SUMMARY OF OBTAINED RESULTS AND RESEARCH PLANS

KONSTANTIN LOGINOV

The area of my research is birational geometry. This is a subfield of algebraic geometry which studies algebraic varieties up to birational equivalence. Typical problems in this area are rationality problem, that is, the problem to determine whether a given variety is birational to the projective space, the problem of classification of certain types of varieties, e.g. Fano varieties, construction of good birational models, that is, varieties with certain properties birational to the given ones. I am also interested in degeneration problem, that is, using some geometrical properties of a given variety, to deduce some properties of its degeneration, or vice versa. These are the main problems that I work with.

1. Obtained results

1.1. **Boundedness of irrationality for fibers of del Pezzo fibrations.** We consider the rationality problem for the fibers of Mori fiber spaces and their generalizations, so called *klt Fano fibrations*. We work with del Pezzo fibrations, so that a general fiber is a rational del Pezzo surface. A special fiber is birational to $\mathbb{P}^1 \times C$ where C is a smooth projective curve. In [**BL20**] we prove that if (X, tF) is lc where F is a special fiber and $t > 0$ is a real number, then gonality of C is bounded in terms of t, and if $t > 1/2$ then genus $g(C)$ is bounded in terms of t. We work in a general setting of klt Fano fibrations of dimension 3 over a positive-dimensional base. We prove some generalizations to the case of log Calabi-Yau fibrations of dimension 2 and 3.

1.2. **Semistable degenerations of Fano varieties.** A *semistable family* is a family of projective algebraic varieties over a curve germ with a smooth total space such that the special fiber is reduced and has simple normal crossings. Its special fiber is called *semistable degeneration* of its generic fiber. In [**Lo19-2**] we classify semistable degenerations of del Pezzo surfaces and show that monodromy is trivial in all cases. We describe the dual complex of the special fiber and show that the maximal degeneration is unique in dimension ≤ 3.

1.3. **Standard models of del Pezzo fibrations.** In [**Lo18**] we construct a model with mild singularities for a Mori fiber space whose fibers are del Pezzo surfaces of degree 1. We consider varieties defined over an arbitrary field of characteristic 0 that admit an action of a finite group G. We also construct an embedding of such a model into a relative weighted projective space with weights $(1, 1, 2, 3)$ over a curve C. On each fiber this embedding coincides with a classical embedding of degree 1 del Pezzo surface into a weighted projective space with weights $(1, 1, 2, 3)$.

2. Research plans

2.1. **Maximal log Fano varieties.** We say that log smooth projective pair (X, Δ) is a *log Fano pair* if $-K_X - \Delta$ is ample. We say that a log Fano pair is *maximal* if Δ has $\dim X$ components. Our aim is to show that such pair is toric. We want to prove that X is a so called *generalized Bott towers*. This means that X is an iterated projective bundle over a point. This problem is closely related to study of semistable degenerations of Fano varieties. We plan to prove that maximal semistable degeration is unique in any dimension.

2.2. **Boundedness of irrationality for fibers of del Pezzo fibrations.** We plan to answer the Blanc's question for the genus $g(C)$. To do this, one has to find an explicit estimate on t in Shokurov's conjecture which bounds singularities of pairs (X, tF) in terms of singularities of X.

2.3. **K-(un)stability of singular del Pezzo surfaces.** Our goal is to find destabilising families and thus prove K-unstability of (some polarizations on) singular del Pezzo surfaces.

3. Teaching, Fall 2020

- Introduction to Algebraic Geometry, course at Moscow Institute of Physics and Technology.
- Algebra 1, Higher School of Economics, Department of Mathematics, problem sessions.
- Topology 2, Math in Moscow, lectures and problem sessions.