Summary

Artem V. Ripatti

Our Research Project is related to the following independent directions:

- The MIRUP conjecture
- Constructing Hadamard matrices

The MIRUP conjecture

In the classical formulation, the cutting stock problem (CSP) is stated as follows: there are infinite pieces of stock material of fixed length L. We have to produce $m \in \mathbb{N}$ groups of pieces of different lengths $l_1, \cdots, l_m$ and demanded quantities $b_1, \cdots, b_m$ by cutting initial pieces of stock material in such a way that the number of used initial pieces is minimized.

The gap $\Delta(E)$ of a CSP instance is the difference between its optimal function value and optimal value of its continuous relaxation. For most instances of CSP the gap is less than 1 and the maximal gaps recently (in 2020) discovered are $77/64 = 1.203125$ and $59/48 - \varepsilon \approx 1.229166$. They improve the previous record $6/5 = 1.2$ discovered in 2003. Both results are obtained by our previous research and the latter construction is described in our early draft attached to this application.

The MIRUP conjecture states that $\Delta(E) < 2$ for all CSP instances E, but it is still open.

The aim of our research project is to find an instances with gaps larger than currently known and determine new classes of CSP instances possessing MIRUP. We also plan to expand our constructions onto related problems: the skiving stock problem (SSP) and the proper versions of CSP and SSP where the continuous relaxation is replaced with so-called proper relaxation.

Hadamard matrices

A $\{+1, -1\}$-matrix H of order m is Hadamard if $HH^\top = mI_m$, where I_m is the identity matrix of order m. The Hadamard conjecture asserts that a Hadamard matrix of order $m = 4k$ exists for every positive integer k. There are various methods to construct the Hadamard matrices, but the Hadamard conjecture is still open. The minimal orders for which no Hadamard matrix is currently known are: $668, 716, 892, 1004, 1132, \ldots$. We considered only the following constructions here: Turyn Type sequences and Williamson matrices.

We remark that we have no publications on this theme, since we started the work in this direction in the beginning of this (2020) year. However by now we have promising developments in this field.

We developed and implemented a new algorithm for enumerating Turyn Type sequences. While the previous researchers reported 50 000 CPU hours and 8 200 CPU hours for enumerating all Turyn Type sequences of length $n = 32$, our implementation handles it in 52 CPU hours. Using it we are going to discover new Turyn Type sequences for greater values of n than currently known. It will bring us closer for values $n = 56$ and $n = 60$ which lead to the first Hadamard matrices of orders 668 and 716 respectively.

We also developed and implemented a new algorithm for enumerating Williamson matrices (WM). The previous researchers reported 5 400 CPU hours for enumeration of all WMs of order $n = 59$. Our implementation handles it in 38 hours. Moreover, a GPU version of the algorithm is implemented and it handles $n = 59$ in 36 minutes. We are going to enumerate all WMs up to order $n = 75$.

We also developed a $2^{7n/24+o(n)}$ algorithm for enumerating so-called Turyn Type Williamson Matrices (TTWM) which are WMs of a special kind. But we didn't implement it yet. The previsly known algorithm has complexity $2^{n/2+o(n)}$.

Expanding our ideas to some classes of so-called Williamson Type matrices (WTM) is imaginable too.

Teaching experience

The course "Interactive graphics systems" (USATU, 2016–2018), laboratory works for the course "Algorithms and data structures" (USATU, 2018–2021). ACM ICPC coach (USATU, 2015–2020). Informatics teacher, lead a circle on the olympiad informatics (Lyceum 153, 2019-2021). Camps for school kids on the olympiad informatics.