, 2021)

Talk ”On a construction of vector bundles on smooth projective horospherical varieties of Picard rank one”

[2] Algebraic Geometry seminar, Tata Institute of Fundamental Research, December 2021, http://www.math.tifr.res.in/~swarnava/seminar.html

Talk ”Semiorthogonal decompositions for flag varieties over $\mathbb{Z}$ and applications to modular representation theory”

3.2.9 George Shabat

[1] Visit to Sofia, August

Talk “Dessins d’enfants and moduli spaces of curves” (online) at algebra and logic seminar (Institute of Mathematics and Informatics Bulgarian Academy of Sciences)

[2] The International Conference ”Classical and Modern Geometry” in honor of Levon S. Atanasyan on his 100th birthday (November 14, 2021, Moscow Pedagogical State University, Moscow)

Talk “Triangulations of surfaces and Hodge theory”

3.2.10 Stanislav Shaposhnikov

[1] Conference "New Frontiers in High-dimensional Probability and Applications to Machine Learning", Sirius University of Science and Technology, May, 12 – 15.

Talk "On uniqueness of probability solutions to the Fokker-Planck-Kolmogorov equation"

[2] Conference "Conference of international mathematical centres", Sirius educational centre in Sochi, August, 9 – 13.

Talk "Kolmogorov problems on uniqueness of probability solutions to elliptic and parabolic equations"

3.2.11 Evgeni Smirnov

[1] IMPANGA 20 Conference on Schubert Varieties, Bedlewo, Poland (via Zoom), July 16, 2021

Talk: Pipe dreams for Schubert polynomials of the classical groups

[2] School and conference "Lie algebras, invariant theory and representation theory", Samara, August 22, 2021

Talk: Schubert polynomials for classical Weyl groups

[3] Topology of torus actions, Sirius, Sochi, October 26, 2021

Talk: Schubert polynomials for classical Weyl groups

[4] Lomonossov Moscow State University, Mechanics and Mathematics Department, seminar "Algebraic Topology and its Applications" — M.M.Postnikov seminar, March 9 and 16, 2021

Talk: "Arrays with strings and Grothendieck polynomials"

3.2.12 Alexei Sossinsky

International Forum on the Popularizattion and Teaching of Mathematics, Moscow, October 6-13.

Invited talk:"Twenty Years of the French-Russian Math Laboratory".

3.2.13 Vladlen Timorin

[1] Beijing–St Petersburg Colloquium, November 04, 2021.

Talk "The main cubioid and the modulus of renormalization"

3.2.14 Vladimir Zhgoon

My joint work with R.Avdeev was partly announced in his talk Root subgroups on affine spherical varieties. in the conference Topology of the torus actions and related topics. Sirius, Sochi 25-29 October.

3.3 Teaching

3.3.1 Vladimir Bogachev

[1] Introduction to stochastic analysis and the Malliavin calculus. Independent University of Moscow, 3-5 year students, September-December 2021, 2 hours per week.

Program.

1. Gaussian measures on the real line and in multidimensional spaces.
2. Basic concepts and facts from measure theory on metric spaces.
3. Gaussian measures on infinite-dimensional spaces.
4. The case of a Hilbert space.
5. Random processes and their distributions. The Kolmogorov theorem.
6. Wiener measure on the space of continuous paths and on L^2.
7. Stochastic integrals of Wiener and Ito.
8. Stochastic differential equations.
9. Ito's formula.
10. Diffusion processes.
11. The Malliavin method in the finite-dimensional case.
12. The Cameron–Martin formula. Sobolev classes over Gaussian measures.
13. Infinite-dimensional integration by parts formula.
14. The Malliavin method in the infinite-dimensional case.

[2] Calculus. Faculty of Mathematics, Higher School of Economics, I year students, September-December 2021, 4 hours per week.

Program.

1. Uncountability of the set of sequences of zeros and units. Countability of the set of rational numbers. The irrationality of the number $\sqrt{2}$.

2. The set of real number. The upper and lower bound of a set on the real line and their existence.

3. The intersection of a decreasing sequence of compact intervals is not empty. Every cover of a compact interval by open intervals admits a finite subcover.

4. Convergent sequences and their limits. Basic properties of convergent sequences.

5. Cauchy sequences. Boundedness of a Cauchy sequence. The Cauchy sequence about convergence of Cauchy sequences.

6. Existence of a limit of monotonic bounded sequence. The number e.

7. Subsequences and partial limits. The Bolzano-Weierstrass theorem on convergent subsequences in bounded sequences.

8. Open and closed sets on the real line. The Baire theorem.

9. Convergent series. Convergence of absolutely convergent series. Convergence of the Leibniz series $\sum(-1)^n/n$. The Gauss-Cauchy criterion of convergence and investigation of convergence of the series of $1/n^p$.

10. A limit of a function at a point. One-sided limits. The equivalence of the Heine and Cauchy conditions. The Cauchy criterion of the existence of a limit.

11. Continuous functions and their basic properties (operations, compositions). The mean value theorem for continuous functions on a compact interval.

12. The continuity of the inverse function for a continuous monotone function. Discontinuities of monotone functions.

13. Elementary functions. Special limits:
$\lim\limits_{x\to+\infty}(1+1/x)^x$, $\lim\limits_{x\to 0}(e^x-1)/x$, $\lim\limits_{x\to 0}\sin x/x$.

14. Derivative. Basic properties of differentiable functions (differentiability of sums, products and ratios).

15. The Fermat theorem on points of extremum. The Rolle theorem.

16. The Lagrange and Cauchy mean value theorems. Connections between monotonicity on an interval and derivative.

17. L'Hôpital's rule.

18. The derivative of a composition and the inverse function. Derivatives of elementary functions.

19. Taylor's formula.

20. Power series. Taylor expansions for e^x, $\sin x$, $\cos x$, $\ln(1+x)$, $(1+x)^\alpha$.

21. Investigation of local extrema in terms of the first two derivatives.

22. Convex and concave functions. Conditions for convexity in terms of the second derivative. Examples.

23. Metric space. The space $\mathbb{R}^n$. Continuous mappings, equivalent characterizations of continuity. The Lipschitz condition.

24. Compact sets in $\mathbb{R}^n$. Continuous mappings of compact sets.

25. Differential functions of many variables. Derivative and gradient. The mean value theorem. Differentiability of functions with continuous partial derivatives.

3.3.2 Yurii Burman

[1] Topology–2, Independent University of Moscow (the course supported by the Simons–IUM grant), fall 2021, 4 hours per week.

Program:

1. Homology

   1.1. Preparatory chapter: homology of graphs.

   Definition, homotopy invariance. Action of cell maps on homology.

   1.2. Singular complex of a topological space.

   Definintion, $\partial^2 = 0$. Singular homology as a functor. Meaning of H_0. Chain homotopy, homotopy invariance of the singular homology.

   1.3. Mayer–Vietoris sequence.

   Singular chains subject to an open cover. Long exact sequence of homology from a short exact sequence of complexes. Exactness of the Mayer–Vietoris sequence.

   1.4. Homology exact sequences of a pair and of a triple.

   Definition and homotopy invariance of relative homology. Exactness of the homology sequences of a pair.

2. Cellular spaces.

   2.1. Main properties and cell homology.

   Relative homology of a cell pair. Definition of a CW-complex, $\partial^2 = 0$. Cell homology as a functor.

   2.2. Cell homology is isomorphic to the singular homology.

3. Smooth manifolds.

   3.1. Main properties.

   Definition of a smooth manifold and a smooth map. Orientability.

   3.2. Top homology.

   Homological definition of the orientability. Top homology of a connected manifold.

   3.3. Degree of a smooth map.

   Degree of a smooth map of compact oriented manifolds of the same dimension is equal to the sum of signs of the preimages of a regular value.

4. Multiplication in cohomology.

   4.1. Main properties.

   Definition of the cup product, associativity, supercommutativity, functoriality.

   4.2. Künneth formula.

Cross product vs cup product. Axioms for cohomology. A simplified Künneth formula (for spaces with free homology of finite rank). Cross product of cell spaces.

[2] The freshmen's seminar "Basic notions of mathematics", Higher School of Economics, fall 2021, 2 hours per week.

The seminar aims at keeping the recently admitted math majors "mathematically agile" by demonstrating them problems and theories that fall between different areas of mathematics. Some talks are given by the seminar organizers, some by seminar participants.

Examples of talks:

1. Principal section of a 4-dimensional cube.
2. On a formula for roots of equation of degree 3.
3. Law of quadratic reciprocity.
4. Banach–Tarski paradox.
5. Quaternions.
6. The number of triangular domains cut out from the plane by n straight lines.

3.3.3 Alexei Elagin

Homological methods in representation theory of finite dimensional algebras - 2. Independent University of Moscow, III-V year students, September-December 2021, 2 hours per week.

Program

1. **Auslander-Reiten Theory**
   Minimal morphisms and almost split morphisms in additive categories. Source and sink morphisms. Irreducible morphisms. Auslander-Reiten (AR) quiver of an additive category. Almost split short exact sequences in abelian categories and almost exact triangles in triangulated categories. The case of hereditary categories. AR translation and translation quivers. Existence of AR translation for triangulated categories with a Serre functor and for module categories over finite dimensional algebras. Stable module category, the transpose and the dual. AR formula and defect lemma. Structure of an AR quiver for hereditary algebras. Dynkin, tame and wild cases. Preprojective / preinjective components, regular components. Stable tubes and $\mathbb{Z}A_\infty$.

2. **Tilting theory**
   Theorem of Rickard. Equivalence between module categories defined by RHom and $\otimes$ functors. Tilting modules and generalized tilting modules, induced derived equivalences. Partial tilting modules, their summands. Dual of a tilting module. Torsion pairs. Torsion pair defined by a tilting module. Brenner-Butler theorem. T-structures and hearts. Mutations of t-structures. Tilting equivalence as a mutation of t-structure. Example of a heart which is not noetherian neither artinian.

3. **Piecewise hereditary algebras and weighted projective lines.**
   Piecewise hereditary algebras, their properties. Examples. Finiteness of global dimension for endomorphism algebra of a partial tilting module. Partial tilting modules come form exceptional collections. APR tilting modules. Tilted and iterated tilted algebras. Theorem of Happel, Rickard and Schofield: iterated tilted = piecewise hereditary. Weighted projective lines as orbifolds and as graded module categories. Torsion sheaves, line bundles, Serre duality. Canonical algebras. Their derived equivalence to WPL-s. WPL-s as noetherian hereditary categories without projectives and with a tilting complex. Domestic, tubular and wild WPL-s. Their AR quivers. Derived equivalence between tubular WPL-s and tame hereditary algebras. Happel's theorem about two types of "piecewise hereditary" algebras.

3.3.4 Anton Fonarev

Algebra. Independent University of Moscow, I year students, September-December 2021, 2 hours per week.

- Semigroups, monoids, groups.
- Homomorphisms, actions, free groups.
- Sylow theorems.
- Rings, modules.
- PIDs, structure theorem for modules.
- Finitely generated abelian groups.
- Vector spaces, linear maps.
- Jordan normal form.
- Fields, field extensions.
- Normal, separable extensions.
- Galois extensions.
- Galois theory.

3.3.5 Alexei Gorodentsev

[1] Algebra-3, Independent University of Moscow, II year students, September-December 2021, 4 hours per week.
(`http://gorod.bogomolov-lab.ru/ps/stud/algebra-3/2122/list.html`)

Program.

1. Tensor products of modules over a commutative ring.
2. Tensor algebra, symmetric algebra, and exterior algebra of a vector space. Linear support of a tensor, Plücker, Segre, and Veronese varieties.
3. Symmetric and skew symmetric tensors in characteristic zero. Polarization of polynomials.

4. Symmetric functions, transitions between standard bases in the $\mathbb{Z}$-module of symmetric functions.

5. Massives and Young tableaux, Schur polynomials, Littlwood–Richardson rule.

6. Semisimple modules over associative algebras, Schur lemma, double commutator theorem, surjectivity of irreducible representations, isotypic components.

7. Representations of finite groups: the group algebra, minimal idempotents, character theory, ring of representations, induced and coinduced representations, the Frobenius reciprocity.

8. Representations of symmetric groups. Shur-Wayl correspondence.

[2] Algebra-2. Independent University of Moscow, I year students (this is the 2nd term of 1 year course), February–May 2021, 4 hours per week.
(`http://gorod.bogomolov-lab.ru/ps/stud/algebra-1/2021/list.html`)
Program.

1. Groups and their homomorphisms, examples. Group actions: orbit length, the Polia–Bernside enumeration of orbits, regular and adjoint action. Normal subgroups and quotients, p-groups and Silov's theorems.

2. The Jordan–Hölder theorem for groups. Simple groups, examples: A_n, $SO_3(\mathbb{R})$. Free groups, generators and relations. Presentation of symmetric groups.

3. Non-degenerated bilinear forms, orthogonal projections. Anti-symmetric forms.

4. Non degenerated quadratic forms, Witt's theory.

5. Hermitean spaces, distances, angles. Orthogonal diagonalization of normal operators. Polar and SVD decompositions.

6. Complex and real structures. Hermitean extensions of a symplectic structure form the Siegel domain.

[3] Linear Algebra and Geometry (1st term), the National Research University 'Higher School of Economics', Faculty of Math., I year students, September-December 2021, 6 hours per week.
Site: `http://gorod.bogomolov-lab.ru/ps/stud/geom_ru/2122/list.html`.
Lecture notes:
`http://gorod.bogomolov-lab.ru/ps/stud/geom_ru/2122/lec_total.pdf`
Program.

1. Vector spaces. The vector space $\Bbbk^2$: proportional vectors, the determinant 2×2, bases and coordinates, Cramer's rule, the oriented area of parallelogram. The affine space $\Bbbk^2$: affine coordinate systems,collinear points, equations for lines, oriented areas of triangles and polygons. The center of masses and barycentric coordinates.

2. Linear and affine transformations of plane, the differential of an affine map. Linear and affine changes of coordinates, the matrix formalism. Semiaffine and semilinear transformations (the maps sending a line to a line).

3. Inner products and orthonormal bases in a real plane, the Cauchy–Bunyakovsky–Schwartz inequality, the triangle inequality. The Gram determinant. The oriented angle between vectors. Metric properties of line equations, bisectors, angles between lines. Trigonometric identities, geometric descriprion for the complex number multiplication. Euclidean plane = complex line. The similarity transformations of Euclidean plane.

4. Higher dimensional vector spaces: generators and linear relations, the exchange lemma, bases and the dimension. Linear maps, the kernel and the image. Dimensions of sums and intersections of vector subspaces, and dimensions of fibers of linear maps. Direct sums and direct products of vector spaces and subspaces. Affine spaces, the geometry of mutual arrangement of a pair of affine subspaces. Quotient spaces.

5. The matrix formalism for linear expansions of collections of vectors, for linear maps, and for systems of linear equations. The multiplication of matrices. Inverse matrices and base changes. The rank of a matrix. The qualitative theory of linear equations: the Fredholm alternative, the Kronecker–Capelli test, the dimension of solution space. Associative algebras over a field, the matrix algebra, the algebra of linear endomorphisms. Matrices over non-commutative rings, the inverse for an upper unitriangular matrix.

6. The Gauss elimination over a field: finding a basis for a subspace $U \subset \Bbbk^n$ and for the quotient $\Bbbk^n/U$, computing the rank of matrix, finding the inverse matrix, solving systems of linear equations, finding bases in the kernel and image of a linear map. The combinatorial type of a subspace in $\Bbbk^n$ and uniqueness of a basis with a reduced echelon matrix of coordinates.

7. The dual space, examples of linear forms. The embedding $V \hookrightarrow V^{**}$, an advanced example: polynomials and power series, the umbral calculus. The isomorphism $V \simeq V^{**}$ for a finite-dimensional V, dual bases, coordinates of a linear form in the dual basis. The dual space to the linear span of a given set of vectors, the rank of matrix revisited. The dual space to a subspace. Annihilators, the bijection $U \leftrightarrow \operatorname{Ann} U$ inverts the inclusions and transforms the sums to the intersections and vice versa. Dual linear mappings. The systems of linear equations revisited. Finding a basis and/or an affine frame in an intersection and sum of vector and/or affine subspaces.

8. The volume of an oriented parallelepiped, multilinear skew-symmetric and sign-alternating forms, the space of skew-symmetric n-linear forms on an n-dimensional space has dimension 1. The sign of a permutation, an exapmple: the sign of a shuffle permutation. The determinant of a matrix, its simplest properties: multilinearity, invariance under transposition, multiplicativity, the row and column expansions. Techniques for calculating determinants. The ratio of the volume of a simplex to the volume of a parallelepiped (over $\mathbb{R}$).

9. Cramer's rules, the adjunct matrix, the identity $A \cdot A^{\vee} = \det A \cdot E$. Matrices with elements in the algebra of polynomials = polynomials with coefficients in the algebra of matrices, the Cayley–Hamilton identity. Grassmannian polynomials over a field, linear changes of variables and minors, an expansion of a determinant via a set of rows or columns (the Laplase relations).

10. Eigenvalues and eigensubspaces of a linear operator. The characteristic and minimal polynomials. Decomposition of a space into a direct sum of invariant subspaces by the factorization of an annihilating polynomial. Criteria for diagonalizability. Root subspace decomposition. Evaluation of analytic functions at a linear operator via polynomial interpolation of jets at the eigenvalues. Nilpotent operators, a cyclic basis and the cyclic type of nilpotent linear operator. Commuting operators: common eigenvector and simultaneous diagonalization, working example: finite groups of linear transformations. The Jordan decomposition of a linear operator and the Jordan normal form of a matrix over an algebraically closed field.

[4] Linear Algebra and Geometry (2nd term), the National Research University 'Higher School of Economics', Faculty of Math., I year students, January-June 2021, 6 hours per week.
Site: `http://gorod.bogomolov-lab.ru/ps/stud/geom_ru/2021/list.html`.
Lecture notes:
`http://gorod.bogomolov-lab.ru/ps/stud/geom_ru/2021/lec_total.pdf`

Program.

1. Euclidean spaces: the Gram–Schmidt orthogonalization, orthogonals and orthogonal projections, Gram matrices and Gram determinants, Euclidean volume and orientation. Angles between hyperplanes and vectors, distances between affine subspaces. Cross-products. Orthogonal maps: the orthogonal group is generated by reflections, the canonical form of an orthogonal map. Adjoint linear maps: orthogonal diagonalization of a self-adjoint operator, the normal form of an anti-self-adjoint operator. The SVD-decomposition an the polar decomposition.

2. Convex geometry: convex bodies, supporting half-spaces, faces and extreme points, cylinders. Convex polytopes: enumeration of faces, the Farkaš lemma, the Minkowski–Weyl theorem, Motzkin's decomposition, the recession cone and the projective cone of convex polytope, the Gale duality.

3. Bilinear forms: the isometry group of a non-degenerated form, form-operator correspondence, two non-degenarated bilinear forms are equivalent iff their canonical operators are conjugated, left and right orthogonals and orthogonal projections. Diagonalization of symmetric bilinear form. The Darboux normal form for an anti-symmetric bilinear form..

4. Non-degenerated symmetric bilinear forms: Witt's lemma, the isometry group is generated by reflections, orthogonal diagonalization of self-adjoint operators. Quadratic forms: polarization, diagonalization, the hyperbolic and anisotropic components, the signature of a real quadratic form, quadratic forms over finite fields of odd characteristic.

5. Non-degenerated anti-symmetric forms: Lagrangian subspaces, the symplectic group. Grassmannian quadratic forms, the Plücker relations. Pfaffians.

6. Projective spaces: affine charts, local affine and global homogeneous coordinate, topology of real and complex projective spaces of small dimension. Projective subspaces: intersections, complementary subspaces, projections, projective duality. Projective quadrics: tangent spaces and polars, smooth and singular points. Plane conics. Projective hypersurfaces, the projective closure of an affine hypersurface, spaces and linear systems of hypersurfaces. Rational normal curves.

7. Projective transformations, homographies, perspectives, cross-axes. Linear fractional automorphisms of projective line: cross-ratio, harmonic relation, involutions, fixed points. Pascal's theorem. Drawings with one ruler.

8. Smooth projective quadrics: the polar transformation, the conjugation relation, the dual quadric, hyperplane sections, linear subspaces lying on a smooth quadric. The classification of real and complex projective quadrics. Pencils of quadrics: the classification of pencils of conics, simple and regular pencils, projective equivalence of pencils, simultaneous diagonalization of all quadrics in a pencil.

9. Conformal structure on a real affine plane. Conformal geometry of real conics: ellipses, hyperbolas, parabolas, foci, directrices, principal axes, directors. Focal properties and confocal families.

10. Affine and projective linear groups. Classification of affine quadrics: central quadrics, paraboloids, simple cones, cylinders. Topology of real quadrics. Euclidean classification of quadrics.

11. Spheres: inversions, stereographic projections, radical axes, degrees of points. The space of spheres, pencils of spheres, orthogonal geometry and reflexions in the space of spheres. The Möbius groups $\mathbb{P}O(n,1) = M(S^{n-1}) = M(\mathbb{R}^{n-1})$.

3.3.6 Alexander Kalmynin

[1] Pretentious multiplicative number theory. Independent University of Moscow, III+ year students, February-May 2021, 2 hours per week

Pretentious multiplicative number theory is an area of analytic number theory based on the fact that many interesting situations in number theory can be described in terms of similarity between two different multiplicative functions. For example, potential existence of Siegel zeros indicates that certain quadratic characters are close to the Liouville function. This course is devoted to the presentation of basics of pretentious number theory and its most noteworthy applications, from theorems on character sums to the recent proof of Erdős discrepancy problem.

1. Multiplicative functions, Dirichlet convolutions, Dirichlet series. Distribution of primes, Riemann zeta-function. Dirichlet characters, L-functions and primes in arithmetic progressions.
2. Basics of pretentious multiplicative number theory: averages of bounded multiplicative functions, Granville-Soundararajan distance, theorems of Halsz, Delange and Wirsing.
3. Large values of character sums, Pólya-Vinogradov inequality and its improvements, additive problems with multiplicative functions.
4. Short sums of multiplicative functions, correlations of shifts of multiplicative functions and Chowla conjecture.
5. Erdős discrepancy problem.

[2] The Riemann zeta-function. Independent University of Moscow, III+ year students, September-December 2021, 2 hours per week.

Program

Many classical and modern problems in number theory can be reduced to properties of the Riemann zeta-function and its analogues, such as Dirichlet L-functions. This is partially explained by the fact that Dirichlet series are the most natural type of generating functions for sequences with multiplicativity property. In this course, we will mostly concentrate on the Riemann zeta-function, its connection with prime numbers and analytical properties, such as Voronins universality.

1. Summation by parts. Dirichlet series. Zeta-function, its analytic continuation and functional equation. Dirichlet L-functions. Ramanujan τ function.*
2. Perrons formula. Dirichlet divisor problem and its analogues. Voronoi summation formula.*
3. Zeta and primes. Zero-free region, Riemann-von Mangoldt formula. Riemann hypothesis. Correlations of zeros and random matrices.*
4. Approximate functional equation, averages of Dirichlet series and zero density estimates. Density hypothesis. Prime numbers in short intervals. Selbergs theorem on gaps between primes.*

5. Large values of zeta-function. Voronins universality theorem. Random holomorphic functions.*

3.3.7 Konstantin Loginov

[1] Introduction to birational geometry, Steklov Mathematical Institute, fall 2021, 2 hours per week.

1. Overview of the Minimal model program.
2. Birational morphisms, intersection form on a surface, adjunction formula, blow up and blow down of (-1)-curves, Castelnuovo criterion.
3. Examples of surfaces: minimal surfaces, ruled surfaces (e.g. Hirzebruch surfaces), rational surfaces and del Pezzo surfaces.
4. Minimal model program for surfaces: contraction theorem, cone theorem, rationality theorem. Structure of the Mori cone.
5. Singularities of surfaces. Du Val singularities, quotient-singularities. Minimal resolution of singularities. Discrepancy, classes of singularities: lc, klt, canonical and terminal.
6. Gorenstein, factorial, $\mathbb{Q}$-Gorenstein, $\mathbb{Q}$-factorial singularities. Singularities of cones.
7. Higher dimensions: examples of small contractions, flips and flops. Equivalence of existence of flips and finite generation of a canonical algebra.
8. Kollár-Shokurov connectedness principle. Inversion of adjunction.

[2]-[3] Algebraic Geometry, Start-up course, Math in Moscow Program, spring 2021, fall 2021. Lectures and exercise classes, 3 hours per week.

1. Affine space, algebraic subsets, Zariski topology, Noetherian rings, Hilbert's basis theorem.
2. Regular functions, affine and quasi-affine varieties, isomorphism of varieties, pullback on regular functions.
3. Hilbert's Nullstellensatz (weak and strong forms). Points as maximal ideals.
4. Projective and quasi-projective varieties. Image of a projective variety is closed. Segre and Veronese morphisms.

5. Rational maps and a field of rational functions. Rationality.
6. Irreducible varieties. Dimension.
7. Plane curves and their singularities. Tangent space and singular points in general.
8. Finite morphisms. Normal varieties.
9. Geometry of projective quadrics. Quadratic forms and polarizations.
10. Geometry of the Plücker quadric. Geometry of lines in projective space. Cell decomposition.
11. Young diagrams and cell decomposition for arbitrary Grassmannians.

3.3.8 Sergei Lvovski

[1] Riemann Surfaces. Math in Moscow program (jointly IUM and HSE). February–May 2021, 4 hours per week.

1. Definition of Riemann surface. Simplest examples.

2. Riemann surface corresponding to an algebraic equation. Statement of Riemann's existence theorem.

3. Genus of a compact Riemann surface. Mappings of compact Riemann surfaces. Ramification. Sum of orders of a meromorphic function. Riemann–Hurwitz formula.

4. Holomorphic and meromorphic forms: definitions. Sum of orders of a meromorphic form. Residues. Sum of residues.

5. Divisors. The space $L(D)$. Linear equivalence. The canonical class. Riemann–Roch inequality. Mittag-Leffler problem. Statement of the Riemann–Roch theorem.

6. Adèles. Adèles on a compact Riemann surface and proof of Riemann–Roch theorem after A.Weil (modulo Riemann's existence theorem).

7. Linear systems. Holomorphic mapping corresponding to a free linear system. Interpretation in terms of line bundles. Canonical linear system and canonical curves.

8. Wronskians and inflection points of linear systems. Weierstrass points.

9. Poincaré residues. Residue of an n-form on an n-dimensional complex manifold, with a simple pole along a smooth hypersurface. Application to smooth plane curves. Extension to the case of nodal and cuspidal plane curves.

10. Weak Castelnuovo–De Franchis theorem. The Hurwitz bound.

11. Period matrix of a Riemann surfaces. The Abel–Jacobi mapping. Abel theorem, Jacobi theorem, Jacobi inversion. Theta divisor. Riemann's theta function. Theta divisor and its geometric interpretation. Torelli's theorem.

[2] Topology II. Math in Moscow program (jointly IUM and HSE). September–December 2021, 4 hours per week.

1. Chain complexes, homology, chain homotopy.
2. Abstract simplicial complexes and polyhedra.
3. Simplicial homology.
4. Singular homology, its isomorphism with simplicial homology for polyhedra.
5. Relative homology, Mayer–Vietoris sequence, excision.
6. Homology of classical spaces.
7. Topological manifolds. Degree of mapping.
8. Homology with coefficients; cohomology.

[3] Elements of field theory and Galois theory. Hisher School of Economics, Math department. September–December 2021, 4 hours per week.

1. Finite and algebraic extensions of fields. Extensions generated by one algebraic element.
2. Finite fields.
3. Algebraic closure.
4. Normal extensions.
5. Straightedge and compass construction.
6. Roots of unity, cyclotomic fields.
7. Galois froup, Galois correspondence.
8. Constructibility and non-constructibility of regular polygons.
9. Solvable and unsolvable groups. Criterion for solvabilty in radicals (without detailed proof).

3.3.9 Vladimir Medvedev

[1] Analytic geometry, People's Friendship University of Russia, I year students, February-April 2021, 2 hours per week.

Program.

I. Vector algebra. Vectors. Scalar, vector and mixed products of vectors. Length of a vector. Angle between two vectors. Equations of a line and a plane. Distance between a point and a line and a point and a plane. Euclidean space. Hypersurfaces.

II. Ellips, hyperbola and parabola. Metric classification of the curves of second order. Surfaces of second order and their classification.

[2] Linear Algebra. Higher School of Economics, I year students, January-May 2021, 4 hours per week.

Program

I. Introduction: sets, algebra of sets, maps between sets, injections, surjections, bijections, complex numbers, trigonimetric form of a complex number, De Moivre's formula, Fundamental Theorem of Algebra, quantifiers.

II. Systems of linear algebraic equations (SLAE): equivalent systems, consistent and inconsistent systems, geometric sense of a SLAE of two equations with two variables and of two equations with three variables, matrices and the matrix form of SLAE, vector-columns and vector-rows, vectors in $\mathbb{R}^n$, linear combinations of vectors, linear dependence and linear independence of vectors, rank and free variables, Kronecker-Capelli-Rouché Theorem, Gauss eliminations method, space of solutions of a SLAE.

III. Matrices and determinants: algebraic operation with matrices, matrix algebra, matrix equations, matrix exponential, invertible matrices, inverse matrix, Inverse Matrix Theorem, Gauss-Jordan algorithm of finding the inverse matrix, trace of a matrix, determinant of a matrix and its properties, methods and strategies of finding the determinant of a matrix, method of computation of the inverse matrix via the adjoint matrix, Cramer's rule, kernel and image of a matrix.

IV. Vector spaces and linear maps: vector space $\mathbb{R}^n$, abstract vector spaces, linear dependence and linear independence of vectors in abstract vector spaces, vector subspaces of a vector space, Theorem about a Vector Subspace of a Vector Space, vector spaces generated by vectors, basis and coordinates, dimension of a vector space, Rank Theorem, linear maps between vector spaces, kernel and image of a linear map, canonical matrix of a linear map, transformation matrix, isomorphisms of vector spaces, Theorem about isomorphic finite dimensional vector spaces.

V. Scalar products: scalar product of vectors in $\mathbb{R}^n$, abstract scalar products in vector spaces, length of a vector, distance between two vectors, Cauchy-Bunyakovski-Schwarz inequality, triangle inequality, orthogonal vectors, orthogonal and orthonormal basis, the orthogonal complement of vector subspace of a vector space with a scalar product, direct sum of vector spaces, orthogonal projection formula, Gram-Schmidt orthogonalization.

VI. Diagonalization and quadratic forms: eigenvectors and eigenvalues of a matrix, eigenspace, characteristic equation, Cayley-Hamilton theorem, similar matrices, diagonalizable matrices, Theorem about a Diagonalizable Matrix, Jordan's normal form of a matrix, symmetric matrices and their orthodiagonalizability, Spectral Theorem, bilinear and quadratic forms, definite and indefinite forms, canonical form of a quadratic form, Sylvester's criterion, Sylvester's law of inertia, Lagrange's method.

[3] Introduction to Modern Topology. Higher School of Economics, I-IV year students, January-May 2021, 4 hours per week.

Program

I. Intuitive topology.

II. Topology of subsets of $\mathbb{R}^n$.

III. Abstract topological spaces.

IV. Graphs.

V. Surfaces and their classification.

VI. Homotopy. Brower's fixed-point theorem.

VII. Vector fields.

VIII. Coverings and the fundamental group.

IX. The Kakutani fixed-point theorem.

[4] Non-Euclidean Geometry. Math in Moscow, I year students, February-May 2021, 2 hours per week.

Program

I. Toy geometries.

II. Abstract groups and group representations.

III. Finite subgroups of $SO(3)$ and the Platonic bodies.

IV. Discrete subgroups of the isometry group of the plane and tilings.

V. Reflection groups and Coxeter geometries.

VI. Spherical geometry.

VII. The Poicaré disk model of the hyperbolic geometry.

VIII. The Poicaré half plane model.

IX. The Cayley-Klein model.

X. Hyperbolic trigonimetry.

XI. Projective geometry.

XII. Finite geometries.

XIII. The hierarchy of geometries.

[5] Introduction to Geometric Analysis-II. Independent University of Moscow, III-V year students, February-May 2021, 2 hours per week.

Program

I. Basic facts of Riemannian geometry and partial differential equations: Riemann tensor, Ricci tensor, Weyl tensor, scalar curvature, mean curvature, injectivity radius, conformal metrics, change of curvature tensors under the conformal deformation of the metric, PDE on Riemannian manifolds, weak and strong solutions, Green's function, Sobolev spaces on Riemannian manifolds and their embeddings, the critical case in the Sobolev Embedding Theorem.

II. Positive Mass Theorem: Einstein's equations, variation of the scalar curvature, Hilbert-Einstein functional, Jacobi operator, nonpositive scalar curvature manifolds vs positive scalar curvature manifolds, minimal submanifolds and their stability, asymptotically flat manifolds, Arnowitt-Deser-Misner mass, the proof of the Positive Mass Theorem by Schoen-Yau.

III. Yamabe Problem: Yamabe equation and the critical case in the Sobolev Embedding Theorem, variational formulation of the Yamabe problem, Yamabe invariant, solution of the Yamabe problem in 2D and the Uniformization theorem, solution of the Yamabe problem in higher dimension by Aubin and Schoen.

[6] Calculus-II. Higher School of Economics, II year students, September-December 2021, 2 hours per week.

Program

I. Infinite series. Finite sums and products. Harmonic numbers. Convergent and divergent series. Examples of series: telescoping series, geometric series, decimal fractions, p-series, alternating series. Necessary condition for convergence. Integral test. Tail of a series. Series of Nonnegative Terms. Convergence Tests. Series of nonnegative terms. Comparison test. Limit comparison test. Ratio and Root tests, their relationship. Rate of convergence. Gauss test. Alternating series. Absolute and conditional convergence.

Cauchy criterion. Alternating series test. Dirichlet/Abel tests. Absolute and conditional convergence. Sine and cosine sums. Conditionally convergent alternating and trigonometric series. Products of series. Infinite Products. Rearrangement of series, Cauchy's and Riemann's theorems. Product of series, Cauchy products. Convergence and divergence of infinite products, reduction to series. Wallis product. Stirling's formula.

II. Double integrals. Riemann sums. Double integrals over rectangles. Lower and upper Darboux sums. Darboux criterion. Properties of double integrals. Fubini's theorem, reduction to iterated integrals. Double integrals over general regions. Change of variables in double integrals. Polar coordinate system. Triple integrals. Applications of double and triple integrals. Fubini's theorem, reduction to iterated integrals. Change of variables in triple integrals. Cylindrical and spherical coordinate systems. Calculating areas of domains, volumes of solids, areas of surfaces. Improper integrals. Multiple integrals. Exhaustions. Improper double integrals.

III. Uniform convergence. Sums of functions. Pointwise and uniform convergence of functional sequences and series, their relationship. Cauchy criterion for the uniform convergence. Tests for the uniform convergence of series: alternating series test, Weierstrass M-test, Dirichlet/Abel tests. Interchange of limits, continuity of a limit function. Term-by-term integration and differentiation of uniformly convergent series. Riemann zeta function.Power series. Examples of power series. Radius and interval of convergence of power series. Cauchy-Hadamard formula. Uniform convergence of power series. Term-by-term differentiation and integration of power series. Abel's theorem. Products of power series. Uniqueness of power series expansion. Taylor series of common functions. Binomial series. Analytic functions. Complex power series. Euler's formula.Fourier series. Trigonometric series. Fourier coefficients and Fourier series. Parseval's identity. Piecewise functions. Riemann-Lebesgue lemma. Dirichlet kernel. Pointwise convergence of the Fourier series of a 2π-periodic piecewise continuously differentiable function. Applications of Fourier series.

[7] Advanced Riemannian Geometry. Higher School of Economics, III-V year students, September-December 2021, 2 hours per week.

Program

I. Theory of geodesics in Riemannian manifolds.

II. Complete Riemannian manifolds.

III. Riemannian manifolds of nonpositive curvature.

IV. Comparison theorems.

V. Quarter-pinched theorem.

[8] Theory of Minimal Submanifolds-I. Independent University of Moscow, III-V year students, September-December 2021, 2 hours per week.

Program

I. Basic facts of Riemannian geometry and partial differential equations: Riemann tensor, Ricci tensor, scalar curvature, mean curvature, Gauss equation, injectivity radius, conformal metrics, change of curvature tensors under the conformal deformation of the metric, PDE on Riemannian manifolds, weak and strong solutions, Green's function, Sobolev spaces on Riemannian manifolds and their embeddings, the critical case in the Sobolev Embedding Theorem, elliptic regularity, Harnack's inequality, spectrum of a Riemannian manifold.

II. Foundation of the theory of minimal submanifolds: volume functional vs energy functional, harmonic maps, minimal submanifolds of the Euclidean space and the standard spheres, minimal graphs, Gauss map, Bernstein theorem, Plateau problem.

III. Stability theory of minimal submanifolds: second variation of the volume functional, Jacobi operator, Morse index of a minimal submanifold, Fisher-Colbri theorem about the index of a minimal hypersurface, Schoen-Yau theorem about the stable minimal hypersurface, Fisher-Colbri-Schoen theorem, Barbosa-do Carmo theorem about stable domains, minimal submanifolds with ends, Cheng-Tysk theorem, stables cones and their applications.

IV. Minimal submanifolds of higher codimension: Kähler geometry, Virtinger's inequality, special lagrangian submanifolds, Bernstein theorem in higher codimension, harmonic maps in grassmanians, calibrations.

3.3.10 Taras Panov

[1] Cobordism and torus actions, Independent University of Moscow, advanced course, September-December 2021, 2 hours per week.

Program

1. Bordism of manifolds.
2. Thom spaces and cobordism functors.
3. Oriented and complex bordism.
4. Characteristic classes and numbers.
5. Structure results and generators of the cobordism rings.
6. Elements of the theory of formal group laws.
7. Formal group law of geometric cobordisms.
8. Hirzebruch genera (complex case).
9. Landweber exact functor theorem.
9. Hirzebruch genera (oriented case). Elliptic cohomology.
10. Geometric and homotopic equivariant complex bordism.
11. The universal toric genus.

[2] Linear algebra and geometry, Faculty of Mathematics and Mechanics, Moscow State University, I year students, February–May 2021, 4 hours per week.

Program:

1. Vector spaces.
2. Linear operators.
3. Geometry of Euclidean and Hermitian spaces.
4. Operators in Euclidean and Hermitian spaces.
5. Bilinear and sesquilinear functions.

6. Tensors.
http://higeom.math.msu.su/people/taras/teaching/panov-linalg.pdf

[3] Introduction to topology, Faculty of Mathematics and Mechanics, Moscow State University, II year students, September–December 2021, 2 hours per week.

Program:
1. Necessary facts from point-set topology.
2. Operations on topological spaces.
3. Homotopy and homotopy equivalence.
4. Cellular (CW) complexes.
5. Fundamental group.
6. Van Kampen Theorem.
7. Fundamental group of CW complexes.
8. Coverings.

http://higeom.math.msu.su/people/taras/teaching/panov-topology1.pdf

3.3.11 Alexei Penskoi

[1] Differential Geometry. National Research University — Higher School of Economics, II year students, February-May 2021, 4 hours per week (lecture 2 hours + exersise class 2 hours).

Program

1. Curves and surfaces in the plane and the three-dimensional space. Curvature, torsion, Frenet frame. First and second fundamental forms. Principal curvatures, mean curvature and Gauß curvature. Mean curvature normal vector. Euler formula for the normal section curvature.

2. Surfaces in n-dimensional space. First and second fundamental forms. Connections in the tangent and normals bundles on a surface. Second fundamental form and Weingarten operator. Gauß-Weingarten derivational equations. Gauß-Bonnet theorem for surfaces.

3. Basic theory of Lie groups ans algebras.

4. Vector bundles and gluing cocycles. Structure group. Euclidean and hermitian bundles. Natural operations with bundles. Orientable bundles.

5. Connections in vector bundles. Connection local form, Christoffel symbols. Connections in euclidean and hermitian bundles. Connections compatible with metrics and their curvature.

6. Riemannian manifolds. Curvature, torsion. Levi-Civita connection. Symmetries of curvature tensor. Ricci tensor. Scalar curvature.

7. Riemannian manifolds II. Geodesics. Geodesic coordinates. Lagrangian approach to geodesics. Second variation.

8. Submanifolds of Riemannian manifolds. First and second fundamental forms.

9. Characteristic classes. Chern-Weil construction of characteristic classes. Chern, Pontryagin and Euler classes and their properties.

10. Principal bundles

[2] Exercise classes for Topology-I at the National Research University — Higher School of Economics: January-March 2021, 2 hours per week (2 hours exercise classes).

[3] Differential Geometry. Independent University of Moscow, II year students, February-May 2021, 4 hours per week (lecture 2 hours + exersise class 2 hours).

Program

1. Curves and surfaces in the plane and the three-dimensional space. Curvature, torsion, Frenet frame. First and second fundamental forms. Principal curvatures, mean curvature and Gauß curvature. Mean curvature normal vector. Euler formula for the normal section curvature.

2. Surfaces in n-dimensional space. First and second fundamental forms. Connections in the tangent and normals bundles on a surface. Second fundamental form and Weingarten operator. Gauß-Weingarten derivational equations. Gauß-Bonnet theorem for surfaces.

3. Basic theory of Lie groups ans algebras.

4. Vector bundles and gluing cocycles. Structure group. Euclidean and hermitian bundles. Natural operations with bundles. Orientable bundles.

5. Connections in vector bundles. Connection local form, Christoffel symbols. Connections in euclidean and hermitian bundles. Connections compatible with metrics and their curvature.

6. Riemannian manifolds. Curvature, torsion. Levi-Civita connection. Symmetries of curvature tensor. Ricci tensor. Scalar curvature.

7. Riemannian manifolds II. Geodesics. Geodesic coordinates. Lagrangian approach to geodesics. Second variation.

8. Submanifolds of Riemannian manifolds. First and second fundamental forms.

9. Characteristic classes. Chern-Weil construction of characteristic classes. Chern, Pontryagin and Euler classes and their properties.

10. Principal bundles

[4] Calculus on manifolds. "Math in Moscow" program at the Independent University of Moscow & NRU HSE for undergraduate students from the U.S. and Canada, February-May 2021, 4 hours per week (lecture 2 hours + exersise class 2 hours).

Program

1. Definition and examples of smooth manifolds.

2. Orientability and orientation.

3. Tangent vectors and tangent space to a manifold at a point. Tangent bundles. Vector fields.

4. Skew-symmetric forms on linear spaces. Wedge product.

5. Differential forms on manifolds. Exterior differential.

6. Smooth maps of manifolds. Diffeomorphisms. The transformation rule under coordinate change for functions, vector fields and differential forms.

7. Integration. Coordinate change in the integral. Integration of differential forms. Stokes theorem. Green's formula, Gauss-Ostrogradskii divergence theorem, Stokes formula for a surface in $\mathbb{R}^3$.

8. Closed and exact forms. The Poincare lemma. De Rham cohomology.

3.3.12 Petr Pushkar

[1] Complex Analysis. Independent University of Moscow, 2 year students, January-May 2021, 4 hours per week.

Program

1. Complex-valued functions. Holomorphic functions. CauchyRiemann equations.
2. Holomorphic forms. Cauchy Theorem. Expansion as a convergent power series.
3. Meromorphic functions. Loran series.
4. Maximum principle, Cauchy's argument principle, open mappings.
5. Residues.
6. Isolated singlar points. The CasoratiWeierstrass theorem
7. Schvarz lemma. Automorphisms.
8. Uniformization theorem.

9. Holomorphic and harmonic functions.

10. Riemmanian surfaces. Elements of elliptic functions theory. Abel theorem.

[2] Ordinary Differential Equations, 2 year students, September-December 2021, 4 hours per week.

Program

1. Tangent vectors, Vector fields
2. Main theorems
3. Linear systems and equations, exponential function, Quasi-polynomials
4. Linearization and Lyapunov stability
5. Obstruction to integrability of distributions
6. First-order differential equations
7. Symplectic and Contact structures and differential equations
8. Examples.

[3] Calculus and Topology and Differential Geometry classes at HSE. Lectures and exercises.

3.3.13 Alexander Samokhin

[1] Complex Analysis. Independent University of Moscow, 2 year students, January-May 2021, 4 hours per week.

Program

1. Complex-valued functions. Holomorphic functions. CauchyRiemann equations.
2. Holomorphic forms. Cauchy Theorem. Expansion as a convergent power series.
3. Meromorphic functions. Loran series.
4. Maximum principle, Cauchy's argument principle, open mappings.
5. Residues.
6. Isolated singlar points. The CasoratiWeierstrass theorem
7. Schvarz lemma. Automorphisms.
8. Uniformization theorem.
9. Holomorphic and harmonic functions.
10. Riemmanian surfaces. Elements of elliptic functions theory. Abel theorem.

[2] Ordinary Differential Equations, 2 year students, September-December 2021, 4 hours per week.

Program

1. Tangent vectors, Vector fields
2. Main theorems
3. Linear systems and equations, exponential function, Quasi-polynomials
4. Linearization and Lyapunov stability
5. Obstruction to integrability of distributions
6. First-order differential equations

7. Symplectic and Contact structures and differential equations

8. Examples.

[3] Calculus and Topology and Differential Geometry classes at HSE. Lectures and exercises.

3.3.14 George Shabat

[1] Algebraic curves. Independent University of Moscow, 3-5 year students, September-December 2021, 2 hours per week.

Program

1. Brief history

1.0. Pythagorean triples in Babylon
1.1. Conic sections in Ancient Greece
1.2. Newton's classification of cubic curves
1.3. Plane projective curves in 19-th century
1.4. Abelian integrals
1.5. Riemann's moduli and Hurwitz spaces

2. Plane curves 2.0. Irreducible curves and fields of rational functions on them
2.1. Birational isomorphism
2.2. Cremona group
2.3. Singularities
2.4. Quadratic transformations
2.5. Dual curves
2.6. Inflexions and cusps
2.7. Curves of small degree

3. Curves in projective spaces
3.0. Smooth models of function fields
3.1. Resolution of curve singularities
3.2. Projections of projective curves
3.3. Smooth curves in 3-space
3.4. Hilbert polynomials

4. Intrinsic theory of curves
4.0. Divisors and linear bundles
4.1. Vector fields and differentiations
4.2. Differentials and canonical class
4.3. Genus
4.4. Picard group

4.5. Canonical and hyperelliptic curves
4.6. Petri theorem

[2] Dessins d'enfants and number theory, Steklov Mathematical Institute, 2-3 year students, September-December 2021, 2 hours per week.
0. Introduction.

Visualization of algebro-geometric objects. Model example: plane trees and Shabat polynomials.The action of absolute Galois group on the plane trees and its faithfillness.

Program.

1. Belyi theory. Transformations of the finite sets $B \subset \overline{\mathbb{Q}}$by the polynomials $P \in \mathbb{Q}[t]$ $B \mapsto P(B) \cup \mathrm{CritVal}(P)$ where $\mathrm{CritVal}(P)$ – is the set of critical values of a polynomial P. Application to the construction of a rational function on an arbitrary curve over the field of algebraic numbers (*Belyi funtions*). Belyi height.

2. Dessins d'enfants theory. Depiction of Belyi functions by the graphs on surfaces. Equivalence between the combinatorial-topological and arithmetico-geometric categories. Edge rotation groups and other Galois invariants. Generalizations.

3. Relations with modular curves. The Belyi tower and the universal cover $\mathcal{H} \to \frac{\mathcal{H}}{\Gamma(2)} \simeq \ddot{\mathbb{C}}$, where $\ddot{\mathbb{C}} = \mathbb{C} \setminus \{0, 1\} \cong \mathbf{P}_1(\mathbb{C}) \setminus \{0, 1, \infty\}$. Familiesof elliptic curves with the level structure and Belyi functions om their bases.

4. Various problems of number theory. Quadratic fields of definitions of plane trees (by N. Adrianov). Polynomial Pell equations (by N.-H. Abel) and families of extensions of quadratic irrationalities into the periodic chain fraction. Isogenies of the elliptic curves and the sublattices of square lattices.

5. Relations with Fermat theorem. Brief history. Equation $x^7 + y^7 = z^7$ (by N. Elkies). Generalized Fermat equations. Elliptic curves of small Belyi height and the equation $x^2 + y^3 = z^{10}$ (by D. Brown); the similarities with E. Wiles' proof.

3.3.15 Stanislav Shaposhnikov

[1] Calculus. Independent University of Moscow, II year students, September-December 2021, 4 hours per week.
Program
1. Inverse function theorem. Implicit function theorem.

2. Smooth surfaces. Tangent space.
3. Topological spaces. Manifold.
4. Smooth functions. Whitney embedding theorem. Sard theorem.
5. Smooth vector fields. Frobenius theorem. Lie theorem.
6. Hausdorff measure. Area formula.
7. Integrating by parts formula.
8. Differential forms. Stokes theorem.

3.3.16 Evgeni Smirnov

[1] Plane partitions and alternating sign matrices

Course outline:

1. Partitions. Generating functions and recurrence relations for partitions. Euler's pentagonal theorem.

2. Plane partitions. Lattice paths. Lindström–Gessel–Viennot trick. MacMahon's formula.

3. Plane partitions with additional symmetries. Symmetric and cyclically symmetric plane partitions. The MacMahon and Macdonald conjectures.

4. Symmetric functions. Schur functions, semistandard tableaux. Proof of the MacMahon conjecture.

5. Hypergeometric series. Proof of the Macdonald conjecture.

[2] Combinatorics, September–December 2021, Independent University of Moscow, 2 hours of lectures per week

Course outline:

1. Generating functions. Rational generating functions, linear recurrencies.
2. Catalan, Schroeder and Motzkin numbers.
3. q-binomial coefficients.
4. Euler's generating function for Young diagrams. Euler's pentagonal theorem and the triple Jacobi identity.
5. Dirichlet generating functions. Lagrange inversion.
6. Bernoulli-Euler triangle and power sums.
7. Partially ordered sets. Moebius function, Moebius inversion.
8. Lindstroem–Gessel–Viennot theorem. Determinant as the sum over nonintersecting families of paths.
9. Matrix identities: Binet–Cauchy formulas and the Lewis Carroll identity.
10. Matrix tree theorem. Kirchhoff's theorem.

3.3.17 Alexei Sossinsky

Knot Theory, IUM students, 3 hours per week, Spring 2021

Program:

1. Oriented knots and links: formal definition as closed polygonal curves, equivalence (= ambient isotopy), knot diagrams, Reidemeister moves, Conway axioms for the Alexander polynomial (without proof of existence), examples of computations.

2. Boxed knots, connected sum (=composition) of knots, the knot semigroup (commutativity and absence of inverse elements), prime knots, unique prime decomposition theorem (without proof).

3. Some simple knot invariants: stick number, crossing number, and unknotting number, tricolorability, Seifert surfaces and the genus of knots.

4. Kauffman bracket of nonoriented knots and links, its properties and its behavior w.r.t. Reidemeister moves.

5. Definition of the Jones polynomial via the Kauffman bracket, main properties, computation via that definition.

6. Axioms for the Jones polynomial, uniqueness, computation via the axioms, knot tables.

7. The braid group: geometric definition, braid generators, pure braids. Group presentations. Artin's theorem on the algebraic presentation of the braid group.

8. Properties of braids, braid ordering, decidability of the word and conjugacy problem for braids.

9. Closure of a braid, Alexande's theorem (any knot is the closure of some braid), Markov moves and Markov's theorem (without full proof).

10. Examples of finite type invariants. Thom-Arnold theory of discriminants, singular knots, the four-term relation, axioms for Vassiliev invariants, examples of computations.

11. The Kontsevich integral and some ideas underlying the proof of the existence of Vassiliev invariants.

3.3.18 Vladlen Timorin

[1] Calculus. Independent University of Moscow, I year students, September-November 2021, 2 hours per week.

Program

- Real numbers: axiomatic description. The nested interval property, the Archimedes axiom. Suprema and infima.

- Metric spaces, distance functions; examples (2D hyperbolic metric, metrics on graphs), induced metric spaces, products, complete metrics, the Contraction Mapping Principle.

- Open and closed subsets in metric spaces, closed vs. complete, continuity, connectedness, the Intermediate Value Theorem.

- Compactness. Motivation: the isoperimetric problem. Compactness vs. completeness. The interval $[0, 1]$ is compact, a product of two compacta is compact. Sequential compactness, ε-nets.

- Homeomorphisms. The Hausdorff metric. The Cantor set and its symbolic model. The Cantor staircase function; application: Peano curves. Uniform convergence, uniform continuity. Continuous extensions; application: the Riemann integral.

- Derivatives and their properties. Theorems of Rolle and Lagrange. The Newton–Leibnitz formula. Higher derivatives. Polynomial interpolation: Lagrange polynomials.

- The remainder term in Lagrange interpolation (possibly with multiple nodes). The Taylor formula and Taylor series. Power series, radius of convergence, the Cauchy–Hadamard formula. Term-wise differentiation and integration of power series.

- Dense sets, Baire's theorem. The Stone–Weierstrass approximation theorem. Approximation of continuous functions by polynomials.

- Approximation by trigonometric polynomials. Fourier series and their convergence; examples.

- Differential 1-forms on the line, integrals of 1-forms, differentials of functions, pullbacks of 1-forms, change of variables formulas in terms of 1-forms. The derivative of a composition and the 1D Inverse Function Theorem.

- The Euler–Maclaurin formula.

[2] Calculus, HSE University, I year students majoring in Biology and Biotechnology, September-December 2021, 6 hours per week.

Program.

- The linear and power functions, scaling laws.

- The natural logarithm function defined as an area under the graph of $1/x$. Properties of the log and log-plots.

- Exponential functions and their properties; finite geometric series; exponential growth.

- Integrals and derivatives; the Newton–Leibnitz formula.

- Changes of variables formulas for derivatives and definite integrals.

- Linear and non-linear models; discrete dynamical systems.
- Complex numbers, trigonometric form, de Moivre's formula.
- The "o" notation, the best linear and quadratic approximations. The complex derivative of the exponential function.
- Taylor polynomials and Taylor series; power series expansions for exp, sin, cos, log.
- Power series expansions for x^α, the Binomial Theorem.
- Maxima and minima, partial derivatives.
- The least squares method.

3.3.19 Vladimir Zhgoon

[1] Lie groups and algebras. Independent University of Moscow Spring semester. 2021

1 Basic definitions and examples: Lie group, Lie subgroup, homomorphism, representation and action of a Lie group.
2 Orbits and stabilizers. Introduction of a smooth structure on a set of adjacent classes. Quotient group.
3 Left-invariant and right-invariant tensors on the Lie group. The existence of an invariant form of volume and metric on a compact Lie group.
4 Various definitions of the Lie algebra corresponding to a given Lie group. Attached view.
5 Tangent homomorphism and tangent representation. Existence and uniqueness theorems for Lie group homomorphisms. Exponential mapping. Description of connected Lie groups with a given Lie algebra.
6 The main classes of Lie groups and algebras are: solvable, nilpotent, compact, simple, semisimple.
7 Engel's theorem, Lie's theorem. Cartan solvability criterion.
8 Cartan subalgebras and their properties.

Literature 1) E.B. Vinberg, A.L. Onishchik. Seminar on Lie groups and algebraic groups, URSS, M., 1995.

[2] Advanced Algebra. Independent University of Moscow, Math in Moscow, Spring 2021, 2 hours per week.

Program

1. Basic group theory. Cosets, quotients, normal subgroups.
2. Existence of the elements of prime order.
3. Actions of the finite groups on sets. Orbits, stabilizers.
4. Actions of p-groups on finite sets and the number of fixed points.
5. Sylow's theorems. Existence, conjugacy, number of Sylow subgroups.
6. Simple groups. Solvable groups. Nilpotent groups.
7. Algebraic extensions of fields.
8. Separable field extensions. Normal extensions.
9. Galois extensions. Galois correspondence.
10. Solvability of roots of the polynomials in radicals.
11. Complements to the course on basic representation theory.
12. Irreducible representations. Schur's lemma.
13. Semisimple algebras and modules.
14. Group algebra. The Maschke theorem.
15. Application of the theory of characters to the structure of finite groups. Solvability of the group of order pnqm

[3] Deformation theory with the view of Mori theory. Higher school of economics. Spring 2021

1) Deformation theory. Deformations of different objects: schemes, sheaves, morphisms etc. Tangent spaces to the space of deformations. Infinitesimal obstructions.
2) Hilbert, Quot, Hom and Chow schemes.
3) Applications to the spaces of rational curves. Bend and break technique.
4) Multiplier ideals. Kawamata-Viehweg vanishing theorem. Shokurov non-vanishing and base-point-freeness theorem. Mori cone theorem.
5) Fulton-Hansen connectedness theorem and its applications to geometry of projective varieties. Zak theorems.

[4] Lie groups and algebras. Independent University of Moscow
Lie groups and algebras. Fall Semester 2021

1 Structural theory of semisimple Lie algebras: Cartan subalgebra, Cartan-Killing form, root system, Weyl group, Cartan matrix, Dynkin scheme.
2 Classification of simple Lie algebras. Chevalley generators, Serre relations. Isomorphisms of small dimensions.
3 A theorem on invariants of groups generated by reflections.
4 Universal enveloping algebra. The Poincare-Birkhoff-Witt theorem. Casimir operator.
5 Levy and Maltsev theorems.
6 Cohomology of Lie algebras. Interpretation of the first and second groups of cohomology.

7 Weyl's formula for the character of an irreducible finite-dimensional representation of a semisimple Lie algebra.

8 Jacobson-Morozov theorem, classification of nilpotent elements.

9 The Kostant and Slodowy slice. The cone of nilpotent elements. Resolution of the closure features of a nilpotent orbit.

Literature 1) E.B. Vinberg, A.L. Onishchik. Seminar on Lie groups and algebraic groups, URSS, M., 1995. 2) J.-P. Serre. Lie algebras and Lie groups. Mir, M., 1969.

[5] Introduction to algebraic numbers and class field theory. HSE, Fall 2021.

1. Adels and idels.
2. An adelic interpretation for the Dirichlet theorem on units and for the theorem on finiteness of a group of classes of ideals.
3. Galois Cohomology. Hilbert's theorem 90.
4. Non-Abelian cohomology and torsors.

   item Shapiro Lemma. The Tate-Nakayama theorem.
5. Brauer group. Central simple algebras.

   item Filtering the Galois group by ramification subgroups.
6. Norm map.
7. Local class field theory.
8. Local characters. The reciprocity laws.
9. The global class field theory.

Yulij Ilyashenko
President of the Independent University of Moscow