Differential Galois Theory

Higher School of Economics

Spring Term 2014

Classical Galois theory was motivated by the question of whether a given polynomial equation can be solved by radicals. It consists in studying symmetries of solutions of polynomial equations, which form a group, the Galois group. In modern Galois theory one studies field extensions that are Galois rather than polynomial equations itself and field theoretic properties can be translated into group theoretic properties.

Differential Galois theory dates back to the late nineteenth century and studies symmetries of solutions of differential equations. Differential fields, i.e. fields equipped with a derivation, play the role of fields in classical Galois theory. The analogue of a splitting field in the context of Galois theory of linear differential equations is a so-called Picard-Vessiot field. The differential Galois group of a linear differential equation is a linear algebraic group, i.e. an algebraic subgroup of GL_n. The theory of algebraic groups was in fact motivated by the study of differential equations, as group theory was motivated by Galois theory. Given a Picard-Vessiot extension for a linear differential equation, there is a Galois correspondence between differential intermediate fields and algebraic subgroups of the differential Galois group, which allows to answer solvability questions about the given differential equation.

This course treats mainly differential Galois theory of linear differential equations, the so-called Picard-Vessiot theory. If time permits, we also give an alternative approach to Galois theory of linear differential equations using category theory and in particular Tannaka duality, which is due to Deligne, and possibly cover parts of the theory of non-linear differential equations.

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Schedule (preliminary): Tuesday 14:00 - 15:20, room 302, Faculty of Mathematics, HSE (Vavilova 7), first class: January 28th.

Interested persons who cannot attend the course due to time constraints are kindly requested to contact the instructor.

Prerequisite: A background in algebra is desirable. An understanding of algebraic geometry is helpful but not necessary.