REPORT ON DOBRUSHIN PROFESSOR FELLOWSHIP 2017

IOSIF KRASIL'SHCHIK

Papers

[1] With P. Holba, O.I. Morozov, P. Vojčák

2D reductions of the equation $u_{yy} = u_{tx} + u_y u_{xx} - u_x u_{xy}$ and their nonlocal symmetries

Accepted by J. of Ninlinear Math. Phys.

We consider the 3D equation $u_{yy} = u_{tx} + u_y u_{xx} - u_x u_{xy}$ and its 2D symmetry reductions: (1) $u_{yy} = (u_y + y)u_{xx} - u_x u_{xy} - 2$ (which is equivalent to the Gibbons-Tsarev equation) and (2) $u_{yy} = (u_y + 2x)u_{xx} + (y - u_x)u_{xy} - u_x$. Using the corresponding reductions of the known Lax pair for the 3D equation, we describe nonlocal symmetries of (1) and (2) and show that the Lie algebras of these symmetries are isomorphic to the Witt algebra.

Accepted by Theor. and Math. Phys.

[2] With H. Baran, O.I. Morozov, P. Vojčák

Nonlocal symmetries of integrable linearly degenerate equations: a comparative study

We continue here the study of Lax integrable equations. We consider four 3D equations: (1) the rdDym equation $u_{ty} = u_x u_{xy} - u_y u_{xx}$, (2) the Pavlov equation $u_{yy} = u_{tx} + u_y u_{xx} - u_x u_{xy}$; (3) the universal hierarchy equation $u_{yy} = u_t u_{xy} - u_y u_{tx}$, and (4) the modified Veronese web equation $u_{ty} = u_t u_{xy} - u_y u_{tx}$. For each equation, and expanding the known Lax pairs in formal series in spectral parameter, we construct two differential coverings [**?**] and give a full description of nonlocal symmetry algebras associated to these coverings. For all the four pairs of coverings, the obtained Lie algebras of symmetries manifest similar (but not the same) structures: the are (semi) direct sums of the Witt algebra, the algebra of vector fields on the line, and loop algebras; all of them contain a component of finite grading. We also discuss actions of recursion operators on shadows of nonlocal symmetries.

[3] With A.M. Verbovetsky and R. Vitolo

The symbolic computation of integrability structures for partial differential equations

Accepted by Springer Texts & Monographs in Symbolic Computation

We present a unified mathematical approach for the symbolic computation of integrability structures of partial differential equations, like Hamiltonian operators, recursion operators for symmetries and cosymmetries, symplectic operators. The computations are carried out within the computer algebra system `Reduce` by the packages `CDE` and `CDIFF`.

Scientific conferences and seminar talks

[1] International Conference "Physics and Mathematics of Nonlinear Phenomena (PMNP2017)", Gallipoli (Italy), June 2017, 17–24

Talk "Nonlocal symmetries of integrable linearly degenerate equations: a comparative study"

[2] International Conference "Geometry and Algebra of PDEs", Tromsø, (Norway), June 2017, 6–10

Talks

(1) Toward a geometry of nonlocal Hamiltonian structures (with A. Verbovetsky),
(2) Valentin Lychagin. Amarcord (sentimental recollections)

Teaching

[1] Calculus I, Russian State University for the Humanities, I year students, September–December 2017, 4 hours per week.

Program.

(1) Sets and maps.
(2) Real numbers, Euclidean plane, Cartesian coordinates polar coordinates.
(3) Limits and their properties. The "remarkable limits" $\lim_{x\to 0}\frac{\sin x}{x}$ and $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$.
(4) Functions in one real variable. Continuity. Basic properties of continuous functions. Continuity of elementary functions.
(5) Geometrical and dynamical meaning of the first derivative. The rigorous definition. First differential.
(6) Basic properties of first derivative.
(7) Derivatives of of the composite and inverse functions.
(8) Derivatives of the elementary functions.
(9) Higher derivatives. The Taylor formula.
(10) L'Hôpital's rules.
(11) The Newton method of solving equations $f(x) = 0$.
(12) Derivatives and extrema.
(13) Drawing graphs of functions using differential calculus.

[2] Calculus I, Russian State University for the Humanities, I year students, February–June 2017, 4 hours per week.

Program.

(1) Areas
(2) The fundamental theorem of calculus
(3) Indefinite integrals
(4) The substitution rule
(5) Trigonometric integrals
(6) Trigonometric substitution
(7) Integration of rational functions
(8) Integration by parts
(9) Definite integrals
(10) Applications of integration
(11) Areas between curves

(12) Volumes
(13) Arc length
(14) Approximate integration
(15) Improper integrals

[3] Ordinary differential equations, Russian State University for the Humanities, II year students, October–December 2017, 4 hours per week.

Program.

(1) Problems in geometry and mechanics leading to ODEs.
(2) General definition of ODEs and their solutions. Types of solutions.
(3) The phase portrait and isolines.
(4) Initial data and Cauchy problem. The uniqueness and existence theorem.
(5) First-order equations.
    (a) Separation of variables.
    (b) Homogeneous equations.
    (c) Inhomogeneous equations.
    (d) Equations in total differentials. Integrating factor.
(6) Second-order equations.
    (a) Equations $f(x, y', y'') = 0$.
    (b) Equations $f(y, y', y'') = 0$.
(7) Linear equations of arbitrary order.
    (a) Homogeneous equations. Fundamental set of solutions. Wronskian.
    (b) Solving inhomogeneous equations by the Lagrange method.
    (c) Equations with constant coefficients. The characteristic polynomial.
        (i) The case of simple real roots.
        (ii) The case of multiple real roots.
        (iii) The case of simple complex roots.
        (iv) The case of multiple complex roots.
(8) Solving ODEs by means of power series.