REPRESENTATIONS OF QUIVERS AND DEFORMED PREPROJECTIVE ALGEBRAS

IVAN LOSEV

Preliminary program

(1) Kac's theorem on indecomposable representations.
(2) Deformed preprojective algebras.
(3) Application: additive Deligne-Simpson problem.
(4)* Application: deformations of Kleinian singularities.

The course will be taught in Russian.

Meeting times and locations: Higher School of Economics, the week of May 12-16.

Monday, May 12, 17:00-≈20:00, 211.
Tuesday, May 13, 12:00-≈15:00, 1001.
Thursday, May 15, 14:00-≈17:00, 209.
Friday, May 16, 17:00-≈20:00, 209.

Description:

A quiver is a directed graph. A representation of a quiver is a collection of vector spaces, one for each vertex, and linear maps between them, one for each arrow. A fundamental problem in this theory is to study representations of quivers up to natural equivalences, given by base changes. This is a generalization of classification problems from Linear Algebra (normal forms of matrices). This problem trivially reduces to describing equivalence classes of indecomposable representations.

A basic result here is a theorem of Kac, [1], that describes the dimension vectors, where indecomposable representations can occur, and also numbers of parameters needed to describe equivalence classes of such representations. The answer is stated in terms of the combinatorics related to Kac-Moody Lie algebras, generalizations of finite dimensional semisimple Lie algebras.

The original proof of Kac was very non-elementary, for example, it used counting of points over finite fields and Weil conjectures proved by Deligne. Crawley-Boevey, [2], made a partially successful attempt to produce a more elementary. His partial proof was based on the study of so called deformed preprojective algebras and their representations.

The deformed preprojective algebras have other nice applications that I'm going to describe. First, Crawley-Boevey in [3] applied them to solving the additive Deligne-Simpson problem. This problem asks when there are matrices from prescribed conjugacy classes summing to 0 and without a common stable subspace. Second, [4], they can be viewed as a tool to produce deformations (both commutative and noncommutative) of Kleinian singularities that can be regarded as an upgraded version of the McKay correspondence.

Target audience: 3rd year undergraduate students. Everybody else is most welcome too, of course.

Prerequisites: Basic algebraic geometry, representation theory, homological algebra, category theory. Familiarity with the structure of semisimple Lie algebras (incl. root systems and Weyl groups), some basic Invariant theory (group actions and quotients) and Hamiltonian mechanics/Symplectic geometry will be useful as well.

References

[1] V. Kac, *Infinite root systems, representations of graphs and invariant theory*, Invent. Math. 56 (1980), 57-92.

[2] W. Crawley-Boevey, *Geometry of the moment map for representations of quivers*, Compositio Math. 126 (2001), 257293.

[3] W. Crawley-Boevey, *On matrices in prescribed conjugacy classes with no common invariant subspace and sum zero*, Duke Math. J. 118 (2003), 339-352.

[4] W. Crawley-Boevey and M.P. Holland, *Noncommutative deformations of Kleinian singularities*, Duke Math. J. 92 (1998), 605-635.