“Young Russian Mathematics” award
Scientific report for 2018
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1. Scientific Results and Papers

1.1. Cross-frequency interactions in human brain

The primary result of year 2018 is the final paper [4] of three year long research on cross-frequency interactions in human brain. In collaboration with Vadim Nikulin (Max Planck Institute for Human Cognitive and Brain Sciences, Leipzig, and HSE, Moscow), Boris Gutkin (ENS Paris and HSE, Moscow), and two Master students, Igor Dubinin (MIPT, Moscow) and Alexandra Myasnikova (HSE, Moscow), we carried out a project in which we developed a new method for finding synchronized activity in human brain cortex based on electro- and magnetoencephalography (E/MEG) recordings.

In 2012, Nikulin et al. [2] proposed a method for finding cross-frequency phase synchronized components in multi-channel E/MEG signal. This method could treat only integer frequency ratios, i.e., $f_1 : f_2 = 1 : q$, which significantly limited the applicability of the method. We developed a new mathematical model which extended the range of uses to any rationally related frequency bands f_1 and f_2, $f_1 : f_2 = p : q$, p, q are integers. In practice, the numbers p, q for biological resonances $p : q$ are typically within $p, q \leq 5$. Numerical experiments with synthetic EEG signals show that the new method works extremely reliably in this p, q range. For $p = 1$, the core idea of the new method reduces to the one used in [2] but we also made several improvements on the top of this.

Then we developed a MATLAB toolbox which allows to apply the new method to E/MEG signals from behavioral, cognitive or any other experiments. The open source code is available at https://bitbucket.org/dsvolk/rhythms-xfreq

The algorithm is well-tested on simulated data and it is now ready to use. It was also significantly impoved and optimized since it was first made available in September 2017.

The algorithm was applied to two kinds of experimental data: Resting State and Steady State Visual Evoked Potentials. In the former experiment, the subject is resting with their eyes closed which typically elicits a rhythm in prefrontal cortex within alpha frequency band. In the latter experiment, the subject is presented with a computer screen flickering at certain frequencies. These stimuli are known to evoke a steady rhythm in the visual cortex at a corresponding frequency. Our algorithm was able to successfully lock onto the rhythms and propose candidate synchronized rhythms in other parts of the brain, see [4].

2. New Projects and Collaborations

Since Fall 2017, my scientific interest gradually shifted towards Bayesian Methods in Machine Learning and Statistical Hypothesis Testing. I participated in DeepBayes: Summer School on Deep Learning and Bayesian Methods and in the scientific seminar on Bayesian methods led by D. Vetrov at HSE and Samsung Research. My current ambition is to apply these methods to datasets coming from real-life applications.

3. Participation in Schools and Conferences

- Conference OpenTalks.AI, Moscow, Russia. Feb 2018

- Conference DataFest, Moscow, Russia. Apr 2018
- Conference Machines Can See, Moscow, Russia. Jun 2018
- School DeepBayes: Summer School on Deep Learning and Bayesian Methods, Moscow, Russia. Aug 2018
- Conference Sberbank Data Science Day, Moscow, Russia. Oct 2018

4. Teaching

In academic year 2017–2018 I was supervising final Master theses of two students, Igor Dubinin from MIPT and Alexandra Myasnikova from HSE who took part in the research project 1.1 and co-authored the paper [4] as a part of their theses.

I also gave an invited lecture on mathematical models in neuroscience at Kazan Federal University.

5. Final report for 2016–2018

5.1. Proposed project: Expanding Attractors

In the proposed project, we planned to investigate a new interesting class of non-invertible partially hyperbolic dynamical systems with one strongly unstable direction and one central. The class is of particular interest because it is a toy model for heat transfer dynamics on one-dimensional lattice of units with individual highly chaotic behavior.

In this class, the averaged “slow” dynamics has one attracting fixed point which marks the location of the “attractor” on the torus for the global system. This attractor is a region where the exponentially big portion of the torus mass is concentrated after sufficiently long time. The surprising feature of this example is that in the attractor, for the most of the initial conditions after some long but finite time the central vectors in the tangent space are expanded. Thus, since most of the mass spend most of their time in the attracting region, and the central vectors are likely to be expanded there, this suggested (but was far from formally implying) that the central Lyapunov exponents are positive in this example. We confirmed this effect in numerical experiments.

In this project we were going to prove this phenomenon rigorously, to explore the rich geometry behind the example (e.g., non-absolutely continuous foliations aka “Fubini nightmare”) and then to extend the results to a bigger open class of dynamical systems.

5.2. Overview of Work Done

5.2.1 Expanding Attractors

The primary goal of the proposed project was successfully completed.

We proved that our systems can (generically) exhibit a “sink” with all the Lyapunov exponents positive. This phenomenon seemed to be very counterintuitive at a first glance but it turned out to be related to the lack of absolutely continuity of the central foliation. Hereby we provided an explicit construction for a natural class of dynamical foliations which do not admit Fubini-type theorems.

We also obtained a detailed statistical description of our systems on “short”, “medium” and “long” time scales. As the number of iterations of the dynamical system tends to infinity, it converges to a stochastic process in a certain sense, and this stochastic process admits an explicit formulation in terms of the original system. This directly relates to the underlying idea that the system is a toy model for certain stochastic model from theoretical physics. The aforementioned results are published in paper [1].

5.2.2 Dynamics of Piecewise Translations

Piecewise translations (PWTs) and a wider class of maps, piecewise isometries, have many applications in computer science, machine learning and electrical engineering, see [3]. In dimension 1 invertible PWTs which preserve Lebesgue measure are interval exchange transformations (IETs) which are classic objects in ergodic theory but still attract a lot of interest. They have deep connections with polygon billiard maps, measurable foliations, translation flows, Abelian differentials, Teichmüller flows and other areas.

The study of piecewise isometries in 2 or more dimensions is still in its relative infancy. We made one of the first steps towards understanding the general PWTs in arbitrary dimension. Namely, we could show that *any* piecewise translation with $m = d+1$ pieces in $\mathbb{R}^d$ with rationally independent translation vectors is finite type, i.e., converges onto its "attractor" after finitely many iterations. The result is presented in preprint [3] along with some other results of ergodic type.

5.2.3 Cross-frequency interactions in human brain

When we completed the primary goal of the proposed project 5.1, our interest shifted from theoretical research in Dynamical Systems to more applied Computational Neuroscience and Neuroimaging. In a joint project with Vadim Nikulin, Boris Gutkin, and master students Igor Dubinin and Alexandra Myasnikova, we developed a new method for detection of cross-frequency neuronal interaction in human brain based on EEG recordings, see Subsection 1.1.

5.2.4 Bayesian Methods in Machine Learning

The dynamics of piecewise translations, see Subsection 5.2.2, are related to Markov Random Fields (MRFs). MRFs are high-dimensional joint probability distributions with known Bayesian-like relationships between some of the dimensions. The task of learning MRFs (i.e., calculating the Bayesian inference or obtaining a Maximum Likelihood (ML) estimate) is computationally intractable in general.

Welling [5] proposed a "herding" approach to learning MRFs which instead of computing the exact ML, introduces a dynamical system which (asymptotically) samples from the states of high likelihood. This dynamical system is in fact a piecewise translation map in a high-dimensional space. Our study of piecewise translation maps was motivated by this application.

Since then, our interest in Bayesian inference and Machine Learning transcended the study of toy examples, and we dedicated a bigger part of the last year to study this field with the ambition to apply the theory to real-world datasets, see Section 3.

References

[1] De Simoi J., Liverani C., Poquet C., Volk D., Fast–Slow Partially Hyperbolic Systems Versus Freidlin–Wentzell Random Systems. *Journal of Statistical Physics*, 14:1-30, Jul 2016

[2] Nikulin V., Nolte G., and Curio G., Cross-frequency decomposition: A novel technique for studying interactions between neuronal oscillations with different frequencies. *Clinical Neurophysiology*, 123(7):1353–1360, 2012.

[3] Volk D., Attractors of Piecewise Translation Maps. *arXiv preprint arXiv:1708.03780*, 2017

[4] Volk D., Dubinin I., Myasnikova A, Gutkin B., Nikulin V., Generalized Cross-Frequency Decomposition: A Method for the Extraction of Neuronal Components Coupled at Different Frequencies. *Frontiers in Neuroinformatics*, 12, 2018, doi 10.3389/fninf.2018.00072

[5] Welling M., Herding Dynamic Weights for Partially Observed Random Field Models. In *Proc. of the Conf. on Uncertainty in Artificial Intelligence*, 2009